

Bounds and Approximations for Optimum Combining of Signals in the Presence of Multiple Cochannel Interferers and Thermal Noise

Marco Chiani, *Senior Member, IEEE*, Moe Z. Win, *Senior Member, IEEE*, Alberto Zanella, *Member, IEEE*, Ranjan K. Mallik, *Senior Member, IEEE*, and Jack H. Winters, *Fellow, IEEE*

Abstract—We derive an upper bound and investigate some approximations on the symbol error probability (SEP) for coherent detection of M -ary phase-shift keying, using an array of antennas with optimum combining in wireless systems in the presence of multiple uncorrelated equal-power cochannel interferers and thermal noise in a Rayleigh fading environment. Our results are general and valid for an arbitrary number of antenna elements as well as an arbitrary number of interferers. In particular, the exact SEP is derived for an arbitrary number of antennas and interferers; the computational complexity of the exact solution depends on the minimum number of antennas and interferers. Moreover, closed-form approximations are provided for the cases of dual optimum combining with an arbitrary number of interferers, and of two interferers with an arbitrary number of antenna elements. We show that our bounds and approximations are close to Monte Carlo simulation results for all cases considered in this paper.

Index Terms—Adaptive arrays, antenna diversity, cochannel interference, eigenvalue distribution, optimum combining, Wishart matrices.

I. INTRODUCTION

ADAPTIVE ARRAYS can significantly improve the performance of wireless communication systems by weighting and combining the received signals to reduce fading effects and suppress interference. In particular, with optimum combining, the received signals are weighted and combined to maximize the output signal-to-interference-plus-noise ratio (SINR). In the presence of interference, this technique provides substantial improvement in performance over maximal ratio combining where the received signals are combined to max-

imize the desired signal-to-noise ratio (SNR) only. However, determining the performance of optimum combining is more difficult than with maximal ratio combining.

In this regard, closed-form expressions for the bit-error probability (BEP) of binary phase-shift keying (BPSK) have been derived for the single-interferer case with Rayleigh fading of the desired signal in [1] and [2], and with Rayleigh fading of the desired signal and interferer in [3]. An exact BEP expression, which requires numerical integration, for BPSK and a single interferer is also given in [4].

With multiple interferers of arbitrary power, Monte Carlo simulation has been used to determine the BEP in [2]. In [5], upper bounds on the BEP of optimum combining were derived given the average powers of the interferers. However, these bounds are generally not tight.

To avoid Monte Carlo simulation, the exact BEP expression was derived in [6] for the case of equal-power interferers, which permits analytical tractability. However, the results are limited to the case of BPSK and no thermal noise. Approximations for the BEP have been presented in [7] and [8] for binary modulation in the presence of thermal noise. However, the approximation of [7] still requires Monte Carlo simulation to derive mean eigenvalues (a table is provided in [7] for some cases), and the approximation of [8] is valid only for the case when the number of interferers is less than the number of antenna elements.

In this paper, starting from the eigenvalues distribution of complex Wishart matrices, we first give the exact expression of the symbol-error probability (SEP) for coherent detection of M -ary phase-shift keying (MPSK) using optimum combining in the presence of multiple uncorrelated equal-power interferers, as well as thermal noise, in a Rayleigh fading environment. Evaluation of this expression involves multiple numerical integrals. Then, based on some new results on the eigenvalues distribution of complex Wishart matrices, we derive new closed-form upper bounds. We show that these bounds are generally tighter than those of [5]. Moreover, we extend the approaches in [7] and obtain new closed-form approximations of the SEP that do not require Monte Carlo simulation and are close to simulation results.

In Section II, we describe the system model, and in Section III, derive the exact SEP of optimum combining with multiple interferers. Upper bounds are derived in Section IV, and approximate formulas are given in Section V. In Section VI, we compare our analytical results with simulations, and in Section VII, we present a summary and conclusions.

Paper approved by P. Y. Kam, the Editor for Modulation and Detection for Wireless Systems of the IEEE Communications Society. Manuscript received October 25, 2001; revised May 27, 2002 and August 17, 2002. The work of M. Chiani and A. Zanella was supported in part by the Ministero dell'Istruzione, dell'Università e della Ricerca Scientifica (MIUR) and in part by Consiglio Nazionale delle Ricerche (CNR), Italy. This paper was presented in part at the IEEE Global Telecommunications Conference, San Antonio, TX, November 2001.

M. Chiani and A. Zanella are with CSITE-CNR, Dipartimento Elettronica Informatica e Sistemistica, University of Bologna, 40136 Bologna, Italy (e-mail: mchiani@deis.unibo.it; azanella@deis.unibo.it).

M. Z. Win was with the Wireless Systems Research Department, AT&T Labs-Research, Middletown, NJ 07748 USA. He is now with LIDS, Massachusetts Institute of Technology, Cambridge, MA 02139 USA (e-mail: moewin@mit.edu).

R. K. Mallik is with the Department of Electrical Engineering, Indian Institute of Technology, Delhi, Hauz Khas, New Delhi 110016 India (e-mail: rk-mallik@ee.iitd.ernet.in).

J. H. Winters is with Jack Winters Communications, LLC, Middletown, NJ 07748-2070 USA (e-mail: jack@JackWinters.com).

Digital Object Identifier 10.1109/TCOMM.2003.809265

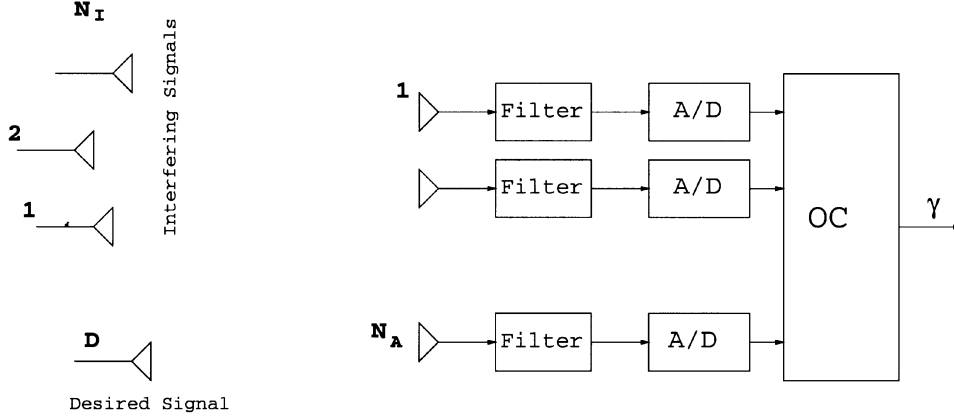


Fig. 1. Baseband model of optimum combining receiver.

II. SYSTEM MODEL

We consider coherent demodulation with optimum combining of multiple received signals in a flat fading environment as in Fig. 1. The fading rate is assumed to be much slower than the symbol rate. Throughout the paper, $(\cdot)^T$ denotes the transposition operator, and $(\cdot)^\dagger$ stands for conjugation and transposition. The received signal at the N_A -element array output consists of the desired signal, N_I interfering signals, and thermal noise. After matched filtering and sampling at the symbol rate, the array output vector at time k can be written as

$$\mathbf{z}(k) = \sqrt{E_D} \mathbf{c}_D b_0 n(k) + \sum_{j=1}^{N_I} \sqrt{E_{I,j}} \mathbf{c}_{I,j} b_j(k) + \mathbf{n}(k), \quad (1)$$

where E_D and $E_{I,j}$ are the mean (over fading) energies of the desired signal and j th interferer, respectively; $\mathbf{c}_D = [\mathbf{c}_{D,1}, \dots, \mathbf{c}_{D,N_A}]^T$ and $\mathbf{c}_{I,j} = [\mathbf{c}_{I,j,1}, \dots, \mathbf{c}_{I,j,N_A}]^T$ are the desired and j th interference propagation vectors, respectively; $b_0(k)$ and $b_j(k)$ (both with unit variance) are the desired and interfering data samples, respectively; and $\mathbf{n}(k)$ represents the additive noise. We model \mathbf{c}_D and $\mathbf{c}_{I,j}$ as multivariate complex-valued Gaussian vectors having $\mathbb{E}\{\mathbf{c}_D\} = \mathbb{E}\{\mathbf{c}_{I,j}\} = \mathbf{0}$ and $\mathbb{E}\{\mathbf{c}_D \mathbf{c}_D^\dagger\} = \mathbb{E}\{\mathbf{c}_{I,j} \mathbf{c}_{I,j}^\dagger\} = \mathbf{I}$, where \mathbf{I} is the identity matrix. The additive noise is modeled as a white Gaussian random vector with independent and identically distributed (i.i.d.) elements with $\mathbb{E}\{\mathbf{n}(k)\} = \mathbf{0}$ and $\mathbb{E}\{\mathbf{n}(k) \mathbf{n}^\dagger(k)\} = N_0 \mathbf{I}$, where $N_0/2$ is the two-sided thermal noise power spectral density per antenna element.

The SINR at the output of the N_A -element array with optimum combining can be expressed [1], [2] as

$$\gamma = E_D \mathbf{c}_D^\dagger \mathbf{R}^{-1} \mathbf{c}_D \quad (2)$$

where the short-term covariance matrix \mathbf{R} , conditioned to all interference propagation vectors, is

$$\mathbf{R} = \mathbb{E}_{\mathbf{n}, b_j(k)} \left\{ \left[\sum_{j=1}^{N_I} \sqrt{E_{I,j}} \mathbf{c}_{I,j} b_j(k) + \mathbf{n}(k) \right] \left[\sum_{j=1}^{N_I} \sqrt{E_{I,j}} \mathbf{c}_{I,j} b_j(k) + \mathbf{n}(k) \right]^\dagger \right\} \quad (3)$$

and $\mathbb{E}_X\{\cdot\}$ denotes expectation with respect to X . Therefore

$$\mathbf{R} = \sum_{j=1}^{N_I} E_{I,j} \mathbf{c}_{I,j} \mathbf{c}_{I,j}^\dagger + N_0 \mathbf{I}. \quad (4)$$

It is important to remark that \mathbf{R} and, consequently, also the SINR γ vary at the fading rate.

The matrix \mathbf{R}^{-1} can be written as $\mathbf{U} \mathbf{\Lambda}^{-1} \mathbf{U}^\dagger$ where \mathbf{U} is a unitary matrix and $\mathbf{\Lambda}$ is a diagonal matrix whose elements on the principal diagonal are the eigenvalues of \mathbf{R} , denoted by $(\lambda_1, \dots, \lambda_{N_A})$. The vector $\mathbf{u} = \mathbf{U}^\dagger \mathbf{c}_D = [u_1, \dots, u_{N_A}]^T$ has the same distribution as \mathbf{c}_D , since \mathbf{U} represents a unitary transformation. The SINR given in (2) can be rewritten as

$$\gamma = E_D \mathbf{c}_D^\dagger \mathbf{U} \mathbf{\Lambda}^{-1} \mathbf{U}^\dagger \mathbf{c}_D = E_D \sum_{i=1}^{N_A} \frac{|u_i|^2}{\lambda_i}. \quad (5)$$

Since \mathbf{R} is a random matrix, its eigenvalues are random variables.

We now investigate the statistical properties of $(\lambda_1, \dots, \lambda_{N_A})$. We will show later that this is related to problems arising in multivariate statistics, regarding the eigenvalue distribution of complex Wishart matrices. Let

$$\mathbf{C}_I \triangleq \begin{bmatrix} | & | & & | \\ \mathbf{c}_{I,1} & \mathbf{c}_{I,2} & \dots & \mathbf{c}_{I,N_I} \\ | & | & & | \end{bmatrix} \quad (6)$$

be a $(N_A \times N_I)$ random matrix composed of N_I interference propagation vectors as columns. For equal-power interferers, i.e., $E_{I,j} = E_I$ for $j = 1, \dots, N_I$, (4) can be rewritten as

$$\mathbf{R} = E_I \tilde{\mathbf{R}} + N_0 \mathbf{I} \quad (7)$$

where $\tilde{\mathbf{R}} = \mathbf{C}_I \mathbf{C}_I^\dagger$ is a $(N_A \times N_A)$ random matrix. The eigenvalues of \mathbf{R} can be written in terms of eigenvalues of $\tilde{\mathbf{R}}$, denoted by $(\tilde{\lambda}_1, \tilde{\lambda}_2, \dots, \tilde{\lambda}_{N_A})$, as

$$\lambda_i = E_I \tilde{\lambda}_i + N_0, \quad i = 1, \dots, N_A \quad (8)$$

where the joint probability density function (pdf) of the N_A eigenvalues of $\tilde{\mathbf{R}}$ are given by the following theorem.

Theorem 1: The joint pdf of the first $N_{\min} \triangleq \min\{N_A, N_I\}$ ordered eigenvalues $\tilde{\lambda} = [\tilde{\lambda}_1, \tilde{\lambda}_2, \dots, \tilde{\lambda}_{N_{\min}}]^T$ of $\tilde{\mathbf{R}}$, with $\tilde{\lambda}_1 \geq \tilde{\lambda}_2 \geq \dots \geq \tilde{\lambda}_{N_{\min}}$, is

$$f_{\tilde{\lambda}}(\tilde{\lambda}_1, \tilde{\lambda}_2, \dots, \tilde{\lambda}_{N_{\min}}) = K \prod_{i=1}^{N_{\min}} e^{-\tilde{\lambda}_i} \tilde{\lambda}_i^{N_{\max} - N_{\min}} \times \prod_{i=1}^{N_{\min}-1} \left[\prod_{j=i+1}^{N_{\min}} (\tilde{\lambda}_i - \tilde{\lambda}_j)^2 \right] \quad (9)$$

where $N_{\max} \triangleq \max\{N_A, N_I\}$ and K is a normalizing constant given by

$$K = \frac{\pi^{N_{\min}(N_{\min}-1)}}{\tilde{\Gamma}_{N_{\min}}(N_{\max}) \tilde{\Gamma}_{N_{\min}}(N_{\min})} \quad (10)$$

with

$$\tilde{\Gamma}_{N_{\min}}(n) = \pi^{N_{\min}(N_{\min}-1)/2} \prod_{i=1}^{N_{\min}} (n - i)!. \quad (11)$$

The additional $N_A - N_{\min}$ eigenvalues of $\tilde{\mathbf{R}}$ are identically equal to zero.

Proof: See Appendix B. ■

As a consequence of *Theorem 1*, we have the following corollary.

Corollary 1 (Reciprocity Principle): The statistical distributions of the eigenvalues of $\tilde{\mathbf{R}}$, for the case of m antennas and p interferers with $m \leq p$, are equal to that of the (nonzero) eigenvalues of $\tilde{\mathbf{R}}$ for the case of p antennas and m interferers.¹

Using the distribution theory for transformations of random vectors [9] together with (8), the joint pdf of $\lambda = [\lambda_1, \dots, \lambda_{N_{\min}}]^T$ with $\lambda_1 \geq \dots \geq \lambda_{N_{\min}} \geq N_0$ is

$$f_{\lambda}(\lambda_1, \dots, \lambda_{N_{\min}}) = \frac{1}{E_I^{N_{\min}}} \times f_{\tilde{\lambda}}\left(\frac{\lambda_1 - N_0}{E_I}, \frac{\lambda_2 - N_0}{E_I}, \dots, \frac{\lambda_{N_{\min}} - N_0}{E_I}\right) \quad (12)$$

where $f_{\tilde{\lambda}}(\cdot)$ is given by *Theorem 1*. The additional $N_A - N_{\min}$ eigenvalues of \mathbf{R} are identically equal to N_0 .

III. EVALUATION OF THE EXACT SEP

The SEP for optimum combining in the presence of multiple cochannel interferers and thermal noise in a fading environment is obtained by averaging the conditional SEP over the (desired and interfering signal) channel ensemble. This can be accomplished by

$$P_e = \mathbb{E}_{\gamma} \{ \Pr \{ e \mid \gamma \} \} = \int_0^{\infty} \Pr \{ e \mid \gamma = x \} f_{\gamma}(x) dx \quad (13)$$

where $\Pr \{ e \mid \gamma \}$ is the SEP conditioned on the random variable γ , and $f_{\gamma}(\cdot)$ is the pdf of the combiner output SINR. Note that γ depends on the desired and interference propagation vectors. Although the evaluation of (13) involves a single integration for averaging over the channel ensemble, it requires the knowledge

¹This proves the equality, observed also numerically by Monte Carlo simulation in [7, Table I], of the expectations of the nonzero eigenvalues of $\tilde{\mathbf{R}}$ when the number of antennas is exchanged with the number of interferers.

of the pdf of γ , which can be quite difficult to obtain. This is alleviated by using the chain rule of conditional expectation as

$$P_e = \mathbb{E}_{\lambda} \left\{ \underbrace{\mathbb{E}_{\mathbf{u}} \left\{ \Pr \left\{ e \mid \gamma = E_D \sum_{i=1}^{N_A} \frac{|u_i|^2}{\lambda_i} \right\} \right\}}_{P_{e|\lambda}} \right\} \quad (14)$$

where we first perform $\mathbb{E}_{\mathbf{u}} \{ \cdot \}$ (i.e., average over the channel ensemble of the desired signal) to obtain the conditional SEP, conditioned on the random vector λ , denoted by $P_{e|\lambda}$. We then perform $\mathbb{E}_{\lambda} \{ \cdot \}$ to average out the channel ensemble of the interfering signals.

The j th interfering data samples, $b_j(k)$ $j = 1, \dots, N_I$, can be modeled as zero-mean, unitary variance Gaussian random variables. Note that the Gaussian assumption gives a good approximation when the interfering contribution is due to a large number of interferers sampled at a random time, and generally it represents a worst case [10]; here, it will be used regardless of the number of interferers. In the following, we assume that $b_0(k)$ is an MPSK data sample. With the previous assumption together with the Gaussianity of $\mathbf{n}(k)$, $\Pr \{ e \mid \gamma \}$ for coherent detection of MPSK is given by [11], [12]

$$\Pr \{ e \mid \gamma \} = \frac{1}{\pi} \int_0^{\Theta} \exp \left(-\frac{c_{\text{MPSK}}}{\sin^2 \theta} \gamma \right) d\theta \quad (15)$$

where $c_{\text{MPSK}} = \sin^2(\pi/M)$ and $\Theta = \pi(M-1)/M$. Using (15), $P_{e|\lambda}$ can be written as

$$P_{e|\lambda} = \frac{1}{\pi} \int_0^{\Theta} \mathbb{E}_{\mathbf{u}} \left\{ \exp \left(-\frac{c_{\text{MPSK}}}{\sin^2 \theta} E_D \sum_{i=1}^{N_A} \frac{|u_i|^2}{\lambda_i} \right) \right\} d\theta = \frac{1}{\pi} \int_0^{\Theta} \psi_{\gamma|\lambda} \left(-\frac{c_{\text{MPSK}}}{\sin^2 \theta} \right) d\theta \quad (16)$$

where $\psi_{\gamma|\lambda}(\cdot)$ is the characteristic function (cf) of γ , conditioned on λ , given by

$$\psi_{\gamma|\lambda}(j\nu) = \frac{1}{\prod_{i=1}^{N_A} \left(1 - \frac{j\nu E_D}{\lambda_i} \right)} \quad (17)$$

and we have used the fact that \mathbf{u} is Gaussian with i.i.d. elements. Therefore, the conditional SEP, conditioned on λ , in the general case of N_A antennas and N_I interferers, becomes

$$P_{e|\lambda} = \frac{1}{\pi} \int_0^{\Theta} A(\theta) \prod_{i=1}^{N_{\min}} \left[\frac{\sin^2 \theta}{\sin^2 \theta + c_{\text{MPSK}} \frac{E_D}{\lambda_i}} \right] d\theta \quad (18)$$

where

$$A(\theta) \triangleq \left[\frac{\sin^2 \theta}{\sin^2 \theta + c_{\text{MPSK}} \frac{E_D}{N_0}} \right]^{N_A - N_{\min}}. \quad (19)$$

Using (9), (12), (14), and (18), the unconditional SEP for optimum combining becomes

$$P_e = \mathbb{E}_{\lambda} \left\{ P_{e|\lambda} \right\} = \int_{N_0}^{\infty} \dots \int_{\lambda_3}^{\infty} \int_{\lambda_2}^{\infty} P_{e|\lambda} \cdot f_{\lambda}(\lambda) d\lambda_1 d\lambda_2 \dots d\lambda_{N_{\min}}. \quad (20)$$

Equation (20) is exact and valid for arbitrary numbers of antennas and interferers; however, it requires the evaluation of

nested N_{\min} -fold integrals, which can be cumbersome to evaluate for large N_{\min} . To give an idea of the amount of time needed for $N_{\min} = 2$ (which allows us to investigate either dual combining with an arbitrary number of interferers or an arbitrary number of antennas with two interferers), the computation of (20) on a 450-MHz PC requires about 100 s.

Since the computation time for the numerical evaluation of (20) increases with the number of antennas and interferers, rigorous bounds, as in [5], or approximate expressions, as in [7], are useful; unfortunately, the bounds in [5] are generally not very tight, and the approximation in [7] requires Monte Carlo simulation. This motivates the need to derive simpler and tighter bounds or approximate expressions in closed form.

IV. UPPER BOUNDS ON SEP

In this section, we derive a new upper bound for the SEP based on the knowledge of the pdf of the trace of the covariance matrix \mathbf{R} .

Theorem 2: The SEP is upper bounded by

$$P_e \leq \frac{1}{\pi} \int_0^\Theta A(\theta) \mathbb{E}_Y \{B(Y, \theta)\} d\theta \quad (21)$$

where $A(\theta)$ is defined in (19), and Y is a chi-square distributed random variable with $2N_A N_I$ degrees of freedom (DOFs), having pdf given by

$$f_Y(y) = \begin{cases} \frac{1}{\Gamma(N_A N_I)} y^{(N_A N_I)-1} e^{-y}, & \text{if } y \geq 0 \\ 0, & \text{otherwise.} \end{cases} \quad (22)$$

In (22), $\Gamma(x)$ is the gamma function [13, eq.(8.310), p. 942], and

$$B(y, \theta) = \left[\frac{\left(y + \frac{N_{\min} N_0}{E_I}\right) \sin^2 \theta}{\left(y + \frac{N_{\min} N_0}{E_I}\right) \sin^2 \theta + \frac{c_{\text{MPSK}} N_{\min} E_D}{E_I}} \right]^{N_{\min}}. \quad (23)$$

For a single-interferer scenario, (21) is an equality, i.e., it gives the exact SEP for $N_I = 1$.

Proof: By applying the result in Appendix C to (18), we have

$$\begin{aligned} P_{e|\boldsymbol{\lambda}} &= \frac{1}{\pi} \int_0^\Theta A(\theta) \prod_{i=1}^{N_{\min}} \left[\frac{\sin^2 \theta}{\sin^2 \theta + \frac{c_{\text{MPSK}} E_D}{\lambda_i}} \right] d\theta \\ &\leq \frac{1}{\pi} \int_0^\Theta A(\theta) \left[\frac{\sin^2 \theta}{\sin^2 \theta + \frac{c_{\text{MPSK}} E_D N_{\min}}{\sum_{i=1}^{N_{\min}} \lambda_i}} \right]^{N_{\min}} d\theta \end{aligned} \quad (24)$$

where the equality is verified for $N_{\min} = 1$, therefore

$$P_e \leq \frac{1}{\pi} \int_0^\Theta A(\theta) \mathbb{E}_{\boldsymbol{\lambda}} \left\{ \left[\frac{\sin^2 \theta}{\sin^2 \theta + \frac{c_{\text{MPSK}} E_D N_{\min}}{\sum_{i=1}^{N_{\min}} \lambda_i}} \right]^{N_{\min}} \right\} d\theta. \quad (25)$$

Note, from (8), that

$$\begin{aligned} \sum_{i=1}^{N_{\min}} \lambda_i &= E_I \sum_{i=1}^{N_{\min}} \tilde{\lambda}_i + N_{\min} N_0 \\ &= E_I \sum_{i=1}^{N_A} \tilde{\lambda}_i + N_{\min} N_0 \end{aligned} \quad (26)$$

where we have used the fact that $N_A - N_{\min}$ eigenvalues of $\tilde{\mathbf{R}}$ are identically equal to zero by *Theorem 1*, and hence

$$\sum_{i=1}^{N_{\min}} \lambda_i = E_I \text{tr} [\tilde{\mathbf{R}}] + N_{\min} N_0. \quad (27)$$

In order to evaluate the expectation in (25), we observe that $\text{tr} [\tilde{\mathbf{R}}] = \text{tr} [\mathbf{C}_I \mathbf{C}_I^\dagger] = \sum_{i,j} |c_{I,j,i}|^2$. Hence, the random variable $Y \triangleq \text{tr} [\tilde{\mathbf{R}}]$ is chi-square distributed with $2N_A N_I$ DOFs, with pdf given by (22). This completes the proof of the theorem. \blacksquare

The expectation $\mathbb{E}_Y \{B(Y, \theta)\}$ is evaluated in Appendix D as shown in (28) at the bottom of the next page, where $E_1(x)$ is the exponential integral defined by (57) in Appendix D.

The bound (21) allows the evaluation of SEP for coherent detection of MPSK modulation with optimum combining; the numerical evaluation of it only requires a fraction of a second on a PC. Note that the inequality in (24) becomes equality for the case of single interferer (as well as for single antenna), and our bound gives the exact results.

V. APPROXIMATIONS ON THE SEP

In this section, some new results on the SEP approximations will be presented. Here, we start from the approximation proposed in [7], and we derive a methodology which allows us to eliminate the need for Monte Carlo simulation in the cases of dual optimum combining with an arbitrary number of interferers, and of two interferers with an arbitrary number of antenna elements. We prove that the approximation proposed in [8] is an upper bound of [7]; furthermore, we generalize the result of [8], and the generalized results are now applicable for the case $N_I \geq N_A$ in addition to $N_I < N_A$.

A. Approximation via Expected Eigenvalues

In [7], it is proposed to approximate the unconditional cf of γ as $\mathbb{E}_{\boldsymbol{\lambda}} \{\psi_{\gamma|\boldsymbol{\lambda}}(j\nu)\} \approx \psi_{\gamma|\mathbb{E}_{\boldsymbol{\lambda}}\{\boldsymbol{\lambda}\}}(j\nu)$.

By adopting this approximation in (20), the SEP for MPSK is approximated as follows:

$$P_e \approx F(\boldsymbol{\beta}_A) \quad (29)$$

where $F(\boldsymbol{\beta})$ is given by

$$F(\boldsymbol{\beta}) = \frac{1}{\pi} \int_0^\Theta A(\theta) \prod_{i=1}^{N_{\min}} \left[\frac{\sin^2 \theta}{\sin^2 \theta + c_{\text{MPSK}} \frac{E_D}{\beta_i}} \right] d\theta \quad (30)$$

and the i th element of $\boldsymbol{\beta}_A$ is

$$\beta_{A,i} = E_I \mathbb{E} \{\tilde{\lambda}_i\} + N_0, \quad i = 1, \dots, N_{\min}. \quad (31)$$

Discussion

Since \mathbf{R} is semidefinite positive, the eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_{N_A}$ are real and nonnegative. Therefore, for each θ , it is easy to verify that the function

$$g(\lambda_1, \lambda_2, \dots, \lambda_{N_{\min}}; \theta) \triangleq A(\theta) \prod_{i=1}^{N_{\min}} \left[\frac{\sin^2 \theta}{\sin^2 \theta + c_{\text{MPSK}} \frac{E_D}{\lambda_i}} \right] \quad (32)$$

is \cap -concave in *each* λ_i when the other variables are fixed, but, despite this, the function is neither globally convex nor concave.

Approximation (29) is obtained by replacing the expected value of $g(\cdot)$ with the function evaluated at the expected values of the λ_i 's, i.e.,

$$\mathbb{E} \{g(\lambda_1, \lambda_2, \dots, \lambda_{N_{\min}}; \theta)\} \approx g(\mathbb{E} \{\lambda_1\}, \mathbb{E} \{\lambda_2\}, \dots, \mathbb{E} \{\lambda_{N_{\min}}\}; \theta). \quad (33)$$

Now, if the function $g(\cdot)$ were concave (convex), applying Jensen's inequality will produce an upper (lower) bound, but, since (32) is neither concave nor convex, Jensen's inequality [14] cannot be applied. However, (29) gives good agreement with the exact SEP expression (20) for typical parameters of interest. This may be due to the fact that, in the region where the pdf of the eigenvalues λ is not negligible, (32) behaves essentially as an affine function.

Integrating both sides of (33) over θ and scaling by $1/\pi$, we obtain

$$\begin{aligned} P_e &= \mathbb{E} \{P_{e|\lambda}\} \\ &\approx \frac{1}{\pi} \int_0^\Theta g(\mathbb{E} \{\lambda_1\}, \mathbb{E} \{\lambda_2\}, \dots, \mathbb{E} \{\lambda_{N_{\min}}\}; \theta) d\theta \\ &= \frac{1}{\pi} \int_0^\Theta A(\theta) \prod_{i=1}^{N_{\min}} \left[\frac{\sin^2 \theta}{\sin^2 \theta + c_{\text{MPSK}} \frac{E_D}{\beta_{A,i}}} \right] d\theta \\ &= F(\beta_A). \end{aligned} \quad (34)$$

Note that, given the expectation of the eigenvalues $\mathbb{E} \{\tilde{\lambda}_i\}$, the last integral can be also derived in closed form by using a

canonical decomposition method [15], [16]. In the following, (34) will be denoted as approximation A, and we will show in Section VI that it is in good agreement with the exact analysis of (20) as well as simulation results. In general, approximation A requires knowledge of $\mathbb{E} \{\tilde{\lambda}_i\}$. In [7], the expectation of the eigenvalues for some specific cases were calculated via Monte Carlo simulation. For the case of dual optimum combining ($N_A = 2$) with arbitrary N_I , or the case of two interferers ($N_I = 2$) with an arbitrary number of antenna elements, $\mathbb{E} \{\tilde{\lambda}_i\}$ is obtained easily in a closed form using the reciprocity principle given in *Corollary 1*, together with the results of Appendix E.

B. Approximation via Equal Expected Eigenvalues

The determination of $\mathbb{E} \{\tilde{\lambda}_i\}$, in general, requires the evaluation of multiple integrals for each of the $(N_{\min} - 1)$ eigenvalues. This can be alleviated, at the expense of tightness, by the following bound.

Theorem 3: $F(\beta_A)$ is upper bounded as follows:

$$F(\beta_A) \leq F(\beta_B) \quad (35)$$

where $F(\beta)$ is given in (30) and the i th element of β_B is

$$\beta_{B,i} = E_I N_{\max} + N_0, \quad i = 1, \dots, N_{\min}. \quad (36)$$

Proof: The integrand of (34) can be written as

$$g(y_1, \dots, y_{N_{\min}}; \theta) = A(\theta) \prod_{i=1}^{N_{\min}} \left[\frac{\sin^2 \theta}{\sin^2 \theta + \frac{c_{\text{MPSK}} E_D}{y_i}} \right] \quad (37)$$

where $y_i = \mathbb{E} \{\lambda_i\}$ for $i = 1, \dots, N_{\min}$. By using (45) of Appendix C, with $n = N_{\min}$, we get

$$g(y_1, \dots, y_{N_{\min}}; \theta) \leq A(\theta) \left[\frac{\sin^2 \theta}{\sin^2 \theta + \frac{c_{\text{MPSK}} E_D N_{\min}}{\sum_{i=1}^{N_{\min}} y_i}} \right]^{N_{\min}}. \quad (38)$$

$$\begin{aligned} \mathbb{E}_Y \{B(Y, \theta)\} &= \frac{1}{\Gamma(N_A N_I)} \times \left\{ \sum_{n=0}^{N_{\min}} \sum_{m=n}^{N_A N_I - 1} \binom{N_{\min}}{n} \binom{N_A N_I - 1}{m} (-1)^{N_A N_I - 1 - (m-n)} \left(\frac{c_{\text{MPSK}} N_{\min} E_D}{E_I \sin^2 \theta} \right)^n (m-n)! \right. \\ &\quad \times \sum_{l=0}^{m-n} \frac{1}{l!} \left(\frac{c_{\text{MPSK}} N_{\min} E_D}{E_I \sin^2 \theta} + \frac{N_{\min} N_0}{E_I} \right)^{N_A N_I - 1 - m + l} \\ &\quad + \sum_{n=1}^{N_{\min}} \sum_{m=0}^{n-1} \binom{N_{\min}}{n} \binom{N_A N_I - 1}{m} \frac{(-1)^{N_A N_I}}{(n-m-1)!} \left(\frac{c_{\text{MPSK}} N_{\min} E_D}{E_I \sin^2 \theta} \right)^n \\ &\quad \times \left\{ \sum_{l=1}^{n-m-1} (-1)^l (l-1)! \left(\frac{c_{\text{MPSK}} N_{\min} E_D}{E_I \sin^2 \theta} + \frac{N_{\min} N_0}{E_I} \right)^{N_A N_I - 1 - m - l} \right. \\ &\quad \left. + \left[\left(\frac{c_{\text{MPSK}} N_{\min} E_D}{E_I \sin^2 \theta} + \frac{N_{\min} N_0}{E_I} \right)^{N_A N_I - 1 - m} \right. \right. \\ &\quad \left. \left. \times e^{+(c_{\text{MPSK}} N_{\min} E_D / E_I \sin^2 \theta + N_{\min} N_0 / E_I)} \right. \right. \\ &\quad \left. \left. \times E_1 \left(\frac{c_{\text{MPSK}} N_{\min} E_D}{E_I \sin^2 \theta} + \frac{N_{\min} N_0}{E_I} \right) \right] \right\} \end{aligned} \quad (28)$$

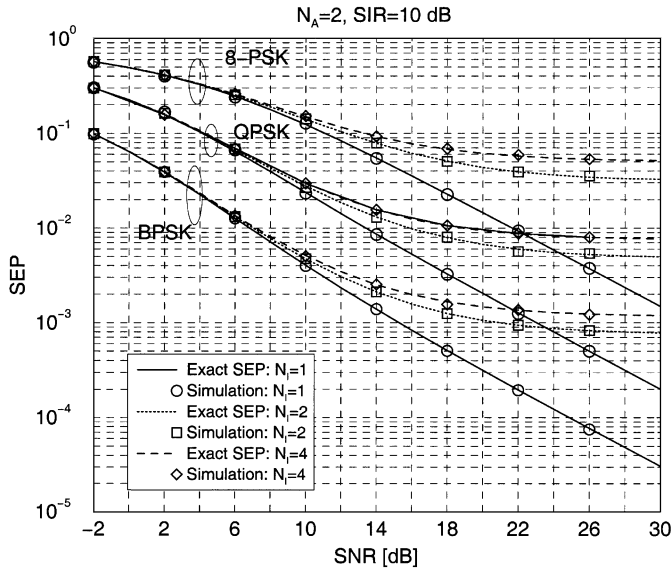


Fig. 2. SEP for coherent detection of BPSK, quaternary PSK, and 8-PSK using dual optimum combining ($N_A = 2$) for $N_I = 1, 2,$ and 4 and $SIR = 10$ dB. Excellent agreement between exact analysis and simulation can be observed.

Using (8)

$$\sum_{i=1}^{N_{\min}} y_i = E_I \sum_{i=1}^{N_{\min}} \mathbb{E} \{ \tilde{\lambda}_i \} + N_{\min} N_0 \quad (39)$$

$$= E_I N_{\min} N_{\max} + N_{\min} N_0 \quad (40)$$

where we have used (66) from Appendix E in deriving (40). Therefore

$$g(y_1, \dots, y_{N_{\min}}; \theta) \leq A(\theta) \left(\frac{\sin^2 \theta}{\sin^2 \theta + \frac{C_{\text{MPSK}} E_D}{E_I N_{\max} + N_0}} \right)^{N_{\min}} \quad (41)$$

Finally, by using (30), (36), and (41), it is straightforward to show that (34) is upper bounded by $F(\beta_B)$. ■

The above theorem provides a rigorous proof that the approximate solution for $N_A > N_I$ proposed in [8], based on heuristic assumptions, represents an upper bound of the solution proposed in [7]. It also provides the generalization of the approximation of [8], which is now valid for arbitrary numbers of antennas and interferers. In the following, we will denote (30) together with (36) as the approximation B. Note that approximation B does not require knowledge of $\mathbb{E} \{ \tilde{\lambda}_i \}$.

VI. NUMERICAL RESULTS

In this section, we evaluate the exact SEP [given by (20)], the upper bound [given by (21) together with (28)], the approximation A [given by (30) together with (31)] and the approximation B [given by (30) together with (36)] derived in previous sections, and compare them with Monte Carlo simulation results. The simulations were performed over 10 000 trials. We investigate the effect of SNR defined as E_D/N_0 , signal-to-interference ratio (SIR) defined as $E_D/(N_I \cdot E_I)$, the number of interferers, and the number of antenna branches on the SEP. Unless otherwise stated, we consider the coherent detection of 8-PSK with optimum combining.

We first consider coherent detection of BPSK, quaternary PSK and 8-PSK using dual optimum combining ($N_A = 2$).

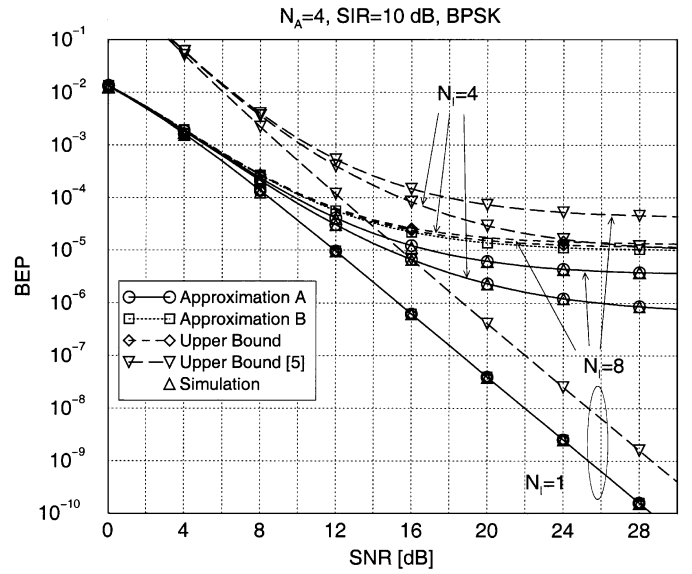


Fig. 3. Comparison between upper bound derived in Section IV with the only previously known upper bound given by [5, eq. (13)] for the case of BPSK, $SIR = 10$ dB, $N_A = 4$, $N_I = 1, 4,$ and 8 . Note that our upper bound is 4.8 and 5.3 dB (at BEP of 10^{-3}) tighter and 4.8 and 7.4 dB (at BEP of 10^{-4}) tighter than [5, eq. (13)] for $N_I = 4$ and 8 , respectively.

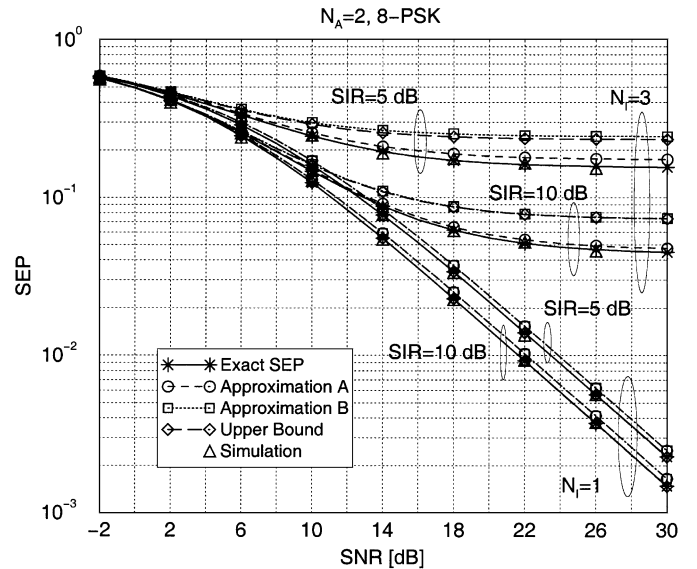


Fig. 4. SEP as a function of SNR for coherent detection of 8-PSK using dual optimum combining ($N_A = 2$) for the case of $N_I = 1$ and 3 with $SIR = 5$ and 10 dB.

Fig. 2 shows the SEP as a function of SNR, for $N_I = 1, 2,$ and 4 , and $SIR = 10$ dB. The results show excellent agreement between exact analysis and simulation. The curves also exhibit an error floor when the number of interferers N_I is greater than the array DOFs, i.e., $N_A - 1$. Next, we compare in Fig. 3 the upper bound derived in Section IV with the only previously known upper bound given by [5, eq. (13)]. Note that our upper bound is 4.8 and 5.3 dB (at BEP of 10^{-3}) tighter and 4.8 and 7.4 dB (at BEP of 10^{-4}) tighter than [5, eq. (13)] for $N_I = 4$ and 8 , respectively.

Fig. 4 shows the SEP with dual optimum combining for the case of $N_I = 1$ and 3 with $SIR = 5$ and 10 dB. Note that there is the error floor for the case of $N_I = 3$ which decreases as

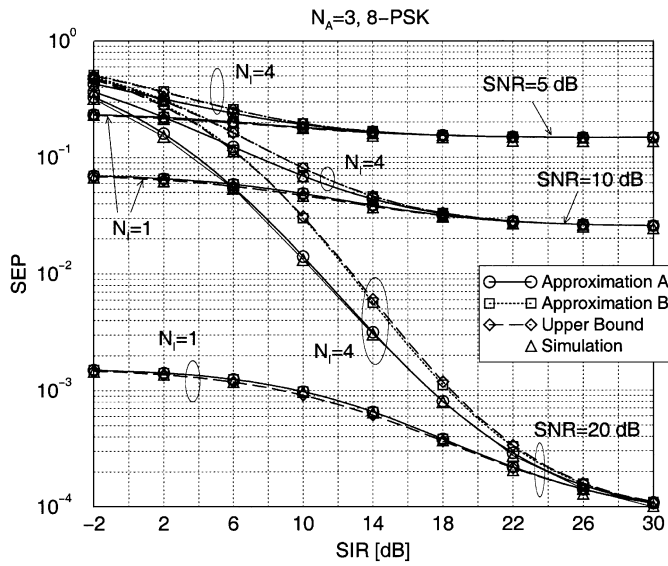


Fig. 5. SEP as a function of SIR for 8-PSK, $N_A = 3$, $N_I = 1$ and 4, SNR = 5, 10, and 20 dB.

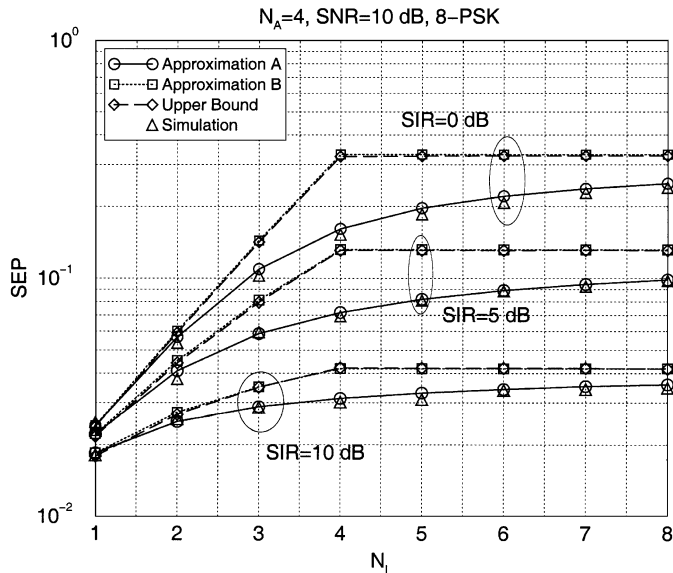


Fig. 6. SEP versus the number of interferers N_I for case of 8-PSK, $N_A = 4$, SNR = 10 dB, SIR = 0, 5, and 10 dB.

SIR increases. In order to further investigate the dependence of SEP on SIR, the SEP is plotted as a function of SIR in Fig. 5 for the case of $N_A = 3$, with $N_I = 1$ and 4, and SNR = 5, 10, and 20 dB. Note that when the SIR is comparable with the SNR, the number of interferers plays a marginal role. Finally, the asymptotic SEP is limited by the thermal noise.

The SEP versus the number of interferers is plotted in Fig. 6 for $N_A = 4$, SNR = 10 dB, and three different values of SIR (0, 5, and 10 dB). It can be seen that, when the array is overloaded, the performance does not depend significantly on the number of interferers; this behavior is accentuated for small values of SIR. The SEP versus the number of antenna branches is plotted in Fig. 7 for SNR = 10 dB, SIR = 5 and 10 dB, and $N_I = 3$. The figure shows that the system is able to exploit the spatial

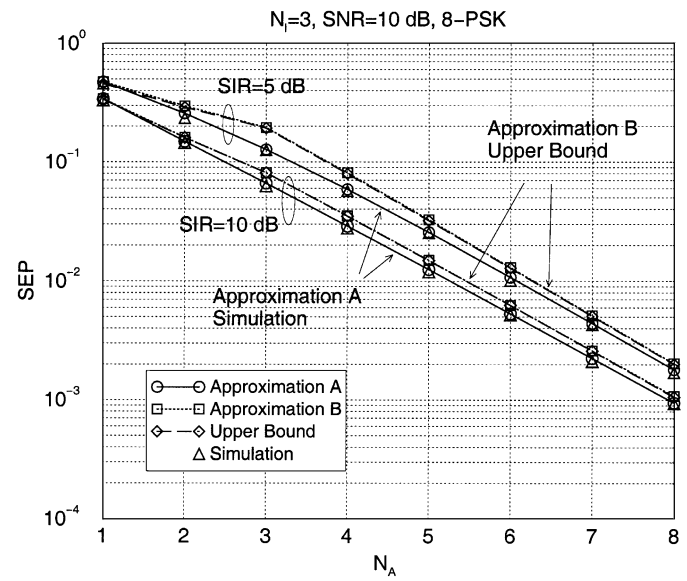


Fig. 7. SEP versus the number of antenna branches N_A for 8-PSK, $N_I = 3$, SNR = 10 dB, SIR = 5 and 10 dB.

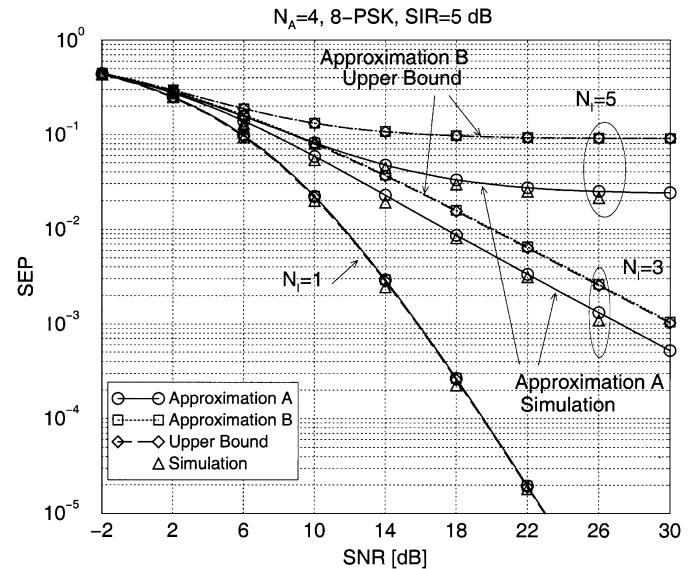


Fig. 8. SEP versus SNR for 8-PSK, $N_A = 4$, $N_I = 1, 3$, and 5; and SIR = 5 dB.

diversity provided by the increasing number of antennas (the SEP in logarithmic scale is approximately linear in N_A). Note that our upper bound is quite close to the simulation results.

Fig. 8 shows the SEP as a function of SNR for $N_A = 4$, SIR = 5 dB, and $N_I = 1, 3$, and 5. As expected, we note the presence of error floor in the overloaded case ($5 = N_I > N_A - 1 = 3$). Moreover, when $N_I < N_A$, the remaining DOFs (diversity order) is $L_{Div} = N_A - N_I$ and we expect an asymptotic behavior for SEP proportional to $1/(SNR)^{L_{Div}}$. This implies that the curve of the SEP versus SNR approaches, for large SNR, a straight line on a semilogarithmic scale with slope $-(N_A - N_I)/10$ decade/dB. Indeed, slopes of 3/10 decade/dB for $N_I = 1$, and 1/10 decade/dB for $N_I = 3$ can be observed from Fig. 8. Similar results are shown in Fig. 9 for $N_A = 4$,

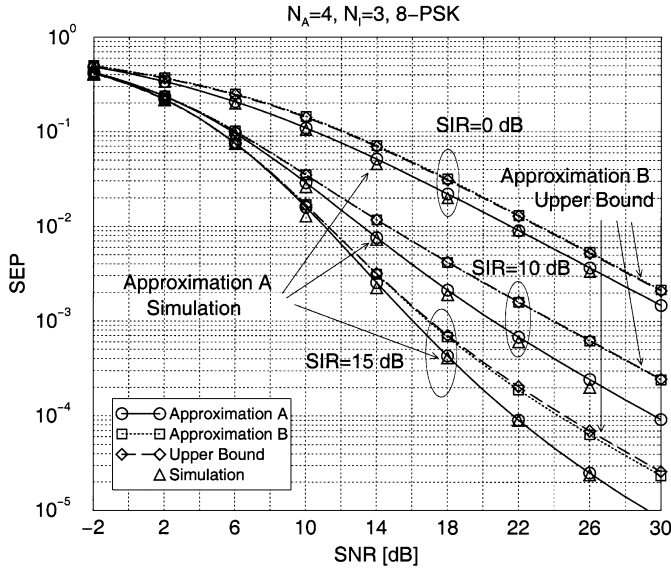


Fig. 9. SEP versus SNR for 8-PSK, $N_A = 4$, $N_I = 3$, SIR = 0, 10, and 15 dB.

$N_I = 3$, and SIR = 0, 10, and 15 dB, and the asymptotic behavior of SEP for large SNR can be seen for all values of SIR.

VII. CONCLUSION

In this paper, we derived the exact SEP for optimum combining of signals in the presence of multiple equal-power interferers and thermal noise. Both cases $N_A \leq N_I$ and $N_A > N_I$ were investigated and, to validate the analysis, results were compared to Monte Carlo simulation results. The exact analytical SEP requires the solution of a multiple integral whose complexity depends on the smaller of N_A and N_I . This led us to derive upper bounds and approximations for reduced computational complexity.

For the case of a single interferer (as well as for a single antenna) our bound becomes the exact result, and agrees with known results for the single-interferer case given in [3] and [4]. Finally, the performance of the upper bound and the approximate formulas have been assessed by comparison with simulations.

The results show that, for typical cases considered in this paper, our new upper bound is at least 4.8 dB tighter than the only other available bound in the literature. The results also show that the approximation based on the knowledge of the expectation of the eigenvalues is close to Monte Carlo simulation results; to this end, we derived a closed-form expression for the expectation of the eigenvalues in the cases of dual optimum combining with an arbitrary number of interferers, and of two interferers with an arbitrary number of antennas. Finally, the results show that the upper bound and approximation B provide similar accuracy.

APPENDIX A

DISTRIBUTION OF EIGENVALUES OF THE WISHART MATRIX

Let us define $\mathbf{A} \in M_{m,p}$, with $m \leq p$, where $M_{m,p}$ is the set of the $(m \times p)$ complex matrices, and $\tilde{\mathbf{W}}(m,p) = \mathbf{A}\mathbf{A}^\dagger$.

If all the ij th elements of \mathbf{A} , a_{ij} , are complex values with real and imaginary part each belonging to a normal distribution $\mathcal{N}(0,1/2)$, then the $(m \times m)$ Hermitian matrix $\tilde{\mathbf{W}}(m,p)$ is called Wishart. Moreover, the joint pdf of the ordered eigenvalues $\tilde{\boldsymbol{\lambda}} = [\tilde{\lambda}_1, \tilde{\lambda}_2, \dots, \tilde{\lambda}_m]^T$ of $\tilde{\mathbf{W}}(m,p)$, with $\tilde{\lambda}_1 \geq \tilde{\lambda}_2 \geq \dots \geq \tilde{\lambda}_m$, can be found in [17] as

$$f_{\tilde{\boldsymbol{\lambda}}}(\tilde{\lambda}_1, \tilde{\lambda}_2, \dots, \tilde{\lambda}_m) = \frac{\pi^{m(m-1)}}{\tilde{\Gamma}_m(p)\tilde{\Gamma}_m(m)} \prod_{i=1}^m e^{-\tilde{\lambda}_i} \tilde{\lambda}_i^{p-m} \prod_{i=1}^{m-1} \left[\prod_{j=i+1}^m (\tilde{\lambda}_i - \tilde{\lambda}_j)^2 \right] \quad (42)$$

with

$$\tilde{\Gamma}_m(p) = \pi^{m(m-1)/2} \prod_{i=1}^m (p-i)! \quad (43)$$

APPENDIX B

PROOF OF THEOREM 1

In this appendix, we will prove *Theorem 1* using the results of Appendix A, and derive the distribution of the eigenvalues of the matrix $\tilde{\mathbf{R}}$ of (7) for arbitrary N_A and N_I . Let us consider the cases $N_A \leq N_I$ and $N_A > N_I$, separately. The proof for the former case is straightforward application of Appendix A, but to prove the latter case, we need the following theorem.

Theorem 4: Suppose that $\mathbf{A} \in M_{m,p}$ and $\mathbf{B} \in M_{p,m}$ with $m \leq p$, the $(p \times p)$ matrix $\mathbf{B}\mathbf{A}$ has the same m eigenvalues as the $(m \times m)$ matrix $\mathbf{A}\mathbf{B}$, counting multiplicity, together with an additional $p - m$ eigenvalues identically equal to zero.

Proof of Theorem 4: See [18, p. 53]. ■

Proof of Theorem 1: [Case I. $N_A \leq N_I$]: When $N_A \leq N_I$, $\tilde{\mathbf{R}}$ can be related directly to a Wishart matrix, since the entries of the random matrix \mathbf{C}_I are i.i.d. Gaussian random variables with zero-mean, independent real and imaginary parts, each with variance 1/2. So, we can write

$$\tilde{\mathbf{R}} = \sum_{j=1}^{N_I} \mathbf{c}_{I,j} \mathbf{c}_{I,j}^\dagger = \mathbf{C}_I \mathbf{C}_I^\dagger = \tilde{\mathbf{W}}(N_A, N_I) \quad (44)$$

where $\tilde{\mathbf{W}}(N_A, N_I)$ is a $(N_A \times N_A)$ complex Wishart matrix. Thus, the joint pdf of the eigenvalues of $\tilde{\mathbf{R}}$ is given by (42) and (43) with $m = N_A$ and $p = N_I$. ■

Proof of Theorem 1: [Case II. $N_A > N_I$]: When $N_A > N_I$, $\tilde{\mathbf{R}}$ can still be related to the Wishart matrix, by means of *Theorem 4*. In fact, by introducing the $(N_I \times N_A)$ matrix $\mathbf{A} \triangleq \mathbf{C}_I^\dagger$ and the $(N_A \times N_I)$ matrix $\mathbf{B} \triangleq \mathbf{C}_I$, then the $(N_A \times N_A)$ matrix $\mathbf{B}\mathbf{A} = \mathbf{C}_I \mathbf{C}_I^\dagger = \tilde{\mathbf{R}}$ has the same N_I eigenvalues as the $(N_I \times N_I)$ matrix $\mathbf{A}\mathbf{B} = \mathbf{C}_I^\dagger \mathbf{C}_I$, and the additional $N_A - N_I$ eigenvalues are equal to zero. Moreover, since $N_A > N_I$, $\mathbf{C}_I^\dagger \mathbf{C}_I = \tilde{\mathbf{W}}(N_I, N_A)$ is a $(N_I \times N_I)$ complex Wishart matrix, and therefore, $\tilde{\mathbf{R}}$ has total of N_A eigenvalues, where N_I eigenvalues have the joint pdf given by (42) with $m = N_I$ and $p = N_A$, and the additional $N_A - N_I$ eigenvalues are identically equal to zero.

APPENDIX C
AN INEQUALITY

Here we prove the following inequality:

Theorem 5: For any $K_1 \geq 0$, $K_2 \geq 0$, and $x_i \in \mathbb{R}^+$, where $\mathbb{R}^+ = \{r \in \mathbb{R} : r > 0\}$

$$\prod_{i=1}^n \left[\frac{K_1}{K_1 + \frac{K_2}{x_i}} \right] \leq \left[\frac{K_1}{K_1 + \frac{K_2 n}{\sum_{i=1}^n x_i}} \right]^n. \quad (45)$$

Proof: We find the maximum of the function

$$h(x_1, x_2, \dots, x_n) = \prod_{i=1}^n \left[\frac{K_1}{K_1 + \frac{K_2}{x_i}} \right] \quad (46)$$

subject to the constraint

$$C(x_1, \dots, x_n) = \sum_{i=1}^n x_i - S = 0. \quad (47)$$

To this aim, by using the Lagrange's multipliers, we introduce a parameter ϵ and the function

$$H(x_1, \dots, x_n) = h(x_1, \dots, x_n) + \epsilon C(x_1, \dots, x_n) \quad (48)$$

and set the partial derivatives of $H(x_1, \dots, x_n)$ to zero. We can observe that the condition $\partial H(\cdot)/\partial x_j = 0$ requires that

$$\epsilon = -h(x_1, \dots, x_n) \left[\frac{K_2}{K_1 x_j^2 + K_2 x_j} \right] \quad \forall j = 1, \dots, n. \quad (49)$$

This equation is satisfied by choosing $x_j = S/n \quad \forall j$, that, for $x_i \in \mathbb{R}^+$, provides a maximum of the function in (46). To see that this cannot be a minimum, it is sufficient to let one x_i going to zero, keeping finite S . In this case (46) goes to zero, whereas the right member of (45) remains finite. ■

APPENDIX D
CALCULATION OF $\mathbb{E}_y \{B(Y, \theta)\}$

Let $K_3 = N_{\min} N_0 / E_I$, and $K_4 = (c_{\text{MPSK}} N_{\min} E_D / E_I) / \sin^2 \theta$, and then (23) can be written as

$$B(y, \theta) = \left[\frac{y + K_3}{y + K_3 + K_4} \right]^{N_{\min}}. \quad (50)$$

Using (22) and letting $z = y + K_3 + K_4$ gives results as shown in (51) at the bottom of the page. Note (52) and (53) at the bottom of the page. Multiplying (52) and (53) gives (54) at the bottom of the page. Substituting (54) into (51), and noting that $K_3 + K_4 > 0$ we obtain (55) at the bottom of the page,

$$\mathbb{E}_Y \{B(Y, \theta)\} = \int_0^\infty B(y, \theta) f_Y(y) dy = \frac{1}{\Gamma(N_A N_I)} \int_{K_3 + K_4}^\infty \left(1 - \frac{K_4}{z}\right)^{N_{\min}} (z - K_3 - K_4)^{N_A N_I - 1} e^{-(z - K_3 - K_4)} dz \quad (51)$$

$$\left(1 - \frac{K_4}{z}\right)^{N_{\min}} = \sum_{n=0}^{N_{\min}} \binom{N_{\min}}{n} (-K_4)^n z^{-n} \quad (52)$$

$$(z - K_3 - K_4)^{N_A N_I - 1} = \sum_{m=0}^{N_A N_I - 1} \binom{N_A N_I - 1}{m} (-K_3 - K_4)^{N_A N_I - 1 - m} z^m \quad (53)$$

$$\begin{aligned} & \left(1 - \frac{K_4}{z}\right)^{N_{\min}} (z - K_3 - K_4)^{N_A N_I - 1} \\ &= \sum_{n=0}^{N_{\min}} \sum_{m=n}^{N_A N_I - 1} \binom{N_{\min}}{n} \binom{N_A N_I - 1}{m} (-1)^{N_A N_I - 1 - (m-n)} (K_4)^n (K_3 + K_4)^{N_A N_I - 1 - m} z^{m-n} \\ &+ \sum_{n=1}^{N_{\min}} \sum_{m=0}^{n-1} \binom{N_{\min}}{n} \binom{N_A N_I - 1}{m} (-1)^{N_A N_I - 1 - (m-n)} (K_4)^n (K_3 + K_4)^{N_A N_I - 1 - m} z^{-(n-m)} \end{aligned} \quad (54)$$

$$\begin{aligned} \mathbb{E}_Y \{B(Y, \theta)\} &= \frac{e^{+(K_3 + K_4)}}{\Gamma(N_A N_I)} \\ &\times \left\{ \sum_{n=0}^{N_{\min}} \sum_{m=n}^{N_A N_I - 1} \binom{N_{\min}}{n} \binom{N_A N_I - 1}{m} (-1)^{N_A N_I - 1 - (m-n)} (K_4)^n (K_3 + K_4)^{N_A N_I - 1 - m} \times \Gamma_c(m - n + 1, K_3 + K_4) \right. \\ &+ \left. \sum_{n=1}^{N_{\min}} \sum_{m=0}^{n-1} \binom{N_{\min}}{n} \binom{N_A N_I - 1}{m} (-1)^{N_A N_I - 1 - (m-n)} (K_4)^n (K_3 + K_4)^{N_A N_I - 1 - m} \times E_{n-m}(K_3 + K_4) \right\} \quad (55) \end{aligned}$$

where $\Gamma_c(k+1, x)$ is the complementary incomplete gamma function defined by [13 (8.350.2), p. 949]

$$\Gamma_c(k+1, x) \triangleq \int_x^\infty u^k e^{-u} du, \quad x > 0, \quad k = 0, 1, 2, \dots \quad (56)$$

and $E_k(x)$ is given by

$$E_k(x) \triangleq \int_x^\infty \frac{e^{-u}}{u^k} du, \quad x > 0, \quad k = 1, 2, 3, \dots \quad (57)$$

The special case of $k = 1$ is known as the exponential integral.

Integrating (56) and (57) by parts, the recurrence relations can be obtained as

$$\Gamma_c(k+1, x) = x^k e^{-x} + k \Gamma_c(k, x), \quad k = 1, 2, 3, \dots \quad (58)$$

$$E_k(x) = x^{-(k-1)} \frac{e^{-x}}{(k-1)} - \frac{1}{(k-1)} E_{k-1}(x), \quad k = 2, 3, 4, \dots \quad (59)$$

By solving the recurrence relations (58) and (59), we get

$$\Gamma_c(k+1, x) = k! e^{-x} \sum_{l=0}^k \frac{x^l}{l!}, \quad x > 0, \quad k = 0, 1, 2, \dots \quad (60)$$

$$E_k(x) = \frac{(-1)^{k-1}}{(k-1)!} \times \left\{ e^{-x} \sum_{l=1}^{k-1} (-1)^l (l-1)! x^{-l} + E_1(x) \right\}, \quad x > 0, \quad k = 1, 2, 3, \dots \quad (61)$$

Using (60) and (61) results in (62) as shown at the bottom of the page.

$$\begin{aligned} \mathbb{E}_Y\{B(Y, \theta)\} &= \frac{1}{\Gamma(N_A N_I)} \\ &\times \left\{ \sum_{n=0}^{N_{\min}} \sum_{m=n}^{N_A N_I - 1} \binom{N_{\min}}{n} \binom{N_A N_I - 1}{m} (-1)^{N_A N_I - 1 - (m-n)} (K_4)^n \times (m-n)! \sum_{l=0}^{m-n} \frac{1}{l!} (K_3 + K_4)^{N_A N_I - 1 - m + l} \right. \\ &+ \sum_{n=1}^{N_{\min}} \sum_{m=0}^{n-1} \binom{N_{\min}}{n} \binom{N_A N_I - 1}{m} \frac{(-1)^{N_A N_I}}{(n-m-1)!} (K_4)^n \\ &\left. \times \left[\sum_{l=1}^{n-m-1} (-1)^l (l-1)! (K_3 + K_4)^{N_A N_I - 1 - m - l} + (K_3 + K_4)^{N_A N_I - 1 - m} e^{+(K_3 + K_4)} E_1(K_3 + K_4) \right] \right\} \quad (62) \end{aligned}$$

APPENDIX E

SOME RESULTS ON MEAN EIGENVALUES OF $\tilde{\mathbf{W}}(m, p)$

Since (42) is a product of several terms in the form $e^{-a\lambda} \lambda^n$, the expected value of the eigenvalues of $\tilde{\mathbf{W}}(m, p)$ can be written in a closed form for all values of m and p .

As an example, we found the following results:

- $m = 1, p \geq 1$: in this case it is straightforward to verify that $\mathbb{E}\{\tilde{\lambda}_1\} = p$;
- $m = 2, p \geq 2$: after some algebra, we get (63) as shown at the bottom of the page.

It can be shown that $\mathbb{E}\{\tilde{\lambda}_1\}$ can be further simplified to

$$\mathbb{E}\{\tilde{\lambda}_1\} = \frac{p(p^2 + 3)}{2^p} + \sum_{k=2}^{p-1} \frac{1}{2^{2p-k-1}} \binom{2p-k-1}{p-2} \binom{k}{2}. \quad (64)$$

To derive $\mathbb{E}\{\tilde{\lambda}_2\}$, we first observe that

$$\sum_{i=1}^m \tilde{\lambda}_i = \text{tr}[\tilde{\mathbf{W}}(m, p)] \quad (65)$$

where $\text{tr}[\cdot]$ is the trace of the matrix. Then

$$\begin{aligned} \sum_{i=1}^m \mathbb{E}\{\tilde{\lambda}_i\} &= \mathbb{E}\left\{ \sum_{i=1}^m \tilde{\lambda}_i \right\} \\ &= \mathbb{E}\left\{ \text{tr}[\tilde{\mathbf{W}}(m, p)] \right\} \\ &= \mathbb{E}\left\{ \sum_{i=1}^m \sum_{j=1}^p |a_{i,j}|^2 \right\} = m \cdot p \quad (66) \end{aligned}$$

$$\begin{aligned} &\mathbb{E}\{\tilde{\lambda}_1\} \\ &= \frac{1}{(p-1)!(p-2)!} \\ &\times \left[\sum_{k=1}^{p-1} \left(\frac{(p-1)!}{(p-1-k)!} + \frac{(p+1)!}{(p+1-k)!} - \frac{2(p)!}{(p-k)!} \right) \frac{(2p-1-k)!}{2^{2p-k}} - \frac{p!(p-1)!}{2^{p-1}} + (p+1)! \left(\frac{(p-1)!}{2^p} + \frac{(p-2)!}{2^{p-1}} \right) \right] \quad (63) \end{aligned}$$

where $a_{i,j}$ is the ij th element of \mathbf{A} with $\tilde{\mathbf{W}}(m,p) = \mathbf{A}\mathbf{A}^\dagger$, and the last equality is due to the following normalization (see Appendix A):

$$\mathbb{E} \{ |a_{i,j}|^2 \} = 1, \quad i = 1, \dots, m \quad j = 1, \dots, p.$$

Finally, by using (66), we get

$$\mathbb{E} \{ \tilde{\lambda}_2 \} = 2p - \mathbb{E} \{ \tilde{\lambda}_1 \} \quad (67)$$

where $\mathbb{E} \{ \tilde{\lambda}_1 \}$ is given by (64).

ACKNOWLEDGMENT

The authors wish to thank L. A. Shepp and G. J. Foschini for helpful discussions. We also thank O. Andrisano, T. E. Darcie, A. M. Odlyzko, and A. R. Calderbank for providing the fertile research environment where collaboration such as this can thrive.

REFERENCES

- [1] V. M. Bogachev and I. G. Kiselev, "Optimum combining of signals in space-diversity reception," *Telecommun. Radio Eng.*, vol. 34/35, pp. 83–85, Oct. 1980.
- [2] J. H. Winters, "Optimum combining in digital mobile radio with cochannel interference," *IEEE J. Select. Areas Commun.*, vol. SAC-2, pp. 528–539, July 1984.
- [3] V. A. Aalo and J. Zhang, "Performance of antenna array systems with optimum combining in a Rayleigh fading environment," *IEEE Commun. Lett.*, vol. 4, pp. 387–389, Dec. 2000.
- [4] A. Shah, A. M. Haimovich, M. K. Simon, and M. S. Alouini, "Exact bit-error probability for optimum combining with a Rayleigh fading Gaussian cochannel interferer," *IEEE Trans. Commun.*, vol. 48, pp. 908–912, June 2000.
- [5] J. H. Winters and J. Salz, "Upper bounds on the bit-error rate of optimum combining in wireless systems," *IEEE Trans. Commun.*, vol. 46, pp. 1619–1624, Dec. 1998.
- [6] A. Shah and A. M. Haimovich, "Performance analysis of optimum combining in wireless communications with Rayleigh fading and cochannel interference," *IEEE Trans. Commun.*, vol. 46, pp. 473–479, Apr. 1998.
- [7] T. D. Pham and K. G. Balmain, "Multipath performance of adaptive antennas with multiple interferers and correlated fadings," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 342–352, Mar. 1999.
- [8] E. Villier, "Performance analysis of optimum combining with multiple interferers in flat Rayleigh fading," *IEEE Trans. Commun.*, vol. 47, pp. 1503–1510, Oct. 1999.
- [9] P. J. Bickel and K. Doksum, *Mathematical Statistics: Basic Ideas and Selected Topics*, 1st ed. Oakland, CA: Holden-Day, 1977.
- [10] M. Chiani, "Analytical distribution of linearly modulated cochannel interferers," *IEEE Trans. Commun.*, vol. 45, pp. 73–79, Jan. 1997.
- [11] R. F. Pawula, S. O. Rice, and J. H. Roberts, "Distribution of the phase angle between two vectors perturbed by Gaussian noise," *IEEE Trans. Commun.*, vol. COM-30, pp. 1828–1841, Aug. 1982.
- [12] M. K. Simon, S. M. Hinedi, and W. C. Lindsey, *Digital Communication Techniques: Signal Design and Detection*, 1st ed. Englewood Cliffs, NJ: Prentice-Hall, 1995.
- [13] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products*, 5th ed. San Diego, CA: Academic, 1994.
- [14] T. A. Cover and J. A. Thomas, *Elements of Information Theory*, 1st ed. New York: Wiley, 1991.
- [15] M. Z. Win, R. K. Mallik, G. Chrisikos, and J. H. Winters, "Canonical expressions for the error probability performance for M -ary modulation with hybrid selection/maximal ratio combining in Rayleigh fading," in *Proc. IEEE Wireless Communication and Networking Conf.*, vol. 1, Sept. 1999, pp. 266–270.
- [16] M. Z. Win and J. H. Winters, "On maximal ratio combining in correlated Nakagami channels with unequal fading parameters and SNRs among branches: An analytical framework," in *Proc. IEEE Wireless Communication and Networking Conf.*, vol. 3, Sept. 1999, pp. 1058–1064.
- [17] A. T. James, "Distributions of matrix variates and latent roots derived from normal samples," *Ann. Math. Statist.*, vol. 35, pp. 475–501, 1964.
- [18] R. A. Horn and C. R. Johnson, *Matrix Analysis*, 1st ed. Cambridge, MA: Cambridge Univ. Press, 1990.



Marco Chiani (M'94–SM'02) was born in Rimini, Italy, on April 4, 1964. He received the Dr. Ing. degree (with honors) in electronic engineering and the Ph.D. degree in electronic engineering and computer science from the University of Bologna, Bologna, Italy, in 1989 and 1993, respectively.

From 1994 he has been with the Dipartimento di Elettronica, Informatica e Sistemistica, University of Bologna, Cesena, Italy, where he is currently Professor and Chair for Telecommunications. His research interests include the areas of communications

theory, coding, and wireless networks.

Dr. Chiani is an Editor for IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, IEEE TRANSACTIONS ON COMMUNICATIONS, and is Chair of the Radio Communications Committee, IEEE Communications Society. He was on the Technical Program Committee of the IEEE Conferences GLOBECOM 1997, ICC 1999, ICC 2001, and ICC 2002.



Moe Z. Win (S'85–M'87–SM'97) received the B.S. degree (*magna cum laude*) from Texas A&M University, College Station, and the M.S. degree from the University of Southern California (USC), Los Angeles, in 1987 and 1989, respectively, in electrical engineering. As a Presidential Fellow at USC, he received both the M.S. degree in applied mathematics and the Ph.D. degree in electrical engineering in 1998.

In 1987, he joined the Jet Propulsion Laboratory (JPL), California Institute of Technology, Pasadena.

From 1994 to 1997, he was a Research Assistant with the Communication Sciences Institute at USC, where he played a key role in the successful creation of the Ultra-Wideband Radio Laboratory. Since 1998, he has been with the Wireless Systems Research Department, AT&T Laboratories-Research, Middletown, NJ. His main research interests are the application of communication, detection, and estimation theories to a variety of communications problems, including time-varying channels, diversity, equalization, synchronization, signal design, ultrawide-bandwidth communication, and optical communications.

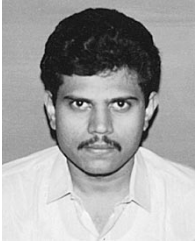
Dr. Win is a member of Eta Kappa Nu, Tau Beta Pi, Pi Mu Epsilon, Phi Theta Kappa, and Phi Kappa Phi. At Texas A&M University, he was a University Undergraduate Fellow, and he received the Academic Excellence Award, among others. At USC, he received several awards including the Outstanding Research Paper Award and the Phi Kappa Phi Student Recognition Award. He was the recipient of the IEEE Communications Society Best Student Paper Award at the Fourth Annual IEEE NetWorld+ Interop'97 Conference. He currently serves as the Technical Program Chair for the IEEE Communication Theory Symposium of ICC-2004. He served as the Tutorial Chair for the IEEE Semiannual International Vehicular Technology Conference (Fall-2001), the Technical Program Chair for the IEEE Communication Theory Symposium of Globecom-2000, IEEE Conference on Ultra Wideband Systems and Technologies (2002); and Technical Program Vice-Chair for the IEEE International Conference on Communications (2002). He is the current Editor for *Equalization and Diversity* for the IEEE TRANSACTIONS ON COMMUNICATIONS and a Guest Editor for the 2002 IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, Special Issue on Ultra Wide Band Radio in Multiaccess Wireless Communications.



Alberto Zanella (S'99–M'00) was born in Ferrara, Italy, on 23 December 1971. He received the Dr. Ing. degree (with honors) in electronic engineering from the University of Ferrara, Ferrara, Italy, in 1996, and the Ph.D. degree in electronic and computer science from the University of Bologna, Bologna, Italy, in 2000.

In 2001, he joined the Consiglio Nazionale delle Ricerche-Centro di Studio per l'Informatica e i sistemi di Telecomunicazioni (CNR-CSITE), now a section of CNR-Istituto di Elettronica e di Ingegneria dell'Informazione e delle Telecomunicazioni (IEIIT) as a Researcher. His research interests include cellular and mobile radio systems, WLANs, radio resource management (handover algorithms, power control, frequency hopping, and dynamic channel allocation), adaptive equalization, smart antennas, and multiple-input multiple-output systems.

Dr. Zanella was on the Technical Program Committee of the IEEE ICC 2002.



Ranjan K. Mallik (S'88–M'93–SM'02) received the B.Tech. degree from the Indian Institute of Technology, Kanpur, in 1987 and the M.S. and Ph.D. degrees from the University of Southern California, Los Angeles, in 1988 and 1992, respectively, all in electrical engineering.

From August 1992 to November 1994, he was a scientist at the Defence Electronics Research Laboratory, Hyderabad, India, working on missile and EW projects. From November 1994 to January 1996, he was a faculty member of the Department of Elec-

tronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur. In January 1996, he joined the faculty of the Department of Electronics and Communication Engineering, Indian Institute of Technology, Guwahati. Since December 1998, he has been with the Department of Electrical Engineering, Indian Institute of Technology, Delhi, where he is an Assistant Professor. His research interests are in communication theory and systems, difference equations, and linear algebra.

Dr. Mallik is a member of Eta Kappa Nu. He is also a member of the IEEE Communications and Information Theory Societies, the American Mathematical Society, and the International Linear Algebra Society, a life member of the Indian Society for Technical Education, and an associate member of The Institution of Engineers (India).



Jack H. Winters (S'77–M'81–SM'88–F'96) received the B.S.E.E. degree from the University of Cincinnati, Cincinnati, OH, in 1977, and the M.S. and Ph.D. degrees in electrical engineering from The Ohio State University, Columbus, in 1978 and 1981, respectively.

From 1981 to early 2002, he was with AT&T Bell Laboratories, and then AT&T Labs-Research, Middletown, NJ, where he was Division Manager of the Wireless Systems Research Department. Since early 2002, he has been consulting for several wireless and optical communication companies. He has studied signal processing techniques for increasing the capacity and reducing signal distortion in fiber optic, mobile radio, and indoor radio systems, and is currently studying smart antennas, adaptive arrays, and equalization for indoor and mobile radio systems.

Dr. Winters is an IEEE Distinguished Lecturer for both the IEEE Communications and Vehicular Technology Societies, Area Editor for Transmission Systems for the IEEE TRANSACTIONS ON COMMUNICATIONS, and New Jersey Inventor of the Year for 2001.