

# Slow Adaptive $M$ -QAM With Diversity in Fast Fading and Shadowing

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**Abstract**—This paper investigates the performance of adaptive  $M$ -ary quadrature amplitude modulation (QAM) with antenna subset diversity. We consider a slow adaptive modulation (SAM) technique that adapts the constellation size to the slow variation of the channel due, for example, to shadowing. The proposed SAM technique is more practical than conventional fast adaptive modulation (FAM) techniques that require adaptation to fast-fading variations. Our results show that the SAM technique can provide a substantial increase in throughput with respect to fixed schemes while maintaining an acceptable low bit-error outage. We also compare SAM and FAM techniques, showing that the throughput of SAM can be, in many practical cases, close to that of FAM, despite the fact that SAM is less complex and requires a lower feedback rate. For example, using a set of possible modulations {4,16,64}-QAM with dual-branch maximal ratio combining reception, 5% outage at a bit-error probability of  $10^{-2}$  and a median signal-to-noise ratio of 22 dB, SAM is capable of improving the mean spectral efficiency of fixed schemes from about 1.9 to 4.7 b/s/Hz, which is close to the 5.5 b/s/Hz achieved by FAM.

**Index Terms**—Adaptive modulation, fading channels, mobile communication, quadrature amplitude modulation (QAM).

## I. INTRODUCTION

DIGITAL WIRELESS communication systems must cope with quality of service (QoS) variation due to channel evolution. Common measures of QoS are bit-error probability (BEP) and bit-error outage (BEO), i.e., the probability that the BEP exceeds a maximum tolerable level (see, for example, [1] and [2]).<sup>1</sup> One possible method for coping with channel variation is to use adaptive modulation, which can provide higher spectral efficiency (SE) (i.e., the average b/s/Hz) while keeping

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<sup>1</sup>Unless otherwise stated, the term BEP will be used in the following to denote the mean BEP, where the averaging is performed over the fast fading. Symbols and acronyms used in the paper are summarized in Table I.

TABLE I  
SYMBOLS AND ACRONYMS USED IN THE PAPER IN ORDER OF APPEARANCE

QAM	quadrature amplitude modulation
SAM	slow adaptive modulation
FAM	fast adaptive modulation
QoS	quality of service
BEP	bit error probability, $P_b$
BEO	bit error outage, $P_o$
SE	spectral efficiency
AWGN	additive white Gaussian noise
SNR	signal-to-noise ratio
ASD	antenna subset diversity
MRC	maximal ratio combining
$\gamma$	instantaneous SNR
$\gamma_T$	instantaneous SNR at the combiner output
$\bar{\gamma}$	mean SNR averaged over fast fading
$P_b^*$	target BEP
$\mu_{dB}$	median SNR, mean of $\bar{\gamma}_{dB}$
$\sigma_{dB}$	variance of $\bar{\gamma}_{dB}$
$N$	number of antenna elements
$M_j$	$j^{\text{th}}$ modulation level
$\eta$	mean spectral efficiency [bits/s/Hz]
$L$	number of combined branches
SD	selection diversity

an acceptable QoS. Adaptive techniques have been proposed for many modern wireless packet data services, such as WCDMA, UMTS, IS-95, IS-136, GPRS, and EDGE, to adjust data rate, coding, and modulation depending on channel conditions [3]. However, the performance evaluation of these adaptive systems are almost always obtained through simulation.

The idea of adapting transmission parameters to channel variations dates back almost 40 years. For example, adaptive transmission schemes have been proposed in which symbol energy or duration is adjusted at the transmitter based on its knowledge of fading conditions conveyed by a feedback channel [4], [5]. The capacity of Rayleigh fading channels under adaptive transmission and diversity combining techniques was obtained in [6]. Adaptive quadrature amplitude modulation (QAM) schemes, where the modulation parameters are chosen according to the instantaneous channel conditions, have been proposed in [7]–[11]. The analysis of adaptive modulation in the presence of shadowing only is addressed in [8]. Due to its bandwidth

efficiency,  $M$ -ary QAM ( $M$ -QAM) has been a popular choice for numerous practical applications such as digital video broadcasting [12], wireless local area networks [13], and metropolitan area networks [14]. Although early works on  $M$ -QAM date back to the early 1960s [15]–[18], the evaluation of BEP for arbitrary  $M$  is still of current interest [19]–[21].<sup>2</sup> In particular, the exact BEP expression for the additive white Gaussian noise (AWGN) channel, assuming Gray code bit mapping, is given in [21].

With the above-mentioned adaptive techniques, the receiver must reliably and rapidly estimate the instantaneous signal-to-noise ratio (SNR) to track the fast-fading evolution. Based on this estimate, the receiver must send the constellation size to be used reliably and instantaneously back to the transmitter. To reduce the channel load caused by feedback, model-based channel-prediction algorithms can be adopted [24]. With this method, it is essential to predict the channel coefficients several tens to hundreds of symbols ahead. Moreover, in rapidly changing mobile radio environments, the vehicle speed and scattering geometry change continuously, and thus the model parameters need to be recomputed frequently [24]. We denote these adaptation techniques that require the knowledge of the fast-fading channel conditions as fast adaptive modulation (FAM).

As an alternative to FAM, one may consider the slow adaptive modulation (SAM) scheme in which parameters are adapted based on the performance averaged over an interval of few seconds (i.e., the BEP averaged over fast fading). With this technique, the constellation size needs to follow only the slow variation of the channel; hence, it requires slower feedback rate, and consequently, it is simpler to implement than FAM. Indeed, the feedback rate is essentially related to the coherence time of the channel. As an example, for a mobile terminal, the coherence time of the fast fading is inversely proportional to the maximum Doppler frequency: with a carrier frequency of 900 MHz, the coherence time is about 72 and 4 ms for a mobile speed of 3 and 50 km/h, respectively. On the other hand, the coherence time of the shadowing is proportional to the coherence distance (e.g., 100–200 m in a suburban area and tens of meters in an urban area [25]). Assuming a coherence distance of 100 m, this results in a coherence time of about 120 and 7.2 s at 3 and 50 km/h, respectively. Note that the coherence time of the fast fading can be an order of magnitude smaller than the coherence time of the shadowing. The importance of the slow feedback rate is accentuated especially in the design of time-division duplex (TDD) systems due to the inherent delay in the feedback channel.

In this paper, we propose a SAM scheme with  $M$ -QAM using antenna subset diversity (ASD). We analyze the SAM performance for coherent reception of  $M$ -QAM with ASD, which uses only a subset of the total available diversity branches [26]–[29]. As a benchmark, we start by analyzing the capacity of adaptive transmission schemes with maximal ratio combining (MRC) in composite Rayleigh fading and shadowing channels. We then compare SAM, FAM, and fixed-modulation schemes with ASD in terms of both SE and BEO.<sup>3</sup> Our

results show that the SAM technique can provide substantial improvement over a nonadaptive scheme in terms of both SE and BEO. Furthermore, even with the reduced feedback rate and consequent lower complexity, the performance of SAM can be close to FAM in many scenarios. For illustration, numerical results are provided for channels with Rayleigh fading and log-normal shadowing.

The paper is organized as follows. We analyze the capacity of adaptive modulation schemes in Section II, the concept of SAM is introduced in Section III, and the comparison with FAM is made in Section IV. In Section V, numerical results are provided, and conclusions are given in Section VI.

## II. CAPACITY OF ADAPTIVE MODULATION SCHEMES

In this section, we will obtain the capacity (in b/s/Hz) of adaptive transmission schemes with diversity reception in composite Rayleigh fading and shadowing channels. This provides theoretical achievable maximum data throughput of adaptive transmission schemes with arbitrarily low bit-error rate without any constraints such as delay and implementation complexity.

To serve as a benchmark of our proposed SAM scheme, here we evaluate the capacity for MRC in composite Rayleigh fading and shadowing channels. In particular, it has been shown that shadowing in mobile radio systems is well modeled by a log-normal distribution [32], [33]. The presence of log-normal shadowing implies that the SNR  $\bar{\gamma}$ , in decibels, is a Gaussian random variable (RV) with mean  $\mu_{\text{dB}}$  and variance  $\sigma_{\text{dB}}^2$  (i.e.,  $10 \log_{10} \bar{\gamma} \sim \mathcal{N}(\mu_{\text{dB}}, \sigma_{\text{dB}}^2)$ ).<sup>4</sup> In this paper, we are interested in the case of microdiversity in which all branches experience the same shadowing level. In Appendix A, it is shown that the probability density function (pdf) of the instantaneous SNR  $\gamma_{\text{T}}$  at the output of the combiner, when the transmit power is constant, is given by ( $\nu = 10/\ln 10$ )

$$f_{\gamma_{\text{T}}}(\xi) = \int_0^{\infty} \frac{1}{w^N} \frac{\xi^{N-1}}{\Gamma(N)} \exp\left[-\frac{\xi}{w}\right] \times \frac{\nu}{\sqrt{2\pi}\sigma_{\text{dB}}w} \exp\left[-\frac{(10 \log_{10} w - \mu_{\text{dB}})^2}{2\sigma_{\text{dB}}^2}\right] dw \quad (1)$$

for  $\xi \geq 0$  and 0 otherwise, where  $N$  is the number of antenna elements.

The pdf in (1) can be inserted into known integral expressions for capacity evaluation. When channel state information (CSI) is available at the receiver only (CSIR), the capacity is given by [6]

$$C = \int_0^{+\infty} \log_2(1 + \xi) f_{\gamma_{\text{T}}}(\xi) d\xi. \quad (2)$$

If we also have CSI at the transmitter, in addition to CSIR, the transmitter can adjust its power according to the channel variations. In this case, the combiner output SNR would be  $\varrho(\gamma_{\text{T}})\gamma_{\text{T}}$

<sup>2</sup>For a brief history of  $M$ -QAM, see [22] and [23].

<sup>3</sup>We focus our attention to frequency-flat fading channels in order to accentuate the importance of SAM. Our methodologies can be extended to wireless systems operating in frequency-selective fading channels, such as orthogonal frequency-division multiplexing (OFDM) systems [30], [31].

<sup>4</sup>Unless otherwise stated, the term SNR,  $\bar{\gamma}$ , will be used in the following to denote the mean symbol SNR per branch, where the averaging is performed over the fast fading.

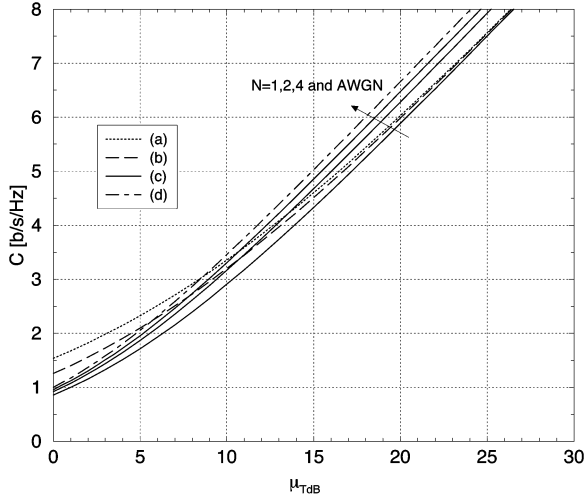


Fig. 1. Mean unconstrained capacity as a function of the median value of the total SNR at the combiner output. (a) CSIR, power control, Rayleigh fading, and log-normal shadowing ( $\sigma_{\text{dB}} = 8$ ) with  $N = 1$ . (b) CSIR, Rayleigh fading, and log-normal shadowing ( $\sigma_{\text{dB}} = 8$ ) with  $N = 1$ . (c) CSIR, Rayleigh fading only with  $N = 1, 2, 4$  (MRC). (d) AWGN with  $N = 1$ .

where  $\varrho(\gamma_T)$  is the ratio of the variable transmit power to its allowed average. It can be shown [6], [34] that the optimal power control policy is

$$\varrho(\gamma_T) = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma_T}, & \gamma_T \geq \gamma_0 \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

where the threshold  $\gamma_0$  in (3) is implicitly given by

$$\int_{\gamma_0}^{+\infty} \left( \frac{1}{\gamma_0} - \frac{1}{\xi} \right) f_{\gamma_T}(\xi) d\xi = 1. \quad (4)$$

Replacing  $\log_2(1 + \xi)$  with  $\log_2(1 + \varrho(\xi)\xi)$  in (2), we arrive at

$$C = \int_{\gamma_0}^{+\infty} \log_2 \left( \frac{\xi}{\gamma_0} \right) f_{\gamma_T}(\xi) d\xi. \quad (5)$$

Fig. 1 shows the unconstrained capacity obtained by (5), (4), (2), and (1) for various channel conditions including fast fading and shadowing. In particular,  $C$  is plotted as a function of the median value of the total SNR at the combiner output (i.e.,  $\mu_{\text{TdB}} = \mu_{\text{dB}} + 10 \log_{10} N$ ), to remove the array gain and, instead, to highlight the diversity gain due to the multiple antennas on the distribution of the decision variable.

From these curves, several observations are worth noting. First, it can be seen, by comparing curves (a) and (b), that knowledge of the CSI at the transmitter has a small impact on the capacity of composite Rayleigh fading and log-normal shadowing channels.<sup>5</sup> Second, the log-normal shadowing also has a limited impact on the capacity of single-branch reception, as can be noted by comparing curves (b) and (c) for  $N = 1$  (the lowest continuous line curve). Note, however, that shadowing describes slow variations in time, and thus practical schemes with limited delay and coding complexity cannot reach these ergodic capacity curves. Therefore, it is worthwhile to consider the notion

<sup>5</sup>This fact is known previously for the case of Rayleigh fading channels [34].

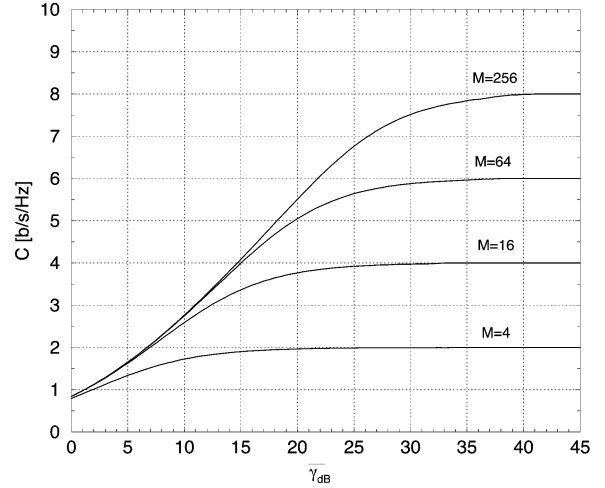


Fig. 2. Mean capacity as a function of SNR for  $N = 1$ , CSIR, Rayleigh fading only with finite  $M$ -QAM constellation.

of outage capacity, where the outage is due to shadowing. Third, the use of multiple antennas can effectively turn a Rayleigh fading channel into an AWGN channel from the capacity perspective; this is evident by observing the behavior of curve (c) for increasing  $N$  and the capacity of the AWGN channel in curve (d). Therefore, fast fading can be mitigated by using diversity, while the shadowing (which cannot be mitigated by using forward error correction) can be ameliorated by adapting the constellation size.<sup>6</sup> Note that previous works dealt with either fast fading [6] or shadowing [8]. In this paper, we consider both fast fading and shadowing with a finite number of antennas and constellation size. Many practical scenarios, where both fast fading and shadowing affect wireless systems performance, cannot be investigated by straightforward extension of previous works. Indeed, it requires simultaneous consideration of BEO and BEP.

Note that results in Fig. 1 do not have any constraints on transmitted signals. For the case of finite constellation size and CSIR, the mean capacity in the presence of fading and AWGN can be evaluated using the methodology described in Appendix B. Specifically, for  $M$ -QAM constellations and  $N = 1$ , the mean capacity over a Rayleigh fading channel with CSIR is plotted in Fig. 2 as a function of the SNR. We note again that in the presence of shadowing, the SNR is also an RV, and therefore, it is worthwhile to introduce the concept of outage capacity, related to the slow but random variation of the SNR. Moreover, practical uncoded systems typically operate in the saturation part of each curve. For example, to reach a BEP of  $10^{-3}$ ,  $M$ -QAM systems in the same conditions as in Fig. 2 require SNRs of about 27, 33, 38.3, and 43.5 dB, for  $M = 4, 16, 64$ , and 256, respectively. Thus, when the SNR varies due to shadowing, significant gains can be achieved by adapting the constellation size to the slow variation of the channel, that is, by using the largest possible constellation size (maximum capacity) satisfying the QoS requirement. We will elaborate on this in the following sections.

<sup>6</sup>For both SAM and FAM, we consider the case of constant transmit power. Another possible solution, not studied here, is to deal with fast fading and shadowing by changing transmit power; however, changing only the constellation size, while maintaining the same transmitter power level, has the advantage of keeping the interference to other receivers constant in mobile radio systems [7].

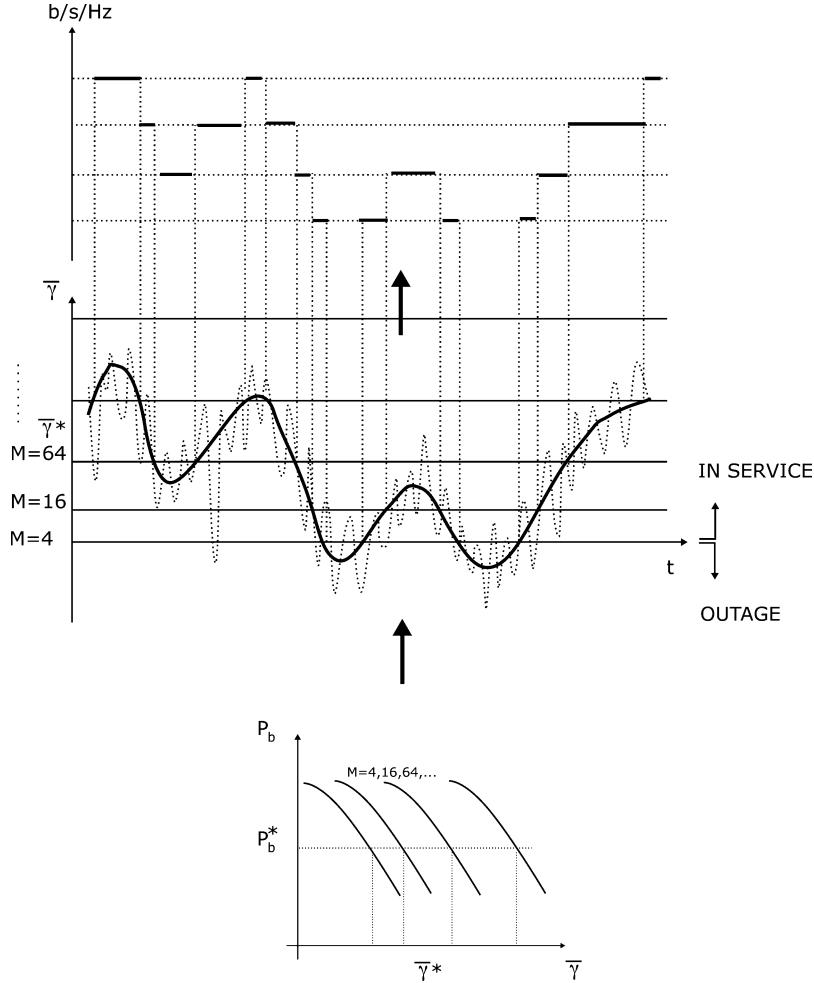


Fig. 3. Methodology for SE and BEO evaluation.

### III. SLOW ADAPTIVE MODULATION

In general, for a given received power, the BEO increases with the constellation size  $M$ . On the other hand, by increasing  $M$ , the system becomes more efficient in spectrum use, that is, the SE increases. When a set of modulation levels  $\{M_0, M_1, \dots, M_J\}$  is used, the minimum BEO and the maximum SE correspond to  $M_0$  and  $M_{\max} = M_J$ , respectively. The system is in outage when even the smallest constellation size does not meet the target BEP. Therefore, SE ranges from zero, when the system is always in outage, to  $\log_2 M_J$  b/s/Hz, when the system never experiences outage.

For a fixed target BEP  $P_b^*$ , the SAM technique enables an increase in SE by increasing the constellation sizes  $M_j$  to the next available  $M_{j+1}$  when the SNR per bit exceeds the SNR required for  $M_{j+1}$  (i.e.,  $\bar{\gamma}_{j+1}^*$  such that  $P_b(\bar{\gamma}_{j+1}^*) = P_b^*$ ). On the other hand, it decreases the constellation size when the SNR is not sufficient to guarantee the target BEP for  $M_j$  (i.e.,  $\bar{\gamma}_j^*$  such that  $P_b(\bar{\gamma}_j^*) = P_b^*$ ). Hence, the transmission system using SAM experiences the same BEO as a fixed (nonadaptive) modulation system with  $M = M_0$ , and can achieve a substantial increase in SE due to the fact that SAM uses the largest possible constellation size (ranging from  $M_0$  to  $M_J$ ).

Fig. 3 illustrates the SAM concept, adapting the constellation size  $M$  to follow the variations of SNR (due to slow fading, i.e.,

shadowing). In particular, for a fixed target BEP, SNR thresholds for different constellation sizes are obtained from BEP expressions for various diversity techniques. By comparing the SNR with these thresholds,  $M$  is adapted to provide the best possible throughput while satisfying QoS requirements. In the following subsections, we derive the mean SE and the BEO to quantify the performance of SAM.

#### A. Normalized SE

For a fixed target BEP  $P_b^*$ , the SE is a discrete RV with distribution that depends on the SNR thresholds, and hence also on the BEP expression of the given system configuration. In particular, the SE has value  $\log_2 M_j$  for SNR in the range  $(\bar{\gamma}_j^*, \bar{\gamma}_{j+1}^*]$ , with probability  $\mathbb{P}\{\bar{\gamma}_j^* < \bar{\gamma} \leq \bar{\gamma}_{j+1}^*\}$ . For example, for a set of constellation sizes ranging from  $M = 4$  ( $j = 0$ ) to  $M = 1024$  ( $j = J = 4$ ), the distribution of the SE is given by

$$\mathbb{P}\{\text{SE} = k\} = \begin{cases} \mathbb{P}\{\bar{\gamma} \leq \bar{\gamma}_0^*\}, & k = 0 \\ \mathbb{P}\{\bar{\gamma}_0^* < \bar{\gamma} \leq \bar{\gamma}_1^*\}, & k = 2 \\ \mathbb{P}\{\bar{\gamma}_1^* < \bar{\gamma} \leq \bar{\gamma}_2^*\}, & k = 4 \\ \mathbb{P}\{\bar{\gamma}_2^* < \bar{\gamma} \leq \bar{\gamma}_3^*\}, & k = 6 \\ \mathbb{P}\{\bar{\gamma}_3^* < \bar{\gamma} \leq \bar{\gamma}_4^*\}, & k = 8 \\ \mathbb{P}\{\bar{\gamma}_4^* < \bar{\gamma}\}, & k = 10. \end{cases} \quad (6)$$

The mean SE is defined as

$$\eta \triangleq \mathbb{E}_{\bar{\gamma}}\{\log_2 M\} \quad [\text{b/s/Hz}] \quad (7)$$

where the averaging is with respect to the distribution of the SNR  $\bar{\gamma}$ . Let  $M_j$  and  $\bar{\gamma}_{\text{dB},j}^*$  be the  $j$ th element from the set of possible constellation sizes and corresponding SNR threshold (in decibels), respectively, to achieve a target BEP. Then,  $\eta$  can be written as

$$\begin{aligned} \eta &= \sum_{j=0}^{J-1} \tilde{M}_j \mathbb{P} \left\{ \bar{\gamma}_{\text{dB},j}^* < \bar{\gamma}_{\text{dB}} \leq \bar{\gamma}_{\text{dB},j+1}^* \right\} \\ &\quad + \tilde{M}_J \mathbb{P} \left\{ \bar{\gamma}_{\text{dB},J}^* < \bar{\gamma}_{\text{dB}} \right\} \\ &= \sum_{j=0}^{J-1} \tilde{M}_j \left[ F_{\bar{\gamma}_{\text{dB}}} \left( \bar{\gamma}_{\text{dB},j+1}^* \right) - F_{\bar{\gamma}_{\text{dB}}} \left( \bar{\gamma}_{\text{dB},j}^* \right) \right] \\ &\quad + \tilde{M}_J \left[ 1 - F_{\bar{\gamma}_{\text{dB}}} \left( \bar{\gamma}_{\text{dB},J}^* \right) \right] \end{aligned} \quad (8)$$

where  $\tilde{M}_k = \log_2 M_k$  and  $F_{\bar{\gamma}_{\text{dB}}}(\cdot)$  is the cumulative distribution function (CDF) of  $\bar{\gamma}_{\text{dB}} = 10 \log_{10} \bar{\gamma}$ .

For a nonadaptive modulation scheme with  $M = M_k$ , system throughput is equal to  $\log_2 M_k$  b/s/Hz during the service period with the target QoS and 0 b/s/Hz during the outage period. Hence, for nonadaptive schemes, the SE is given by

$$\eta = \tilde{M}_k \left[ 1 - F_{\bar{\gamma}_{\text{dB}}} \left( \bar{\gamma}_{\text{dB},k}^* \right) \right]. \quad (9)$$

For log-normal shadowing,  $F_{\bar{\gamma}_{\text{dB}}}(\cdot)$  of (8) and (9) can be written as

$$F_{\bar{\gamma}_{\text{dB}}}(\xi) = Q \left( \frac{\mu_{\text{dB}} - \xi}{\sigma_{\text{dB}}} \right) \quad (10)$$

where  $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-t^2/2} dt$  is the Gaussian- $Q$  function.<sup>7</sup>

### B. Bit-Error Outage

In digital mobile radio systems when a fast process is superimposed on a slow process, the BEP alone is not sufficient to describe the link quality. In this situation, the SNR also varies in time, due, for example, to a combination of mobility, shadowing, and power control. In such environments, a reasonable performance measure relating to the slow variations of the channel is given by the BEO, outage probability based on BEP, that is

$$P_o(P_b^*) = \mathbb{P} \{ P_b(\bar{\gamma}) \geq P_b^* \} \quad (11)$$

where  $P_b(\bar{\gamma})$  is the BEP as a function of SNR,  $\bar{\gamma}$ , and  $P_b^*$  is the maximum tolerable BEP. For log-normal shadowing, this is

$$P_o(P_b^*) = Q \left( \frac{\mu_{\text{dB}} - 10 \log_{10} \bar{\gamma}^*}{\sigma_{\text{dB}}} \right) \quad (12)$$

where  $\bar{\gamma}^*$  satisfies  $P_b(\bar{\gamma}^*) = P_b^*$  [1], [2]. The parameter  $\mu_{\text{dB}}$  corresponds to the median value of the shadowing, and it plays an important role in link-budget evaluation for system design.

Note that our proposed methodology, [i.e., (8), (9), (11), together with (10) and (12)], is valid for different modulation formats and various ASD schemes. The specific choice of modulation format and diversity scheme affect the required SNRs according to the associated BEP expression. In the following, we will consider the coherent detection of  $M$ -QAM with ASD.

<sup>7</sup>The Gaussian- $Q$  function can be related to the complementary error function by  $Q(x) = (1/2)\text{erfc}(x/\sqrt{2})$ .

### C. Canonical BEP Expression With ASD

One method of achieving ASD is generalized diversity combining (GDC) [35], [36] where  $L$  branches out of  $N$  are selected for MRC. If the selected branches are the  $L$  strongest, GDC results in hybrid-selection/MRC (H-S/MRC) [35], [36].

By applying the virtual branch technique of [36], we derive in Appendix C the pdf of the SNR at the combiner output as given by

$$f_{\gamma_T|\bar{\gamma}}(\xi) = \sum_{n=1}^{\tilde{N}} \sum_{k=1}^{\mu_n} A_{n,k} g_{n,k}(\xi) \quad (13)$$

where  $A_{n,k}$  is given in (39) and

$$g_{n,k}(\xi) = \begin{cases} \frac{\xi^{k-1}}{\tilde{b}_n^k \Gamma(k)} e^{-\xi/\tilde{b}_n}, & \xi \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (14)$$

with  $\tilde{b}_n$  given by (37). Note that (13) depends on  $\bar{\gamma}$  through  $\tilde{b}_n$  in  $g_{n,k}(\xi)$ .

We now turn our attention to BEP evaluation. The exact instantaneous BEP expression for QAM with arbitrary  $M$  is given in [21] as

$$\begin{aligned} P_b(e|\gamma_T) &= \frac{2}{\sqrt{M} \log_2(\sqrt{M})} \\ &\quad \times \sum_{h=1}^{\log_2(\sqrt{M})} \sum_{i=0}^{(1-2^{-h})\sqrt{M}-1} (-1)^{\lfloor \frac{i \cdot 2^{h-1}}{\sqrt{M}} \rfloor} \\ &\quad \times \left( 2^{h-1} - \left\lfloor \frac{i \cdot 2^{h-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor \right) \\ &\quad \times Q \left( (2i+1) \sqrt{\frac{3\gamma_T}{(M-1)}} \right) \end{aligned} \quad (15)$$

where  $\lfloor x \rfloor$  denotes the largest integer less than or equal to  $x$  and  $\gamma_T$  is the instantaneous symbol SNR at the combiner output. Note that the instantaneous BEP is a linear combination of  $Q$ -functions. By using (13), we obtain the canonical expression for the BEP as

$$\begin{aligned} P_b(\bar{\gamma}) &= \frac{2}{\sqrt{M} \log_2(\sqrt{M})} \sum_{h=1}^{\log_2(\sqrt{M})} \sum_{i=0}^{(1-2^{-h})\sqrt{M}-1} (-1)^{\lfloor \frac{i \cdot 2^{h-1}}{\sqrt{M}} \rfloor} \\ &\quad \times \left( 2^{h-1} - \left\lfloor \frac{i \cdot 2^{h-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor \right) \mathcal{I} \left( \frac{3(2i+1)^2 \tilde{b}_n}{2(M-1)} \right) \end{aligned} \quad (16)$$

where the  $\tilde{b}_n$  depend on  $\bar{\gamma}$  as in (37) and

$$\begin{aligned} \mathcal{I}(x) &= \sum_{n=1}^{\tilde{N}} \sum_{k=1}^{\mu_n} A_{n,k} I_k(x) \\ I_k(x) &= \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{\sin^2 \theta}{\sin^2 \theta + x} \right)^k d\theta. \end{aligned} \quad (17)$$

Note that for  $N$ -branch MRC ( $L = N$ ), with independent and identically distributed (i.i.d.) Rayleigh fading, (13) reduces to

$$f_{\gamma_T|\bar{\gamma}}(\xi) = \begin{cases} \left( \frac{1}{\bar{\gamma}} \right)^N \frac{\xi^{N-1}}{\Gamma(N)} \exp\left(-\frac{\xi}{\bar{\gamma}}\right), & \xi \geq 0 \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

In this case (16) holds with  $\mathcal{I}(x) = I_N(x)$ , while for  $N$ -branch MRC with i.i.d. Nakagami- $m$  fading, it holds with  $\mathcal{I}(x) = I_{mN}(x/m)$ .

#### IV. COMPARISON WITH FAM

In this section, we obtain the SE and BEO for the FAM technique. Recall that the receiver for FAM must reliably and rapidly estimate the instantaneous channel conditions, and the feedback rate to the transmitter for FAM must be higher than that for SAM to track the fast-fading evolution, thus resulting in higher complexity. We compare SAM and FAM under the same QoS requirement governed by a user perspective. For example, in voice or digital video streaming applications, the QoS is governed by the error rate averaged over fast fading.

The SE for FAM can be evaluated using methodology similar to the one for SAM, except that the instantaneous SNR  $\gamma_T$  is compared with the thresholds obtained from the instantaneous BEP. Given the instantaneous SNR thresholds (i.e.,  $\gamma_T^*$  that results in the instantaneous target BEP), the SE for the FAM scheme becomes

$$\begin{aligned} \eta &= \sum_{j=0}^{J-1} \tilde{M}_j \mathbb{P} \{ \gamma_{T,j}^* < \gamma_T \leq \gamma_{T,j+1}^* \} + \tilde{M}_J \mathbb{P} \{ \gamma_{T,J}^* < \gamma_T \} \\ &= \sum_{j=0}^{J-1} \tilde{M}_j [F_{\gamma_T}(\gamma_{T,j+1}^*) - F_{\gamma_T}(\gamma_{T,j}^*)] \\ &\quad + \tilde{M}_J [1 - F_{\gamma_T}(\gamma_{T,J}^*)] \end{aligned} \quad (19)$$

where  $F_{\gamma_T}(\cdot)$  is the CDF of  $\gamma_T$ . Since the instantaneous SNR is subjected to both multipath fading and shadowing, the composite CDF is needed to evaluate (19). Here, we derive the canonical expression for the CDF of the instantaneous SNR for H-S/MRC microdiversity in a composite Rayleigh fading and log-normal shadowing channel.

By using (24) in Appendix A, the pdf of the instantaneous output SNR for H-S/MRC can be written in the canonical form as (recall that  $g_{n,k}(\xi)$  depends on  $\bar{\gamma}$  through  $\tilde{b}_n$ )

$$f_{\gamma_T}(\xi) = \sum_{n=1}^{\tilde{N}} \sum_{k=1}^{\mu_n} A_{n,k} \int_0^{\infty} g_{n,k}(\xi) f_{\bar{\gamma}}(w) dw. \quad (20)$$

From the pdf in (20), we obtain the canonical expression for the CDF as

$$F_{\gamma_T}(\gamma_T^*) = \sum_{n=1}^{\tilde{N}} \sum_{k=1}^{\mu_n} A_{n,k} \int_0^{\gamma_T^*} \int_0^{\infty} g_{n,k}(\xi) f_{\bar{\gamma}}(w) dw d\xi. \quad (21)$$

#### V. NUMERICAL RESULTS

In this section, numerical results are presented in terms of BEP, BEO, and SE for coherent detection of  $M$ -QAM with H-S/MRC and Gray code mapping in composite Rayleigh fading and log-normal shadowing channels. Note that the shadowing level is the same for all branches, since we are dealing with microdiversity.

To evaluate the performance of SAM and nonadaptive schemes, we first obtain the required SNRs from (16) and (17) to achieve a target BEP for different values of diversity order  $N$  and constellation size  $M$ . By using these thresholds together with (8) and (12), we obtain the SE and BEO of the SAM scheme operating at different target BEPs for various diversity orders, constellation sizes, fading, and shadowing parameters. For FAM,

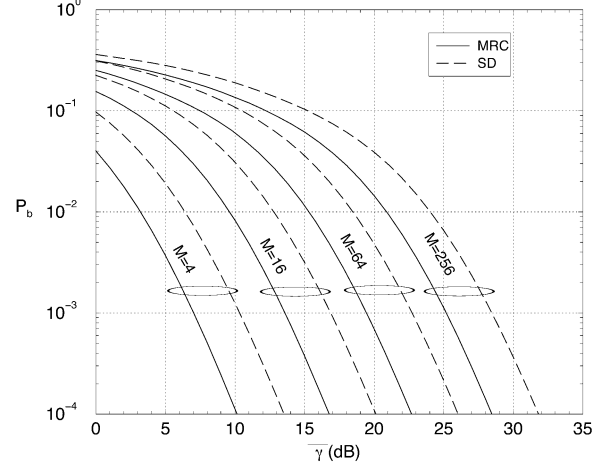


Fig. 4. BEP versus  $\bar{\gamma}$  for MRC and SD  $M$ -QAM with  $N = 4$  and  $M = 4, 16, 64, 256$ .

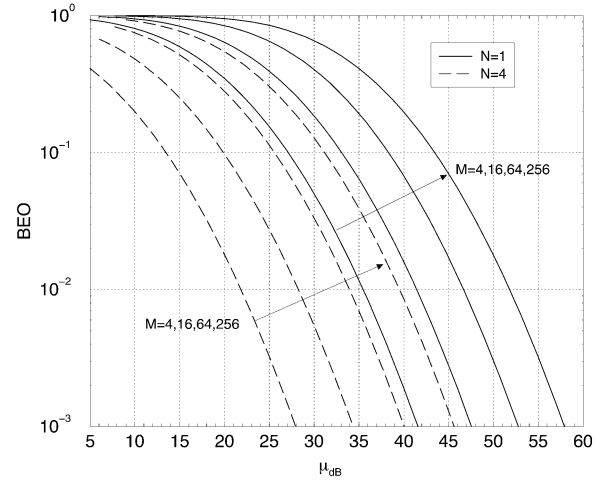


Fig. 5. BEO versus  $\mu_{dB}$  for a nonadaptive scheme with  $P_b^* = 10^{-2}$ ,  $\sigma_{dB} = 8$ ,  $N = 1$ , and  $N = 4$  (MRC), and different constellation sizes  $M$ .

the thresholds for the instantaneous SNR are obtained from the instantaneous BEP; with these thresholds, together with (19) and (26), we obtain the SE and BEO of FAM schemes.

Using the canonical expression for the BEP in (16), obtained by the virtual branch technique, Fig. 4 shows the BEP as a function of SNR for MRC ( $L = 4, N = 4$ ) and ASD ( $L = 1, N = 4$ ) in Rayleigh fading for constellation sizes  $M = 4, 16, 64, 256$ . Note that ASD with  $L = 1$  is known as selection diversity (SD). Fixing a target BEP, one can obtain the SNR thresholds for MRC or SD with different constellation sizes. For example, for target BEP of  $10^{-2}$  and  $M = 4$ , the required SNR is about 3.3 dB with MRC and 6.5 dB with SD, whereas for  $M = 256$ , it is about 21.0 and 24.1 dB, respectively.

In Fig. 5, the BEO as a function of  $\mu_{dB}$  for a nonadaptive scheme with target BEP of  $P_b^* = 10^{-2}$ ,  $\sigma_{dB} = 8$ ,  $N = 1$ , and  $N = 4$  (MRC), and  $M = 4, 16, 64, 256$  is shown. Using this figure, one can obtain the requirement on the parameter  $\mu_{dB}$ , corresponding to the median value of the shadowing level, to achieve the desired BEO. This is useful for the design of digital radio systems with diversity reception. From this, the maximum distance of the radio link can be estimated when the pathloss

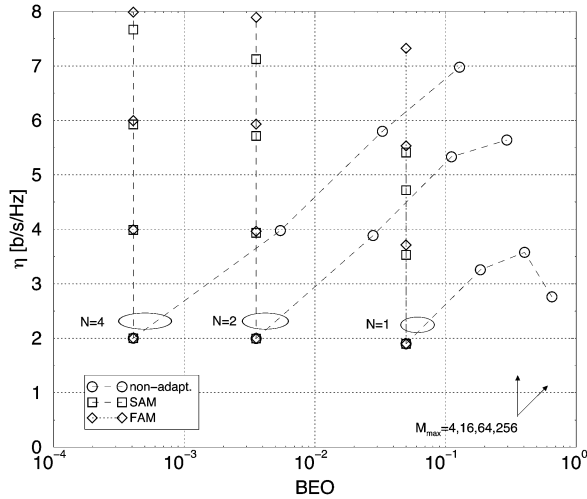


Fig. 6. Comparison between slowly adapting and nonadapting schemes in terms of BEO and mean SE for  $P_b^* = 10^{-2}$ ,  $\sigma_{dB} = 8$ ,  $\mu_{dB} = 30$ , and MRC with different diversity orders  $N$ .

law is known. For example, Fig. 5 shows that for a target BEP  $P_b^* = 10^{-2}$  and  $\sigma_{dB} = 8$ , a BEO equal to 10% is obtained with  $\mu_{dB}$  around 27 and 38 for  $M = 4$  and 64, respectively, with single-branch reception. If four-branch MRC is adopted in the same conditions, the required  $\mu_{dB}$  reduces to about 13 and 25, respectively.

A comparison among SAM, FAM, and fixed-modulation schemes is shown in Fig. 6 in terms of both SE and BEO for  $P_b^* = 10^{-2}$ ,  $\sigma_{dB} = 8$ ,  $\mu_{dB} = 30$ , and different values of  $N$  and  $M_{max}$ . As an example, for BEO of 5% with single-branch reception and  $M$  ranging from 4 to 256, nonadaptive schemes can achieve about 1.9 b/s/Hz, whereas SAM can achieve 5.4 b/s/Hz and FAM can achieve 7.3 b/s/Hz. Fig. 6 also shows that both BEO and SE can be enhanced by using antenna diversity, that is, when the diversity order  $N$  increases the performance of the SAM technique approaches that of FAM, despite the need for lower feedback rate.<sup>8</sup> In fact, for BEO of 5% with dual-branch reception, nonadaptive schemes can achieve about 3.9 b/s/Hz, whereas SAM can achieve 7.1 b/s/Hz and FAM can achieve 7.9 b/s/Hz. For  $N = 4$ , the nonadaptive schemes can achieve about 5.8 b/s/Hz, whereas 7.6 and 7.9 b/s/Hz are achieved by SAM and FAM, respectively. It is important to note that SE for the fixed modulation scheme does not necessarily increase with the constellation size, since larger constellation size can result in more frequent system outages.

In Fig. 7, we compare the outage capacity with the SE obtained with SAM. To make a comparison with SE for BEO  $\leq 5\%$ , we consider outage capacity at 5%,  $C_{5\%}$ , defined as  $\mathbb{P}\{C \leq C_{5\%}\} = 5\%$ . It can be seen that the SE at  $P_b^* = 10^{-3}$  is below the capacity. On the other hand, note that the SE at  $P_b^* = 10^{-2}$  has a behavior similar to that of the outage capacity, slightly exceeding it, due to the fact that error probability is not negligible. In the following, we will focus on the SE at  $P_b^* = 10^{-2}$ .

<sup>8</sup>Similar to the capacity behavior observed in Section II, as  $N$  increases, fast fading diminishes, and one should expect the performance of SAM and FAM to be closer.

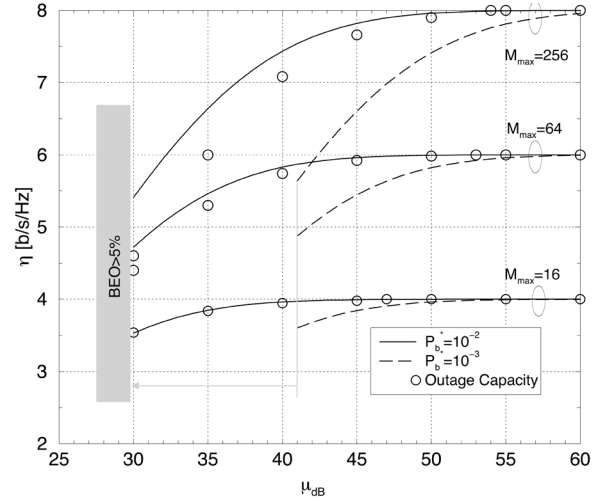


Fig. 7. Comparison between the mean SE and the outage capacity, at an outage level of 5%. The mean SE is reported at both  $P_b^* = 10^{-2}$  and  $P_b^* = 10^{-3}$ ,  $N = 1$ , and  $\sigma_{dB} = 8$ .

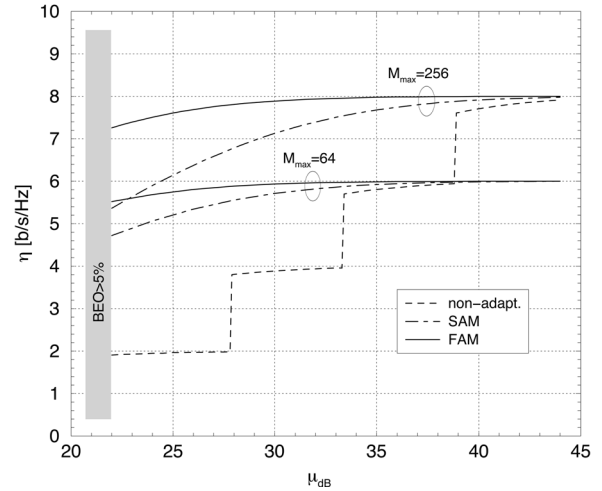


Fig. 8. Mean SE versus  $\mu_{dB}$  for fast, slow, and nonadaptive QAM with maximum constellation size 64 and 256, maximum BEO of 5%,  $P_b^* = 10^{-2}$ , dual-branch MRC, and  $\sigma_{dB} = 8$ .

Another important point is the role played by shadowing parameters on the performance. For example, the maximum constellation size that can be used by SAM depends on  $\mu_{dB}$ , hence the SE also depends on this parameter. Fig. 8 shows the SE versus  $\mu_{dB}$  for maximum constellation size  $M_{max} = 64, 256$ , maximum BEO of 5%,  $P_b^* = 10^{-2}$ ,  $\sigma_{dB} = 8$ , and dual-branch MRC. It is apparent that SAM can provide substantial improvement over nonadaptive schemes. Moreover, the performance of SAM is quite close to that of FAM, despite the need for less complexity with SAM schemes.

To evaluate the influence of diversity order on the SE of SAM, Fig. 9 shows the mean SE as a function of  $\mu_{dB}$  for SAM with  $M_{max} = 16, 64, 256$ , maximum tolerable BEO of 5%,  $P_b^* = 10^{-2}$ ,  $\sigma_{dB} = 8$ , and for  $N = 1, 2, 4, 8$  (MRC). Note that the curves start from a value of  $\mu_{dB}$  that guarantees a BEO not greater than the maximum tolerable. Fixing a desirable SE and maximum BEO, one can obtain the minimum required  $\mu_{dB}$  that satisfies both BEO and SE requirements for different diversity

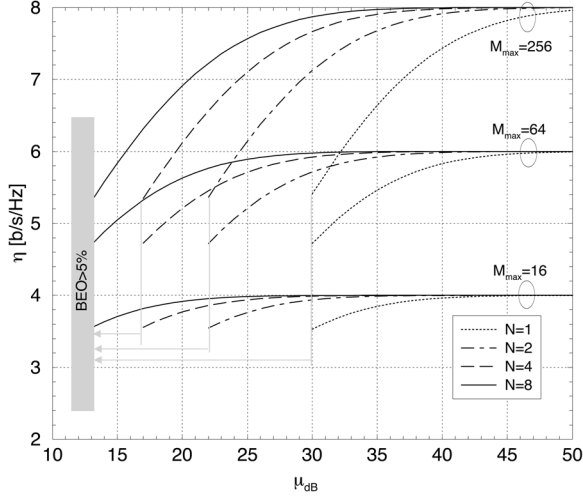


Fig. 9. Mean SE versus  $\mu_{dB}$  for SAM with maximum constellation size 16, 64, 256, maximum BEO of 5%,  $P_b^* = 10^{-2}$ ,  $\sigma_{dB} = 8$ , and MRC with different diversity orders  $N$ .

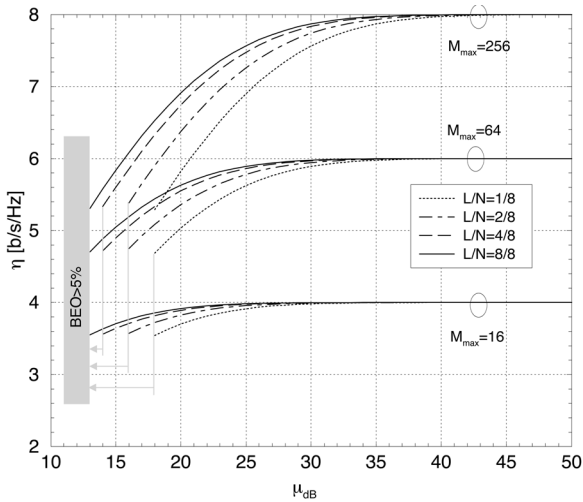


Fig. 10. Mean SE versus  $\mu_{dB}$  for SAM with maximum constellation size 16, 64, 256, maximum BEO of 5%,  $P_b^* = 10^{-2}$ ,  $\sigma_{dB} = 8$ , and H-S/MRC with  $L/N = 1/8, 2/8, 4/8, 8/8$ .

orders. For example, for SE equal to 7 b/s/Hz and maximum BEO of 5%, the minimum required  $\mu_{dB}$  has to be around 37, 29, 24, and 21 dB for  $N = 1, 2, 4, 8$ , respectively. Similar results for H-S/MRC with  $N = 8$  and  $L = 1, 2, 4, 8$  are given in Fig. 10. Comparing Figs. 9 and 10, it can be seen that the throughput of 1/8 H-S/MRC is close that of four-branch MRC.

To summarize, from the above curves, the system designer can obtain the minimum value of  $\mu_{dB}$  for a specified level of performance. Since the actual  $\mu_{dB}$  is tied to the propagation law and location of the user, one can then design the cellular system, in terms of cell size, power levels, etc., that fulfills the specified requirements.

## VI. CONCLUSIONS

In this paper, we analyzed a SAM technique, which adapts the constellation size to the mean SNR. We compared SAM with FAM and nonadaptive modulation schemes in terms of both bit-error outage and normalized throughput (SE) for coherent

detection of  $M$ -QAM with ASD in the presence of Rayleigh fading and log-normal shadowing. It is shown that the SAM technique can provide substantial improvement over nonadaptive schemes in terms of both SE and BEO. In particular, it is shown that by using the SAM technique with modulation levels in  $\{M_{\min}, \dots, M_{\max}\}$ , a substantial increase in SE can be achieved while maintaining the same outage of nonadaptive modulation using  $M = M_{\min}$ . We also showed that the performance of SAM is close to FAM, despite the need for a lower feedback rate, and thus less complexity, with SAM. The proposed methodology is applicable to other modulation formats, diversity techniques, and fading channels. By using the proposed methodology, one can obtain SE and BEO for various channel parameters as well as for different diversity techniques.

## APPENDIX A

### COMPOSITE RAYLEIGH FADING AND LOG-NORMAL SHADOWING WITH MICRODIVERSITY

In this appendix, we derive the marginal pdf and CDF for composite Rayleigh fading and log-normal shadowing with microdiversity (i.e., all branches experience the same shadowing). Specifically, we characterize the combiner output SNR when the transmit power is constant. Starting from the log-normal pdf of the SNR  $\bar{\gamma}$ , with  $\nu = 10/\ln 10$

$$f_{\bar{\gamma}}(w) = \begin{cases} \frac{\nu}{\sqrt{2\pi}\sigma_{dB}w} \exp\left[-\frac{(10\log_{10}w - \mu_{dB})^2}{2\sigma_{dB}^2}\right], & w \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (22)$$

and the conditional pdf of the instantaneous SNR  $f_{\gamma_T|\bar{\gamma}}(\xi)$ , conditioned on a given value of  $\bar{\gamma}$ , we obtain the joint pdf as

$$f_{\gamma_T, \bar{\gamma}}(\xi, w) = f_{\gamma_T|\bar{\gamma}}(\xi) f_{\bar{\gamma}}(w). \quad (23)$$

Using (23), the marginal pdf of the instantaneous SNR is given by

$$f_{\gamma_T}(\xi) = \begin{cases} \int_0^\infty f_{\gamma_T|\bar{\gamma}}(\xi) f_{\bar{\gamma}}(w) dw, & \xi \geq 0 \\ 0, & \text{otherwise.} \end{cases} \quad (24)$$

For  $N$ -branch MRC, (24) results in (1) for  $\xi \geq 0$ , and 0, otherwise. From this pdf one can easily obtain the CDF

$$F_{\gamma_T}(\gamma_T^*) = \int_0^{\gamma_T^*} \int_0^\infty \frac{1}{w^N} \frac{\xi^{N-1}}{\Gamma(N)} \exp\left[-\frac{\xi}{w}\right] \times \frac{\nu}{\sqrt{2\pi}\sigma_{dB}w} \exp\left[-\frac{(10\log_{10}w - \mu_{dB})^2}{2\sigma_{dB}^2}\right] dw d\xi. \quad (25)$$

By changing the order of integration and using a change of variables, we also obtain the following more compact form:

$$F_{\gamma_T}(\gamma_T^*) = 1 - \int_0^\infty \frac{\Gamma(N, \gamma_T^*/w)}{\Gamma(N)} \frac{\nu}{\sqrt{2\pi}\sigma_{dB}w} \times \exp\left[-\frac{(10\log_{10}w - \mu_{dB})^2}{2\sigma_{dB}^2}\right] dw \quad (26)$$

where  $\Gamma(\cdot)$  and  $\Gamma(\cdot, \cdot)$  are the Euler gamma function and the incomplete gamma function, respectively.



We note in passing that the methodology presented above can be extended to other types of fading. For example, the instantaneous SNR at the combiner output of MRC in composite Nakagami- $m$  fading and log-normal shadowing can be written as

$$f_{\gamma_T}(\xi) = \int_0^\infty \left(\frac{m}{w}\right)^{mN} \frac{\xi^{mN-1}}{\Gamma(mN)} \exp\left[-\frac{m\xi}{w}\right] \frac{\nu}{\sqrt{2\pi}\sigma_{\text{dB}}w} \times \exp\left[-\frac{(10\log_{10} w - \mu_{\text{dB}})^2}{2\sigma_{\text{dB}}^2}\right] dw. \quad (27)$$

In this case, the CDF becomes

$$F_{\gamma_T}(\gamma_T^*) = 1 - \int_0^\infty \frac{\Gamma(mN, m\gamma_T^*/w)}{\Gamma(mN)} \frac{\nu}{\sqrt{2\pi}\sigma_{\text{dB}}w} \times \exp\left[-\frac{(10\log_{10} w - \mu_{\text{dB}})^2}{2\sigma_{\text{dB}}^2}\right] dw. \quad (28)$$

#### APPENDIX B

##### CAPACITY WITH FINITE ALPHABET OVER FADING CHANNELS

Here we describe how we obtain the capacity curves for the case of CSIR and finite input alphabet. Let us start with the case of real signals. Let  $X$  denote the generic real transmitted sample, and  $Z$  the additive Gaussian noise with zero mean and variance  $\sigma^2$ . The received sample can then be written as  $Y = aX + Z$ , with  $a$  corresponding to the fading level. The mutual information for a given fading level  $a$  is

$$I(X; Y|a) = - \int_{-\infty}^{\infty} p_{Y|a}(y|a) \log_2 \left[ p_{Y|a}(y|a) \sqrt{2\pi e \sigma^2} \right] dy \quad (29)$$

where  $p_{Y|a}(y|a)$  is the pdf of the random variable  $Y$  given  $a$ , which depends upon the statistical distribution of the additive noise and of the input signal  $X$ . In the case of practical  $M$ -QAM systems, the input  $X$  represents the 1-D projection on the signal constellation points; thus, it can be thought as taking values in the set  $\mathcal{A} = \{\pm\epsilon, \pm 3\epsilon, \dots, \pm(\sqrt{M} - 1)\epsilon\}$ , with  $\epsilon$  being a normalization constant.

Due to symmetry, the capacity is achieved in this case with equiprobable symbols; therefore, the conditional pdf of  $Y$  can be written

$$p_{Y|a}(y|a) = \frac{1}{\sqrt{2\pi}M\sigma^2} \sum_{x \in \mathcal{A}} \exp\left\{-\frac{(y - ax)^2}{2\sigma^2}\right\}. \quad (30)$$

By inserting (30) in (29) and averaging with respect to the distribution of  $a$ , we obtain the curves in Fig. 2.

#### APPENDIX C

##### CANONICAL FORM FOR THE PDF OF THE SNR AT THE COMBINER OUTPUT

The SNR at the combiner output of GDC is given by

$$\gamma_T = \langle \mathbf{a}, \boldsymbol{\gamma}_{[N]} \rangle \quad (31)$$

where  $\mathbf{a}$  is the  $N \times 1$  selection vector with elements 1 or 0, depending on whether the branch is selected or not,  $\boldsymbol{\gamma}_{[N]}$  is the vector of ordered physical diversity branch SNRs, and  $\langle \cdot, \cdot \rangle$  denotes the inner product. For i.i.d. Rayleigh fading, each branch SNR  $\gamma_i$  is exponentially distributed with the same mean  $\bar{\gamma}$  (i.e., microdiversity). Different choices of vector  $\mathbf{a}$  correspond to different diversity-combining schemes within the class of GDC such as MRC ( $a_n = 1$  with  $1 \leq n \leq N$ ), H-S/MRC ( $a_n = 1$  for  $n \leq L$  and 0 otherwise), and SD ( $a_1 = 1$  and  $a_n = 0$  for  $2 \leq n \leq N$ ) [36].

We aim to derive the canonical BEP expression for ASD as a weighted sum of the BEP for single-branch systems. Direct analysis of BEP in terms of physical branch variables involves nested  $N$ -fold integrals, which are, in general, prohibitively difficult to evaluate, if even possible. The problem is made analytically tractable by transforming  $\boldsymbol{\gamma}_{[N]}$ , which are not independent, into a vector of i.i.d. virtual branch variables  $\mathbf{V}_N \triangleq [V_1, \dots, V_N]^t$ , through a linear transformation  $\mathbf{T}_{\text{VB}}$  as  $\boldsymbol{\gamma}_{[N]} = \mathbf{T}_{\text{VB}} \mathbf{V}_N$  where

$$\mathbf{T}_{\text{VB}} = \begin{pmatrix} \bar{\gamma}/1 & \bar{\gamma}/2 & \dots & \bar{\gamma}/N \\ & \bar{\gamma}/2 & \dots & \bar{\gamma}/N \\ & & \ddots & \vdots \\ & & & \bar{\gamma}/N \end{pmatrix} \quad (32)$$

is an upper triangular matrix [36]. The pdf of each  $V_n$  is given by

$$f_{V_N}(v) = \begin{cases} e^{-v}, & v \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (33)$$

with characteristic function

$$\psi_{V_N}(j\nu) = \frac{1}{1 - j\nu}. \quad (34)$$

The SNR at the output of the combiner can now be written as

$$\gamma_T = \langle \mathbf{a}, \mathbf{T}_{\text{VB}} \mathbf{V}_N \rangle = \langle \mathbf{b}, \mathbf{V}_N \rangle \quad (35)$$

where  $\mathbf{b} = \mathbf{T}_{\text{VB}}^t \mathbf{a}$ . Since  $V_n$ 's are i.i.d., the characteristic function of  $\gamma_T$  results in  $\psi_{\gamma_T}(j\nu) = \prod_{n=1}^N \psi_{V_N}(j\nu b_n)$ , and it can be rewritten as

$$\psi_{\gamma_T}(x) = \prod_{n=1}^{\tilde{N}} \left( \frac{c_n}{c_n + x} \right)^{\mu_n} \quad (36)$$

where  $x = j\nu$ ,  $c_n = -1/\tilde{b}_n$ ,  $\tilde{b}_n$  is the set of  $\tilde{N}$  distinct  $b_n$ , each with algebraic multiplicity  $\mu_n$  such that  $\sum_{n=1}^{\tilde{N}} \mu_n = N$ . For example, with H-S/MRC, which combines the  $L$  strongest branches out of  $N$

$$\begin{aligned} \tilde{N} &= N - L + 1 \\ \mu_n &= \begin{cases} L, & n = 1 \\ 1, & n = 2, \dots, \tilde{N} \end{cases} \\ \tilde{b}_n &= \begin{cases} \bar{\gamma}, & n = 1 \\ \bar{\gamma}L/(L + n - 1), & n = 2, \dots, \tilde{N}. \end{cases} \end{aligned} \quad (37)$$

Expression (36) can be rewritten in the form of canonical expansion as [36]

$$\psi_{\gamma_T}(x) = \sum_{n=1}^{\tilde{N}} \sum_{k=1}^{\mu_n} A_{n,k} \left( \frac{c_n}{c_n + x} \right)^k \quad (38)$$

where the coefficients of the expansion are given by

$$A_{n,k} = \frac{\varphi_{n,k}}{c_n^k (\mu_n - k)!} \quad (39)$$

with  $n = 1, \dots, \tilde{N}$  and  $k = 1, \dots, \mu_n$ . In (39),  $\varphi_{n,k}$  denotes the  $(\mu_n - k)$ th derivative of  $\varphi_n(x) \triangleq x^{\mu_n} \psi_{\gamma_T}(x - c_n)$  evaluated at  $x = 0$ . Thus

$$\psi_{\gamma_T}(j\nu) = \sum_{n=1}^{\tilde{N}} \sum_{k=1}^{\mu_n} A_{n,k} \left[ \psi_{V_N}(j\nu \tilde{b}_n) \right]^k. \quad (40)$$

Recognizing that  $[\psi_{V_N}(j\nu \tilde{b}_n)]^k$  in (40) is the characteristic function of a sum of a  $k$  i.i.d. exponential RV with mean  $\tilde{b}_n$ , the pdf of  $\gamma_T$  conditioned on the mean value  $\bar{\gamma}$  can be written in the canonical form as in (13).

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