Energy-Efficient Network Navigation Algorithms

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***Abstract*—Network navigation is an emerging paradigm that enables high-accuracy location awareness in GPS-challenged environments. Two important operations of network navigation, location inference and power control, interrelate with each other, thus motivating the design of joint inference and control algorithms. In this paper, we develop efficient network navigation algorithms with optimized energy allocation. In particular, we first determine the confidence region for lzocation inference based on Fisher information analysis, and then design robust energy allocation strategies that minimize the position errors of the agents within the confidence region. Based on these strategies, both centralized and distributed energy-efficient network navigation algorithms are developed. Simulation results show that the proposed algorithms significantly reduce the position errors compared to the algorithms with uniform or non-robust power control.**

***Index Terms*—Cooperative networks, energy allocation, inference algorithms, localization, navigation.**

I. Introduction

NETWORK NAVIGATION is critical for many future wireless applications in commercial, military and social sectors, and thus have attracted intensive research efforts in recent years [1]–[14]. The objective of a navigation process is to infer the positions of mobile nodes (agents) over a period of time based on various measurements of the agents' position state. For example, commonly used GPS-based navigation systems can determine the trajectories of moving vehicles by consistently making pseudo-range measurements to the GPS satellites [15]. On the other hand, in GPS-challenged environments, such an objective can be achieved by network navigation, where agents cooperate with their neighboring nodes by making inter-node range measurements and exchanging position information [1]–[3].

Network navigation consists of two important operations: location inference and power control. The former performs inference algorithms to determine the agents' positions using

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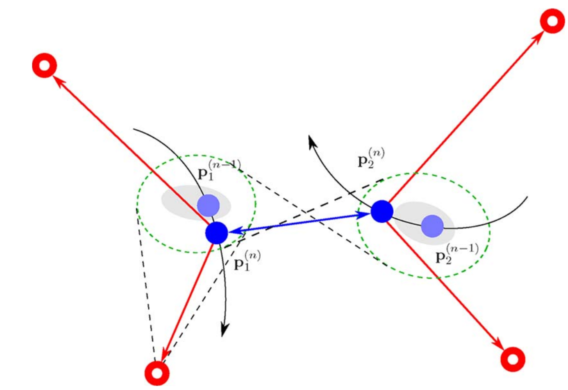


Fig. 1. Network navigation: agents (blue dots) communicate with neighboring agents and anchors (red circles) to infer their positions. The previous positional information along with the step size can be exploited for robust power control in the next time slot. The confidence region of agent k is the grey ellipse *after* the location inference at time t_{n-1}, and becomes the green ellipse *before* the location inference at time t_n due to the movement.

range measurements, and the latter pertains to the allocation of the transmitting energy for the range measurements.[1] These two operations highly interrelate with each other: in the navigation process, the perceiving of the agents' positions provides prior knowledge for the energy allocation in the next time slot, while the energy allocation of the localization network determines the accuracy of agents' position estimates. For example, in Fig. 1, the position estimate and the confidence region[2] of the agents serve as the input of the energy allocation strategy, which in turn affects the localization performance in the next time slot.

Existing studies commonly focus on only one of the operations in network navigation. Extensive efforts have been carried out on the processing of range measurements and the design of range-based localization algorithms [17]–[27]. For example, the authors in [18]–[20] investigated the typical techniques used in the measurements of the distances between nodes, i.e., time-of-arrival (TOA) and received signal strength (RSS). Localization algorithms can be categorized into centralized algorithms (e.g., [21]–[23]) and distributed algorithms (e.g., [25]–[27]). Centralized algorithms generally transform the localization problems into optimization programs for determining the maximum likelihood estimates (MLEs), whereas distributed algorithms typically adopt the methods of successive refinement. Detailed reviews of localization algorithms can be found in [28]–[30]. Though these studies provide various techniques

[1] Energy is considered as a resource for localization in this paper, but our framework can account for the allocation of other transmitting resources such as bandwidth and time for range measurements.

[2] A confidence region, calculated from a set of sample data, is an area that is likely to include an unknown parameter of interest (in this paper, the agent's position) [16].

and algorithms for range-based localization, they typically start from given measurement results and few studies consider the effect of the energy allocation on the localization accuracy and efficiency.

The design of energy allocation strategies in localization networks receives increasing research interest because the allocation of transmitting energy is critical in determining the localization performance [6]–[8], [31]–[36]. Current approaches typically formulate the energy allocation problem as an optimization program, aiming to achieve the optimal tradeoff between the localization performance and the energy allocation. For example, in [37]–[39], the energy allocation problems for radar networks are investigated. The Cramér-Rao lower bound (CRLB) for position errors are adopted as the performance metric and suboptimal solutions of the original problem are obtained via relaxed convex programs. Recent work [40]–[42] proposed robust power allocation problems in wireless localization networks, where the former two focused on noncooperative networks and the latter one considered cooperative networks. However, all of these studies do not investigate the performance improvement of localization algorithms with the proposed power allocation strategies.

The interaction between the two operations motivates the study of joint inference and control for network navigation, where the challenge mainly lies in their coupling effect in time. That is, the control (power control) depends on the position errors of agents, while such a control decision determines their position errors in the next time slot. Moreover, cooperation among the agents adds another layer of difficulty since both the inference and control process become further coupled in space. To the best of the authors' knowledge, there lacks such a framework jointly designing energy allocation and position estimation algorithms in cooperative network navigation. In addition, since the CRLB only characterizes a lower bound for the mean squared position errors [43], it is preferable to adopt the mean squared position errors directly as the metric to evaluate the performance of the proposed algorithms.

In this paper, we develop energy-efficient network navigation algorithms that jointly consider location inference and power control. We first determine the confidence region for location inference based on Fisher information analysis, and develop robust energy allocation strategies to minimize the position errors of the agents within the confidence region. We then propose position estimation methods based on some of the existing localization techniques (e.g., MLE [44] and particle filtering techniques [45]). The main contributions of this paper are as follows.

- We propose a robust energy allocation strategy in the centralized setting and show that these strategies can be transformed into semi-definite programs (SDPs). This is an extension of the work in [40], [41], which consider the robust energy allocation in noncooperative localization networks.
- We propose a robust energy allocation strategy in the distributed setting. Compared to the strategies in our previous work [42], the proposed strategy adopts a tighter upper bound for the CRLB as the performance metric and can be transformed into an second-order cone program (SOCP).
- We develop network navigation algorithms with optimized energy allocation in both centralized and distributed settings, and demonstrate the efficiency and the robustness of the proposed strategies.

The rest of the paper is organized as follows. Section II presents the system model and formulates the network navigation problem with optimized energy allocation. Sections III and IV present the navigation algorithms with energy allocation in the centralized and distributed setting, respectively. Finally, numerical results are presented in Section V and conclusions are drawn in the last section.

Notation: $[\mathbf{A}]_{ij}$ denotes the element in the i^{th} row and j^{th} column of matrix $\mathbf{A}$. $\mathbf{I}_n$ denotes an $n \times n$ identity matrix. $\mathbf{0}_{m,n}$ denotes a $m \times n$ matrix with all 0's. $\mathbf{1}_n$ and $\mathbf{0}_n$ denote n-dimensional vectors with all 1's and 0's, respectively. For $\mathbf{0}_{m,n}$, $\mathbf{1}_n$, and $\mathbf{0}_n$, the subscript will be omitted if clear in the context. $\mathbf{e}_k$ is a unit vector with the k^{th} element being 1 and all other elements being 0's. $\|\cdot\|_0$ denotes the number of non-zero elements. Matrix $\mathbf{J}_{\text{r}}(\phi)$ is denoted as $\mathbf{J}_{\text{r}}(\phi) = [\cos\phi \;\; \sin\phi]^{\text{T}}[\cos\phi \;\; \sin\phi]$. The function $\mathbb{I}\{x\}$ is the indicate function defined to be 0 if $x=0$, and 1 otherwise. For vectors $\mathbf{x}$ and $\mathbf{y}$, the relations $\mathbf{x} \succeq \mathbf{y}$ and $\mathbf{x} \succ \mathbf{y}$ denote that all elements of $\mathbf{x} - \mathbf{y}$ are nonnegative and positive, respectively. For square matrices $\mathbf{A}$ and $\mathbf{B}$, the relation $\mathbf{A} \succeq \mathbf{B}$ denotes that $\mathbf{A} - \mathbf{B}$ is a semidefinite matrix. For sets $\mathcal{A}, \mathcal{B} \subset \mathbb{R}^d$, $\mathcal{A} + \mathcal{B}$ denotes the set $\{\mathbf{a} + \mathbf{b} : \mathbf{a} \in \mathcal{A}, \mathbf{b} \in \mathcal{B}\}$.

II. System Model and Problem Formulation

This section presents the system model and formulates the network navigation problem with optimized energy allocation.

A. Network Model

Consider a time-slotted synchronized navigation network with N_{b} anchors and N_{a} agents. Anchors are fixed nodes with known positions, whereas agents are mobile nodes with unknown positions. Anchors and agents are deployed in a 2-D region of interest $\mathcal{R} \subseteq \mathbb{R}^2$. Let $\mathcal{N}_{\text{a}} = \{1, 2, \ldots, N_{\text{a}}\}$ and $\mathcal{N}_{\text{b}}$ denote the set of agents and anchors, respectively and let $\mathcal{T} = \{t_n\}_{n=1,2,\ldots,N}$ denote the time slot set of interest. The position of node k at time t_n is denoted by $\mathbf{p}_k^{(n)}$. Moreover, let $\phi_{jk}^{(n)}$ and $d_{jk}^{(n)}$ denote the angle and distance from node k to node j at time t_n, respectively.

At each time slot, agents aim to determine their positions based on range measurements to neighboring nodes. Let $x_{jk}^{(n)}$ denote the transmit energy of the ranging signal from node $k \in \mathcal{N}_{\text{a}}$ to node $j \in \mathcal{N}_{\text{a}} \cup \mathcal{N}_{\text{b}}$ at time n. Let $\mathbf{x}^{(n)} = \{x_{jk}^{(n)}\}_{k \in \mathcal{N}_{\text{a}}, j \in \mathcal{N}_{\text{a}} \cup \mathcal{N}_{\text{b}} \setminus \{j\},}$ denote the energy allocation strategy set that consists of $N_{\text{a}}(N_{\text{b}} + N_{\text{a}} - 1)$ variables. Let $\hat{\mathbf{p}}^{(n)} = \{\hat{\mathbf{p}}_k^{(n)}\}_{k \in \mathcal{N}_{\text{a}}}$ denote the agent position estimates at time t_n, respectively.

In network navigation, the energy allocation and position estimation can be interactive. In specific, the energy allocation strategy $\mathbf{x}^{(n)}$ may be determined based on the position estimate $\hat{\mathbf{p}}^{(n_0)}(0 \leq n_0 \leq n)$. Similarly, the position estimates may depend on the energy allocation policies $\mathbf{x}^{(n_0)}(0 \leq n_0 \leq n)$. In this way, the joint energy allocation and position estimation algorithms can run iteratively in the navigation process.

B. Measurement and Dynamic Model

Let $z_{jk}^{(n)}$ denote the measurement made by node k to node j, modeled as

$$z_{jk}^{(n)} = d_{jk}^{(n)} + w_{jk}^{(n)}$$

where $w_{jk}^{(n)}$ is the range error, modeled as a zero-mean Gaussian random variable (RV) with variance given by $\left(x_{jk}^{(n)}\xi_{jk}^{(n)}\right)^{-1}$, in which $\xi_{jk}^{(n)}$ is the equivalent ranging coeficient (ERC) that characterizes the channel quality between node k and node j [41]. Moreover, we assume that $w_{jk}^{(n)}$ is independent over time t_n.

Agents' positions are described as deterministic unknown variables, where no prior dynamic knowledge of the agent's position distribution is available. Let $\mathbf{z}^{(n)}$ denote the set consisting of range measurements at time t_n and the likelihood function for agents' positions can then be written as

$$f\left(\mathbf{z}^{(n)}|\mathbf{p}^{(n)}\right) = \prod_{k\in\mathcal{N}_\mathrm{a}} \prod_{j\in\mathcal{N}_\mathrm{a}\cup\mathcal{N}_\mathrm{b}\setminus\{k\}} f\left(z_{jk}^{(n)}\,|\,\mathbf{p}_j^{(n)}, \mathbf{p}_k^{(n)}\right) \tag{1}$$

where the likelihood function for the nodes' positions with respect to $z_{jk}^{(n)}$ can then be given as follows[3]

$$f\left(z_{jk}^{(n)}|\mathbf{p}_j^{(n)}, \mathbf{p}_k^{(n)}\right) = \begin{cases} 1, & x_{jk}^{(n)} = 0 \\ \sqrt{\frac{x_{jk}^{(n)}\xi_{jk}^{(n)}}{2\pi}}\exp\left\{-\frac{x_{jk}^{(n)}\xi_{jk}^{(n)}\left(z_{jk}^{(n)}-d_{jk}^{(n)}\right)^2}{2}\right\}, & \text{otherwise.}\end{cases}$$

The agents' speed are assumed to be upper bounded by a constant number during the navigation process. Therefore, for each agent, the distance between its positions in two consecutive time instants is upper bounded by a constant, denoted by the step size Δ, i.e.,

$$\left\|\mathbf{p}_k^{(n+1)} - \mathbf{p}_k^{(n)}\right\| \leq \Delta \tag{2}$$

for $k \in \mathcal{N}_\mathrm{a}$ and $n = 1, 2, \cdots, N-1$.

C. Performance Metric and Problem Formulation

The performance of the position estimator can be quantified by the mean squared error (MSE) of the position estimate, given by

$$\sum_{n=1}^{N}\sum_{k\in\mathcal{N}_\mathrm{a}} \mathbb{E}\left\{\left\|\mathbf{p}_k^{(n)} - \hat{\mathbf{p}}_k^{(n)}\right\|^2\right\}.$$

Note that the MSE depends on the agents' true positions, the energy allocation strategy, and the position estimation methods. The latter two factors can be designed so that the MSE is minimized. Hence, the joint energy allocation and position estimation problem in the *centralized setting* is

$$\mathscr{P}: \min_{\{\mathbf{x}^{(n)},\hat{\mathbf{p}}^{(n)}\}} \quad \sum_{n=1}^{N}\sum_{k\in\mathcal{N}_\mathrm{a}} \mathbb{E}\left\{\left\|\mathbf{p}_k^{(n)} - \hat{\mathbf{p}}_k^{(n)}\right\|^2\right\}$$

$$\text{s.t.} \quad x_{jk}^{(n)} \geq 0, \qquad \forall\, t_n \in \mathcal{T} \tag{3}$$

$$\sum_{k\in\mathcal{N}_\mathrm{a}}\sum_{j\in\mathcal{N}_\mathrm{a}\cup\mathcal{N}_\mathrm{b}\setminus\{k\}} x_{jk}^{(n)} \leq X_\mathrm{tot}, \forall\, t_n \in \mathcal{T} \tag{4}$$

where (3) is the nonnegative constraint for the energy and (4) denotes the total transmit energy constraint, which X_tot is the amount of total available energy.

Note that in the *distributed setting*, constraint (4) is replaced by the individual energy constraints

$$\sum_{j\in\mathcal{N}_\mathrm{b}} x_{jk}^{(n)} \leq X_\mathrm{anc}^k, \qquad \forall\, t_n \in \mathcal{T} \quad \forall\, k \in \mathcal{N}_\mathrm{a},$$
$$\sum_{j\in\mathcal{N}_\mathrm{a}\setminus\{k\}} x_{jk}^{(n)} \leq X_\mathrm{agt}^k, \qquad \forall\, t_n \in \mathcal{T} \quad \forall\, k \in \mathcal{N}_\mathrm{a},$$

where X_anc^k and X_agt^k are the total energy associated with agent k for anchor transmission and agent transmission, respectively. Moreover, in the distributed setting, each node has only estimates of local network parameters. This restriction further reduces the feasible set of the inference and control policies.

III. Centralized Network Navigation

This section presents the robust network navigation algorithm in the centralized setting, where both noncooperative and cooperative networks are considered.

A. Uncertainty Model

We first consider the noncooperative network navigation, where the range measurements are made only between anchors and agents. For the ease of exposition, we focus on one of the agents, denoted as agent k.

In the navigation process, the agent maintains its positional information, including the estimated position $\hat{\mathbf{p}}_k^{(n)}$ and its confidence region. The confidence region is defined to be the area in which $\mathbf{p}_k^{(n)}$ belongs to with high confidence. Let $\mathcal{U}_k^{(n)}$ and $\widetilde{\mathcal{U}}_k^{(n)}$ denote the confidence region of agent k before and after the position estimation at time t_n, respectively. The confidence region $\mathcal{U}_k^{(n)}$ depends on the confidence region $\widetilde{\mathcal{U}}_k^{(n-1)}$ and the step size Δ, described as follows.

Consider the maximum likelihood estimation in high signal-to-noise ration (SNR) regimes. The MLE $\hat{\mathbf{p}}_k^{(n-1)}$ can be approximated as a Gaussian random vector with mean $\mathbf{p}_k^{(n-1)}$ and covariance matrix $\mathbf{J}^{-1}\left(\mathbf{p}_k^{(n-1)}\right)$ [43], where $\mathbf{J}\left(\mathbf{p}_k^{(n-1)}\right)$ is the Fisher information matrix (FIM) for $\mathbf{p}_k^{(n-1)}$, given by [31]

$$\mathbf{J}\left(\mathbf{p}_k^{(n-1)}\right) = \sum_{j\in\mathcal{N}_\mathrm{b}} x_{jk}^{(n-1)}\xi_{jk}^{(n-1)}\mathbf{J}_\mathrm{r}\left(\phi_{jk}^{(n-1)}\right).$$

[3]In the rest of this paper, the shorthand $f\left(z_{jk}^{(n)}\,|\,\mathbf{p}_j^{(n)}\right)$ will be used for $f\left(z_{jk}^{(n)}\,|\,\mathbf{p}_j^{(n)}, \mathbf{p}_k^{(n)}\right)$ when $j \in \mathcal{N}_\mathrm{b}$.

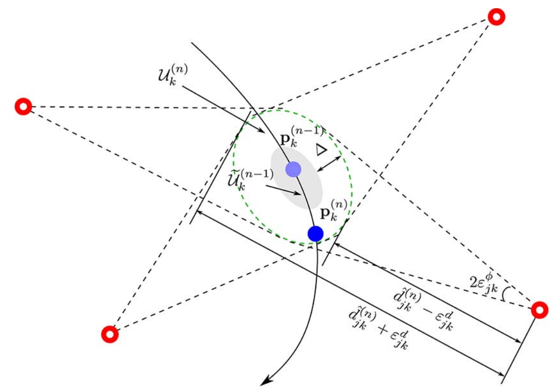


Fig. 2. Illustration of the confidence region in network navigation: the position of agent k, $\mathbf{p}_k^{(n-1)}$, falls into grey ellipse with high confidence, and $\mathbf{p}_k^{(n)}$ falls into green ellipse with high confidence.

Hence, the probability that $\mathbf{p}_k^{(n-1)}$ belongs to the confidence region $\widetilde{\mathcal{U}}_k^{(n-1)}$ is

$$\mathbb{P}\{\mathbf{p}_k^{(n-1)} \in \widetilde{\mathcal{U}}_k^{(n-1)}\} = 1 - \exp(-c/2)$$

where

$$\widetilde{\mathcal{U}}_k^{(n-1)} = \left\{\mathbf{p} : \left(\mathbf{p} - \hat{\mathbf{p}}_k^{(n-1)}\right)^{\mathrm{T}} \mathbf{J}\left(\mathbf{p}_k^{(n-1)}\right)\left(\mathbf{p} - \hat{\mathbf{p}}_k^{(n-1)}\right) \le c\right\} \quad (5)$$

in which c is a positive number that determines the confidence level. For example, in Section V, we consider the confidence level to be 0.95, in which case $c = -2\ln 0.05$. Due to the movement of agent k, the confidence region at time t_n before position estimation is

$$\mathcal{U}_k^{(n)} = \widetilde{\mathcal{U}}_k^{(n-1)} + \{\boldsymbol{\delta} : \|\boldsymbol{\delta}\| \le \Delta\}. \quad (6)$$

The confidence region $\mathcal{U}_k^{(n)}$ then leads to the uncertainty set of the angles and distances as follows

$$\phi_{jk}^{(n)} \in \left[\,\hat{\phi}_{jk}^{(n)} - \varepsilon_{jk}^{\phi}\,,\ \hat{\phi}_{jk}^{(n)} + \varepsilon_{jk}^{\phi}\,\right] \quad (7)$$

$$d_{jk}^{(n)} \in \left[\,\hat{d}_{jk}^{(n)} - \varepsilon_{jk}^{d}\,,\ \hat{d}_{jk}^{(n)} + \varepsilon_{jk}^{d}\,\right] \quad (8)$$

where $\hat{\phi}_{jk}^{(n)}$ and $\hat{d}_{jk}^{(n)}$ denote the angles and distances corresponding to the position $\hat{\mathbf{p}}_k^{(n)}$, and ε_{jk}^{ϕ} and ε_{jk}^{d} denote the maximum uncertainty of the angles and distances, respectively.[4] Fig. 2 shows an example of the confidence region and the uncertainty set of angles and distances.

B. Robust Energy Allocation

The MSE of the position estimate $\hat{\mathbf{p}}_k^{(n)}$ is often intractable. Thus we consider the CRLB for the MSE of the agent k,[5] given by

$$\mathcal{P}\left(\mathbf{p}_k^{(n)}\right) := \mathrm{tr}\left\{\mathbf{J}^{-1}\left(\mathbf{p}_k^{(n)}\right)\right\}.$$

Then the goal of energy allocation operation at time t_n is to minimize the one-time worst-case CRLB in the confidence region, i.e.,

$$\begin{aligned}
\min_{\left\{x_{jk}^{(n)}\right\}} \quad & \max_{\mathbf{p}_k^{(n)} \in \mathcal{U}_k^{(n)}} \mathcal{P}\left(\mathbf{p}_k^{(n)}\right) \\
\text{s.t.} \quad & x_{jk}^{(n)} \ge 0, \qquad j \in \mathcal{N}_{\mathrm{b}} \qquad (9) \\
& \sum_{j \in \mathcal{N}_{\mathrm{b}}} x_{jk}^{(n)} \le X_{\mathrm{tot}}. \qquad (10)
\end{aligned}$$

It is shown in [40] that the worst-case CRLB with respect to angular uncertainty can be bounded by as follows

$$\max_{\mathbf{p}_k^{(n)} \in \mathcal{U}_k^{(n)}} \mathcal{P}\left(\mathbf{p}_k^{(n)}\right) \le \mathrm{tr}\left\{\mathbf{Q}^{-1}\left(\hat{\mathbf{p}}_k^{(n)}\right)\right\}$$

provided that $\mathbf{Q}(\hat{\mathbf{p}}_k^{(n)}) \succeq 0$, where

$$\mathbf{Q}\left(\hat{\mathbf{p}}_k^{(n)}\right) = \sum_{j \in \mathcal{N}_{\mathrm{b}}} x_{jk}\xi_{jk}\left(\mathbf{J}_{\mathrm{r}}(\hat{\phi}_{jk}) - |\sin\varepsilon_{jk}^{\phi}|\,\mathbf{I}\right).$$

The robust energy allocation problem then becomes

$$\begin{aligned}
\mathscr{P}_{\mathrm{R}}^{(n)} : \min_{\{x_{jk}^{(n)}\}} \quad & \mathrm{tr}\left\{\mathbf{Q}^{-1}\left(\hat{\mathbf{p}}_k^{(n)}\right)\right\} \\
\text{s.t.} \quad & (9) \text{ and } (10)
\end{aligned}$$

which can be transformed into an SDP and hence solved efficiently by optimization engine [40].

C. Position Estimation Method

At each time slot t_n, the position estimate $\hat{\mathbf{p}}_k^{(n)}$ is determined by the range measurements $\left\{z_{jk}^{(n)}\right\}_{j\in\mathcal{N}_{\mathrm{b}}}$, or equivalently, by the likelihood function $f\left(\left\{z_{jk}^{(n)}\right\}_{j\in\mathcal{N}_{\mathrm{b}}} | \mathbf{p}_k^{(n)}\right)$.

The MLE is asymptotically efficient, i.e., it can achieve the CRLB for the MSE when the SNR tends to infinity. Therefore, we try to find the MLE of the measurement model (1), i.e., solve the following optimization problem

$$\begin{aligned}
\hat{\mathbf{p}}_k^{*} &= \arg\max_{\mathbf{p}_k^{(n)} \in \mathcal{R}} \log f\left(\left\{z_{jk}^{(n)}\right\}_{j\in\mathcal{N}_{\mathrm{b}}} | \mathbf{p}_k^{(n)}\right) \\
&= \arg\min_{\mathbf{p}_k^{(n)} \in \mathcal{R}} \sum_{j\in\mathcal{N}_{\mathrm{b}}} x_{jk}^{(n)}\xi_{jk}^{(n)}\left(z_{jk}^{(n)} - d_{jk}^{(n)}\right)^2 \qquad (11)
\end{aligned}$$

which is a non-linear least square problem. Such a problem has many mature solvers, for example, the Levenberg-Marquardt method, the Gauss-Newton method, and the method of gradient descent [46]. The performance of the MLE solver largely depends on the initial value. In the navigation process, the

[4]Note that the uncertainty in distance leads to the upper and lower bounds for the ERC ξ, which can be easily treated in the robust strategy design. We omit detailed discussion for brevity and readers can refer to [40]–[42] for more information.

[5]The CRLB is tight for high SNR scenarios and hence an important criteria to evaluate the localization accuracy in the navigation process.

initial value is set to be the estimated positions of agents at the previous time.

Algorithm 1 Noncooperative Network Navigation with Optmized Energy Allocation

Input: $\{\mathbf{p}_j^{(n)}\}_{j\in\mathcal{N}_\mathrm{b}}$, $\hat{\mathbf{p}}^{(0)}$ and $\{\mathcal{U}_k^{(0)}\}_{k\in\mathcal{N}_\mathrm{a}}$.
Output: $\{x_{jk}^{(n)}\}_{j\in\mathcal{N}_\mathrm{b}}$ and $\hat{\mathbf{p}}_k^{(n)}$, $1\le n\le T$.
1: **while** $n \le T$ **do**
2: Estimate the ERCs $\xi_{jk}^{(n)}$, $j\in\mathcal{N}_\mathrm{b}$;
3: Find the confidence region $\mathcal{U}_k^{(n)}$ according to (6);
4: Determine the uncertainty set according to (7) and (8);
5: Solve the SDP $\mathscr{P}_\mathrm{R}^{(n)}$ and obtain the energy allocation strategy $\{x_{jk}^{(n)}\}$;
6: Make range measurements corresponding to the transmit energy $\{x_{jk}^{(n)}\}$;
7: Determine the MLE $\hat{\mathbf{p}}_k^{(n)}$ by solving the non-linear least square problem (11);
8: Update the confidence region $\widetilde{\mathcal{U}}_k^{(n)}$;
9: $n \leftarrow n+1$;
10: **end while**

Alternatively, one can resort to particle methods since the dimension of the unknown parameter $\mathbf{p}_k^{(n)}\in\mathbb{R}^2$ is tractable. The true distribution of the agent can be approximated by a collection of two-dimensional samples and the sample with the highest weight is selected as the estimation of the agent's position.

At time t_n, let $\{\boldsymbol{\xi}^{(i)}\}_{i=1}^{N_\mathrm{s}}$ denote N_s samples and let $\{w^{(i)}\}_{i=1}^{N_\mathrm{s}}$ denote the associated weights, given by

$$w^{(i)} = \sum_{j\in\mathcal{N}_\mathrm{b}} x_{jk}^{(n)}\xi_{jk}^{(n)}\left(z_{jk}^{(n)} - \left\|\boldsymbol{\xi}^{(i)} - \mathbf{p}_j\right\|\right)^2$$

then the MLE for $\mathbf{p}_k^{(n)}$ is approximated by

$$\hat{\mathbf{p}}_k^{(n)} = \underset{\{\boldsymbol{\xi}^{(i)}\}_{1\le i\le N_\mathrm{s}}}{\arg\min}\ w^{(i)}.$$

The performance of the particle methods depends on how the samples are generated. Here, the samples can be deployed either randomly or deterministically at the confidence region.

Remark 1: The non-linear least square methods may converge to a local optimum and results in an biased position estimate. On the other hand, the particle method does not have this issue, but its performance relies on the number of particles, leading to large computation complexity if the dimension of unknown parameters is large.

After obtaining the position estimate of $\hat{\mathbf{p}}_k^{(n)}$, the positional information of agent k should be updated and such information will serve as input for the energy allocation in the next time slot. Details of the navigation algorithm with optimized energy allocation is given in Algorithm 1.

D. Extension to Cooperative Network Navigation

In cooperative network navigation, the network FIM $\mathbf{J}\big(\mathbf{p}^{(n-1)}\big)$ is given by (12), shown at the bottom of the page, where

$$\mathbf{J}^\mathrm{A}\big(\mathbf{p}_k^{(n-1)}\big) = \sum_{j\in\mathcal{N}_\mathrm{b}} x_{jk}^{(n-1)}\xi_{jk}^{(n-1)}\mathbf{J}_\mathrm{r}\big(\phi_{jk}^{(n-1)}\big)$$
$$\mathbf{C}_{kj} = \big(x_{kj}^{(n-1)}\xi_{kj}^{(n-1)} + x_{jk}^{(n-1)}\xi_{jk}^{(n-1)}\big)\mathbf{J}_\mathrm{r}\big(\phi_{jk}^{(n-1)}\big).$$

Therefore, the confidence regions of the agents are highly correlated and complicated for joint evaluation. To address this issue, we decouple the confidence region for each agent by equivalent Fisher information analysis. In particular, the equivalent Fisher information matrix (EFIM) for $\mathbf{p}_k^{(n-1)}$ is given by [33]

$$\mathbf{J}_\mathrm{e}\big(\mathbf{p}_k^{(n-1)}\big) = \Big(\big[\mathbf{J}^{-1}\big(\mathbf{p}^{(n-1)}\big)\big]_{\mathbf{p}_k}\Big)^{-1} \tag{13}$$

where $[\mathbf{A}]_{\mathbf{p}_k}$ denotes the kth 2×2 submatrix on the diagonal of $\mathbf{A}$. Consequently, the confidence region $\widetilde{\mathcal{U}}_k^{(n-1)}$ for agent k can be obtained by replacing $\mathbf{J}\big(\mathbf{p}_k^{(n-1)}\big)$ with $\mathbf{J}_\mathrm{e}\big(\mathbf{p}_k^{(n-1)}\big)$ in (5) and then $\mathcal{U}_k^{(n)}$ can be obtained by (6) accordingly.

The uncertainty set of the angles and distances can be similarly obtained as (7) and (8). Note that compared to the noncooperative case, the uncertainty of the angles and distances among agents needs to be considered.

We next extend the result in [40] to the cooperative network in the centralized setting, providing the upper bound for the worst-case network CRLB.

Proposition 1: Under the uncertainty model (7) and (8), the worst-case CRLB can be upper bounded by

$$\max_{\{\mathbf{p}_k^{(n)}\in\mathcal{U}_k^{(n)}\}_{k\in\mathcal{N}_\mathrm{a}}} \sum_{k\in\mathcal{N}_\mathrm{a}} \mathcal{P}\big(\mathbf{p}_k^{(n)}\big) \le \mathrm{tr}\left\{\mathbf{Q}^{-1}\big(\hat{\mathbf{p}}^{(n)}\big)\right\}$$

in which

$$\mathbf{Q}\big(\hat{\mathbf{p}}^{(n)}\big) = \sum_{k\in\mathcal{N}_\mathrm{a}}\ \sum_{j\in\mathcal{N}_\mathrm{a}\cup\mathcal{N}_\mathrm{b}\setminus\{k\}} x_{jk}^{(n)}\xi_{jk}^{(n)}\,\mathbf{V}_{jk}^{(n)}$$

where

$$\mathbf{V}_{jk}^{(n)} = \begin{cases} \big(\mathbf{e}_k\,\mathbf{e}_k^\mathrm{T}\big)\otimes\Big(\mathbf{J}_\mathrm{r}\big(\hat{\phi}_{jk}^{(n)}\big) - \delta_{jk}^{(n)}\mathbf{I}\Big), & j\in\mathcal{N}_\mathrm{b}\\ (\mathbf{e}_k - \mathbf{e}_j)(\mathbf{e}_k - \mathbf{e}_j)^\mathrm{T}\otimes\Big(\mathbf{J}_\mathrm{r}\big(\hat{\phi}_{jk}^{(n)}\big) - \delta_{jk}^{(n)}\mathbf{I}\Big), & \text{otherwise}\end{cases}$$

$$\mathbf{J}\big(\mathbf{p}^{(n-1)}\big) = \begin{bmatrix} \mathbf{J}^\mathrm{A}\big(\mathbf{p}_1^{(n-1)}\big) + \sum_{j\in\mathcal{N}_\mathrm{a}\setminus\{1\}}\mathbf{C}_{1,j} & -\mathbf{C}_{1,2} & \cdots & -\mathbf{C}_{1,N_\mathrm{a}}\\ -\mathbf{C}_{2,1} & \mathbf{J}^\mathrm{A}\big(\mathbf{p}_2^{(n-1)}\big) + \sum_{j\in\mathcal{N}_\mathrm{a}\setminus\{2\}}\mathbf{C}_{2,j} & & -\mathbf{C}_{2,N_\mathrm{a}}\\ \vdots & & \ddots & \\ -\mathbf{C}_{N_\mathrm{a},1} & -\mathbf{C}_{N_\mathrm{a},2} & & \mathbf{J}^\mathrm{A}\big(\mathbf{p}_{N_\mathrm{a}}^{(n-1)}\big) + \sum_{j\in\mathcal{N}_\mathrm{a}\setminus\{N_\mathrm{a}\}}\mathbf{C}_{N_\mathrm{a},j} \end{bmatrix} \tag{12}$$

in which $\delta_{jk}^{(n)} = |\sin \varepsilon_{jk}^{\phi}|$, provided that $\mathbf{Q}(\hat{\mathbf{p}}^{(n)}) \succeq 0$.

Proof: See Appendix A. □

The centralized energy allocation problem is to minimize the upper bound for the worst-case CRLB as

$$\mathscr{P}_{\text{R-C}}^{(n)} : \min_{\mathbf{x}^{(n)}} \quad \text{tr}\Big\{\mathbf{Q}^{-1}\big(\hat{\mathbf{p}}^{(n)}\big)\Big\}$$

$$\text{s.t.} \quad x_{jk}^{(n)} \geq 0, \quad k \in \mathcal{N}_{\text{a}}, j \in \mathcal{N}_{\text{b}} \cup \mathcal{N}_{\text{a}} \tag{14}$$

$$\sum_{k\in\mathcal{N}_{\text{a}}} \sum_{j\in\mathcal{N}_{\text{a}}\cup\mathcal{N}_{\text{b}}\setminus\{k\}} x_{jk}^{(n)} \leq X_{\text{tot}}. \tag{15}$$

Using the same technique in [40], one can show that the problem $\mathscr{P}_{\text{R-C}}^{(n)}$ can be transformed into the following SDP:

$$\mathscr{P}_{\text{R-C-SDP}}^{(n)} : \min_{\{x_{jk}^{(n)}\}} \quad \text{tr}\{\mathbf{M}\}$$

$$\text{s.t.} \quad \begin{bmatrix} \mathbf{M} & \mathbf{I} \\ \mathbf{I} & \mathbf{Q}\big(\hat{\mathbf{p}}^{(n)}\big) \end{bmatrix} \succeq 0$$

$$(14) \text{ and } (15).$$

With the range measurements $\mathbf{z}^{(n)}$, one can find the MLE of agents' positions as

$$\begin{aligned}\hat{\mathbf{p}}^* &= \arg\max_{\mathbf{p}^{(n)}} \log f\big(\mathbf{z}^{(n)}|\mathbf{p}^{(n)}\big) \\ &= \arg\min_{\mathbf{p}^{(n)}} \sum_{k\in\mathcal{N}_{\text{a}}} \sum_{j\in\mathcal{N}_{\text{a}}\cup\mathcal{N}_{\text{b}}\setminus\{k\}} x_{jk}^{(n)} \xi_{jk}^{(n)} \big(z_{jk}^{(n)} - d_{jk}^{(n)}\big)^2\end{aligned}$$

which is a non-linear least squares problem. Note that the MLE for agents' positions in cooperative network navigation has a higher dimension of unknown parameters, and hence the particle methods proposed in Section III-C may not be amenable due to computation complexity.

Remark 2: In the centralized setting, solving the energy allocation problem and determining the MLE for agents require knowing the ERCs and the confidence regions for all the agents in the network, which is usually impractical to fufill, especially when the network is large and ad-hoc. Hence, the distributed strategies are developed in the next section.

E. Complexity Analysis

The computation complexity of the proposed centralized algorithm consists of two parts: solving $\mathscr{P}_{\text{R-C-SDP}}^{(n)}$ and determining $\hat{\mathbf{p}}^*$. Using the existing optimization engines, the SDP $\mathscr{P}_{\text{R-C-SDP}}^{(n)}$ can be solved in $\mathcal{O}\big(\sqrt{N_{\text{a}}N_{\text{b}} + N_{\text{a}}^2}\big)$ iterations and the amount of work per iteration is $\mathcal{O}\big((N_{\text{a}}N_{\text{b}} + N_{\text{a}}^2)^3\big)$, and the worst-case complexity of solving $\mathscr{P}_{\text{R-C-SDP}}^{(n)}$ is $\mathcal{O}\big(N_{\text{a}}^7 + N_{\text{a}}^{3.5}N_{\text{b}}^{3.5}\big)$ [47].

The computation complexity to determine $\hat{\mathbf{p}}^*$ is affected by many factors, e.g., the number of iterations, the initial point and the solution accuracy. Among these factors, the number of nonlinear items in the summation, denoted as L, plays an important role. For example, one of the widely used techniques, the Gauss-Newton method, is iterative and the asymptotic complexity in each iteration is $\mathcal{O}\big(L^3\big)$ [48]. The next proposition reveals the sparsity of the energy allocation, leading to an upper bound for L in the centralized setting for cooperative navigation.

Proposition 2: There exists an optimal solution $\{x_{kj}^*\}$ for $\mathscr{P}_{\text{R-C-SDP}}^{(n)}$ such that

$$\sum_{k\in\mathcal{N}_{\text{a}}} \sum_{j\in\mathcal{N}_{\text{a}}\cup\mathcal{N}_{\text{b}}\setminus\{k\}} \mathbb{I}\{x_{jk}^*\} \leq N_{\text{a}}(N_{\text{a}} + 3)$$

provided that $\mathscr{P}_{\text{R-C-SDP}}^{(n)}$ has a feasible solution.

Proof: See Appendix C. □

Proposition 2 is an extension of the sparsity property for non-cooperative networks discovered in [49]. Such sparsity property implies that $L \leq N_{\text{a}}(N_{\text{a}} + 3)$ and that the computation complexity of the MLE does not depend the number of anchors.

IV. DISTRIBUTED COOPERATIVE NAVIGATION

This section presents the robust network navigation algorithm in the distributed setting, where each agent has only estimates of local network parameters.

A. Phase Decomposition

Note that in the cooperative case, the CRLB for the MSE of $\hat{\mathbf{p}}_k^{(n)}$ is

$$\mathcal{P}\big(\mathbf{p}_k^{(n)}\big) = \text{tr}\big\{\mathbf{J}_{\text{e}}^{-1}\big(\mathbf{p}_k^{(n)}\big)\big\}$$

and thus optimizing $\mathcal{P}(\mathbf{p}_k^{(n)})$ requires knowing the energy allocation strategies of other agents, which are also to be optimized. Moreover, the likelihood function for agent k is

$$\begin{aligned}&f\big(\{z_{jk}^{(n)}\}_{j\in\mathcal{N}_{\text{a}}\cup\mathcal{N}_{\text{b}}\setminus\{k\}}|\mathbf{p}_k^{(n)}\big) \\ &\quad = \prod_{j\in\mathcal{N}_{\text{b}}} f\big(z_{jk}^{(n)}|\mathbf{p}_k^{(n)}\big) \times \prod_{j\in\mathcal{N}_{\text{a}}\setminus\{k\}} f\big(z_{jk}^{(n)}\,|\,\mathbf{p}_k^{(n)}\big)\end{aligned}$$

which depends on other agents' positions that are unknown to agent k. Therefore, the method proposed in Section III-D cannot be used directly in the distributed setting.

To circumvent the difficulties above, the original problem is decomposed into a sequential two-phase problem. In the first phase (infrastructure phase), each agent k determines the transmitting energy from agent k to anchors, i.e., $\{x_{jk}^{(n)}\}_{j\in\mathcal{N}_{\text{b}}}$, and then make range measurements to anchors and update confidence region. In the second phase (cooperation phase), based on the range measurements made in the first phase, each agent k determines its transmitting energy to neighboring agents, i.e., $\{x_{jk}^{(n)}\}_{j\in\mathcal{N}_{\text{a}}\setminus\{k\}}$, and then make range measurements. Note that with this decomposition, the FIM for agent k at time t_{n-1} is different from that in the centralized setting, given by $\mathbf{J}\big(\mathbf{p}^{(n-1)}\big)$ with $x_{ij}^{(n-1)} = 0$ for $k \notin \{i, j\}$.

B. Infrastructure Phase

In the infrastructure phase, each agent k first determines the anchor energy allocation strategy and makes range measurements to anchors. It then estimates its position based on these range measurements. The energy allocation problem is then to minimize the worst-case CRLB for $\mathbf{p}_k^{(n)}$ with $x_{jk}^{(n)}=0$ for $j \in \mathcal{N}_\mathrm{a} \setminus \{k\}$, i.e.,

$$\mathscr{P}_{\text{R-D-anc}}^{(k)}: \min_{\{x_{jk}\}_{j\in\mathcal{N}_\mathrm{b}}} \max_{\mathbf{p}_k^{(n)}\in\mathcal{U}_k^{(n)}} \operatorname{tr}\left\{\left[\mathbf{J}^\mathrm{A}\left(\mathbf{p}_k^{(n)}\right)\right]^{-1}\right\}$$
$$\text{s.t.} \quad x_{jk}^{(n)} \geq 0, \qquad j\in\mathcal{N}_\mathrm{b}$$
$$\sum_{j\in\mathcal{N}_\mathrm{b}} x_{jk}^{(n)} \leq X_\mathrm{anc}^k$$

which can be solved by the methods proposed in [41]. The solution of $\mathscr{P}_{\text{R-D-anc}}^{(k)}$ is used as the anchor power control strategy for agent k to make the range measurements $\{z_{jk}^{(n)}\}_{j\in\mathcal{N}_\mathrm{b}}$. Then, the MLE of $\mathbf{p}_k^{(n)}$ is given by the non-linear least squares problem

$$\hat{\mathbf{p}}_{\mathrm{A},k}^{(n)} = \arg\min_{\mathbf{p}_k^{(n)}} \sum_{j\in\mathcal{N}_\mathrm{b}} x_{jk}^{(n)}\xi_{jk}^{(n)}\left(z_{jk}^{(n)} - \left\|\mathbf{p}_k^{(n)} - \mathbf{p}_j^{(n)}\right\|\right)^2. \tag{16}$$

With the range measurements in the first phase, the position estimate $\hat{\mathbf{p}}_k^{(n)}$ and confidence region $\mathcal{U}_k^{(n)}$ are updated to $\hat{\mathbf{p}}_{\mathrm{A},k}^{(n)}$ and $\widetilde{\mathcal{U}}_{\mathrm{A},k}^{(n)}$, respectively.

C. Cooperation Phase

In the cooperation phase, each agent k first determines the energy allocation strategy based on the positional information of its neighboring agents obtained in the infrastructure phase.

Proposition 3: The CRLB $\mathcal{P}_\mathrm{d}(\mathbf{p}_k^{(n)})$ is

$$\mathcal{P}_\mathrm{d}(\mathbf{p}_k^{(n)}) = \operatorname{tr}\left\{\left[\mathbf{J}^\mathrm{A}\left(\mathbf{p}_k^{(n)}\right) + \sum_{j\in\mathcal{N}_\mathrm{a}\setminus\{k\}} x_{jk}^{(n)}\chi_{jk}^{(n)}\mathbf{J}_\mathrm{r}\left(\phi_{jk}^{(n)}\right)\right]^{-1}\right\}$$

where

$$\chi_{jk}^{(n)} = \xi_{jk}^{(n)} \Big/ \left(1 + x_{jk}^{(n)}\xi_{jk}^{(n)}\mu_{jk}^{(n)}\right)$$
$$\mu_{jk}^{(n)} = \operatorname{tr}\left\{\left[\mathbf{J}^\mathrm{A}\left(\mathbf{p}_j^{(n)}\right)\right]^{-1}\cdot\mathbf{J}_\mathrm{r}\left(\phi_{jk}^{(n)}\right)\right\}.$$

Proof: Refer to [42] for the proof. □

We next present an upper bound for the worst-case CRLB $\mathcal{P}_\mathrm{d}(\mathbf{p}_k^{(n)})$ over confidence regions as follows.

Proposition 4: For a sufficiently large integer M and $\mathcal{M} = \{0, 1, 2, \ldots, M-1\}$,

$$\max_{\left\{\mathbf{p}_j\in\widetilde{\mathcal{U}}_{\mathrm{A},j}^{(n)}\right\}_{j\in\mathcal{N}_\mathrm{a}}} \mathcal{P}_\mathrm{d}(\mathbf{p}_k^{(n)}) \leq \overline{\mathcal{P}}^M(\mathbf{p}_k^{(n)}) := \max_{m\in\mathcal{M}} \frac{4\cdot f_k^{(n)}}{\left(f_k^{(n)}\right)^2 - \left(g_{k,m}^{(n)}\right)^2}$$

where

$$f_k^{(n)} = \sum_{j\in\mathcal{N}_\mathrm{b}} \xi_{jk}^{(n)}x_{jk}^{(n)} + \sum_{j\in\mathcal{N}_\mathrm{a}\setminus\{k\}} \xi_{jk}^{(n)}y_{jk}^{(n)}$$
$$g_{k,m}^{(n)} = \sum_{j\in\mathcal{N}_\mathrm{b}} h_{jk,m}^{(n)}\xi_{jk}^{(n)}x_{jk}^{(n)} + \sum_{j\in\mathcal{N}_\mathrm{a}\setminus\{k\}} h_{jk,m}^{(n)}\xi_{jk}^{(n)}y_{jk}^{(n)}$$

in which

$$y_{jk}^{(n)} = \frac{x_{jk}^{(n)}}{1 + x_{jk}^{(n)}\xi_{jk}^{(n)}\overline{\mu}_{jk}^{(n)}} \tag{17}$$
$$h_{jk,m}^{(n)} = \max_{|\varepsilon|\leq\varepsilon_{jk}^{\phi}} \frac{\cos\left(2\hat{\phi}_{kj}^{(n)} - (2m+1)\cdot\pi/M + \varepsilon\right)}{\cos\left(\pi/M\right)}$$
$$\overline{\mu}_{jk}^{(n)} = \max_{\left\{\left|\phi_{lj}^{(n)} - \hat{\phi}_{lj}^{(n)}\right|\leq\epsilon_{lj}^{\phi}\right\}_{l\in\{k\}\cup\mathcal{N}_\mathrm{b}}} \mu_{jk}^{(n)}.$$

Proof: See Appendix B. □

We can then obtain a relaxation problem for the agent energy allocation by adopting the above upper bound as the objective function, leading to

$$\mathscr{P}_{\text{R-D-agt}}^{(k)}: \min_{\left\{x_{jk}^{(n)}\right\}_{j\in\mathcal{N}_\mathrm{a}\setminus\{k\}}} \overline{\mathcal{P}}^M\left(\mathbf{p}_k^{(n)}\right)$$
$$\text{s.t.} \quad x_{jk}^{(n)} \geq 0, \quad j\in\mathcal{N}_\mathrm{a}\setminus\{k\} \tag{18}$$
$$\sum_{j\in\mathcal{N}_\mathrm{a}\setminus\{k\}} x_{jk}^{(n)} \leq X_\mathrm{agt}^k. \tag{19}$$

This problem can be transformed into a SOCP as shown in the next proposition.

Proposition 5: The problem $\mathscr{P}_{\text{R-D-agt}}^{(k)}$ is equivalent to the SOCP

$$\min_{\varrho,\left\{x_{jk}^{(n)},y_{jk}^{(n)}\right\}_{j\in\mathcal{N}_\mathrm{a}\setminus\{k\}}} -\varrho$$
$$\text{s.t.} \quad \left\|\left[2\varrho \;\; g_{k,m}^{(n)}\right]^\mathrm{T}\right\| \leq f_k^{(n)} - 2\varrho, \qquad \forall m\in M \tag{20}$$
$$\left\|\left[\sqrt{2} \;\; 1 - y_{jk}^{(n)}\xi_{jk}\overline{\mu}_{jk} \;\; 1 + x_{jk}^{(n)}\xi_{jk}\overline{\mu}_{jk}\right]^\mathrm{T}\right\| \leq 2 + \left(x_{jk}^{(n)} - y_{jk}^{(n)}\right)\xi_{jk}\overline{\mu}_{jk} \tag{21}$$
$$y_{jk}^{(n)} \geq 0, \qquad j\in\mathcal{N}_\mathrm{a}\setminus\{k\}$$
(18) and (19).

Proof: (Outline) First, we can replace the objective function with $1/\varrho$ and add a new constraint $\overline{\mathcal{P}}^M\left(\mathbf{p}_k^{(n)}\right) \leq 1/\varrho$, which is equivalent to the set of constraints (20). Second, due to the monotonicity of $y_{jk}^{(n)}$ in $x_{jk}^{(n)}$, one can verify that in $\mathscr{P}_{\text{R-D-agt}}^{(k)}$ the relationship (17) between $y_{jk}^{(n)}$ and $x_{jk}^{(n)}$ can be replaced by

$$0 \leq y_{jk}^{(n)} \leq \frac{x_{jk}^{(n)}}{1 + x_{jk}^{(n)}\xi_{jk}\overline{\mu}_{jk}^{(n)}}$$

where the second inequality can be transformed into the second-order cone (SOC) form (21). □

Algorithm 2 Distributed Network Navigation

Input: $\{\mathbf{p}_j^{(n)}\}_{j\in\mathcal{N}_\mathrm{b}}$, $\hat{\mathbf{p}}^{(0)}$ and $\{\mathcal{U}_k^{(0)}\}_{k\in\mathcal{N}_\mathrm{a}}$.
Output: $\mathbf{x}^{(n)}$ and $\hat{\mathbf{p}}^{(n)}$, $1 \le n \le T$.

```
while n ≤ T do
  //INFRASTRUCTURE PHASE
  for k ∈ N_a do
    Estimate the ERCs {ξ_jk^(n)}_{j∈N_b∪N_a\{k}};
    Determine the confidence region U_k^(n);
    Solve P_R-D-anc^(k);
    Make range measurements with {x_jk^(n)}_{j∈N_b};
    Determine the MLE p̂_A,k^(n) by solving (16);
    Update the confidence region Ũ_A,k^(n);
    Agent k broadcasts its position range measurements
    with respect to anchors {z_jk^(n)}_{j∈N_b};
  end for
  //COOPERATION PHASE
  for k ∈ N_a do
    Solve P_R-D-agt^(k);
    Make range measurements with {x_jk^(n)}_{j∈N_a\{k}};
    Determine the MLE p̂_k^(n) by solving (22);
    Update the confidence region Ũ_k^(n);
  end for
  n ← n + 1;
end while
```

Solving $\mathscr{P}_{\text{R-D-agt}}^{(k)}$ gives the agent energy allocation strategy of agent k to make range measurements. Together with the range measurements and position estimates of neighboring agents in the infrastructure phase, the MLE of $\mathbf{p}_k^{(n)}$ is

$$\hat{\mathbf{p}}_k^{(n)} = \arg\min_{\mathbf{p}_k^{(n)}} \Bigg[\sum_{j\in\mathcal{N}_\mathrm{b}} x_{jk}^{(n)} \xi_{jk}^{(n)} \big(z_{jk}^{(n)} - \|\mathbf{p}_k^{(n)} - \mathbf{p}_j^{(n)}\|\big)^2 + \sum_{j\in\mathcal{N}_\mathrm{a}\backslash\{k\}} x_{jk}^{(n)} \chi_{jk}^{(n)} \big(z_{jk}^{(n)} - \|\mathbf{p}_k^{(n)} - \hat{\mathbf{p}}_{\mathrm{A},j}^{(n)}\|\big)^2 \Bigg]. \quad (22)$$

Details of the distributed network navigation algorithm are given in Algorithm 2, where both energy allocation and position estimation involve only local network parameters.

D. Complexity Analysis

The computation complexity of the proposed distributed algorithm consists of two parts: solving $\mathscr{P}_{\text{R-D-anc}}^{(k)}$ and $\mathscr{P}_{\text{R-D-agt}}^{(k)}$ and determining $\hat{\mathbf{p}}_{\mathrm{A},k}^{(n)}$ and $\hat{\mathbf{p}}_k^{(n)}$. The complexity of solving $\mathscr{P}_{\text{R-D-anc}}^{(k)}$ (using the SOCP-based method in [41]) and $\mathscr{P}_{\text{R-D-agt}}^{(k)}$ depend on the choice of M. The worst-case complexity of solving $\mathscr{P}_{\text{R-D-anc}}^{(k)}$ and $\mathscr{P}_{\text{R-D-agt}}^{(k)}$ are $\mathcal{O}(M^{1.5}N_\mathrm{b}^{3.5})$ and $\mathcal{O}(M^{1.5}N_\mathrm{a}^{3.5})$, respectively.

The computation complexity to determine $\hat{\mathbf{p}}_{\mathrm{A},k}^{(n)}$ and $\hat{\mathbf{p}}_k^{(n)}$ largely depend on the number of nonlinear items in (16) and (22), respectively. For the infrastructure phase, the sparsity property is proved in [42], [49], showing that there no more than three nonlinear items in (16) as the uncertainty vanishes. However, such sparsity property do not always exist in the cooperation phase and thus the number of nonlinear items in (22) can be up to $(N_\mathrm{a} + N_\mathrm{b} - 1)$.

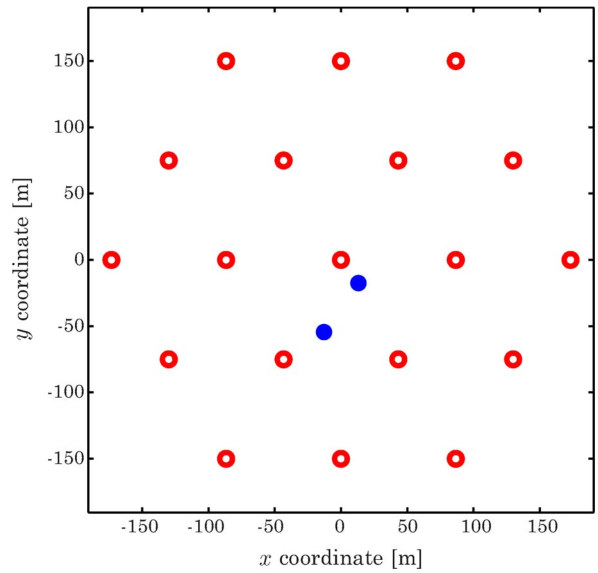


Fig. 3. The cooperative network: anchors (red circles) and agents (blue dots).

V. NUMERICAL RESULTS

This section evaluates the performance of the proposed energy-efficient navigation algorithms. The simulation setup is first described and the numerical results are then presented.

A. Network Setting and Algorithms

We consider a two-dimensional network where 19 anchors are placed in the vertices of equilateral triangles with circumradius of 50 meters (see Fig. 3). Initially, agents are uniformly distributed in the area of $[-100\text{ m}, 100\text{ m}] \times [-100\text{ m}, 100\text{ m}]$. The ERCs are obtained according to the formulas in [33]. The ranging signals with carrier frequency $f_\mathrm{c} = 2.1$ GHz and bandwidth of 100 MHz. The noise power density is -168 dBm/Hz. The ranging signal propagation model is [50]

$$\text{Path loss [dB]} = A + B\log_{10} d\ [\mathrm{m}] + C\log_{10} f_\mathrm{c}\ [\mathrm{GHz}].$$

We also consider $W \sim \mathcal{N}(0, \sigma^2)$ accounting for the shadow fading. The choice of A, B, C, and σ is generated according to the models of Indoor Hotspot (InH) and Urban Macro (UMa), corresponding to agent transmission and anchor transmission, respectively. Moreover, the extended typical urban model is used for the power dispersion profile [51]. For each of the agent, the total power of anchor transmission is 0.4 Watt and the total power of agent transmission is 1.6 Watt. The measured ERC $\hat{\xi}_{jk}^{(n)}$ is assumed to be a RV uniformly distributed in $[0.9\xi_{jk}^{(n)}, 1.1\xi_{jk}^{(n)}]$.

To evaluate the proposed robust algorithm, we compare its performance with three other algorithms. In all these algorithms, the localization part adopts the methods proposed Section III and Section IV.

- Uniform Algorithm: This algorithm adopts the uniform energy allocation strategy, which allocates the

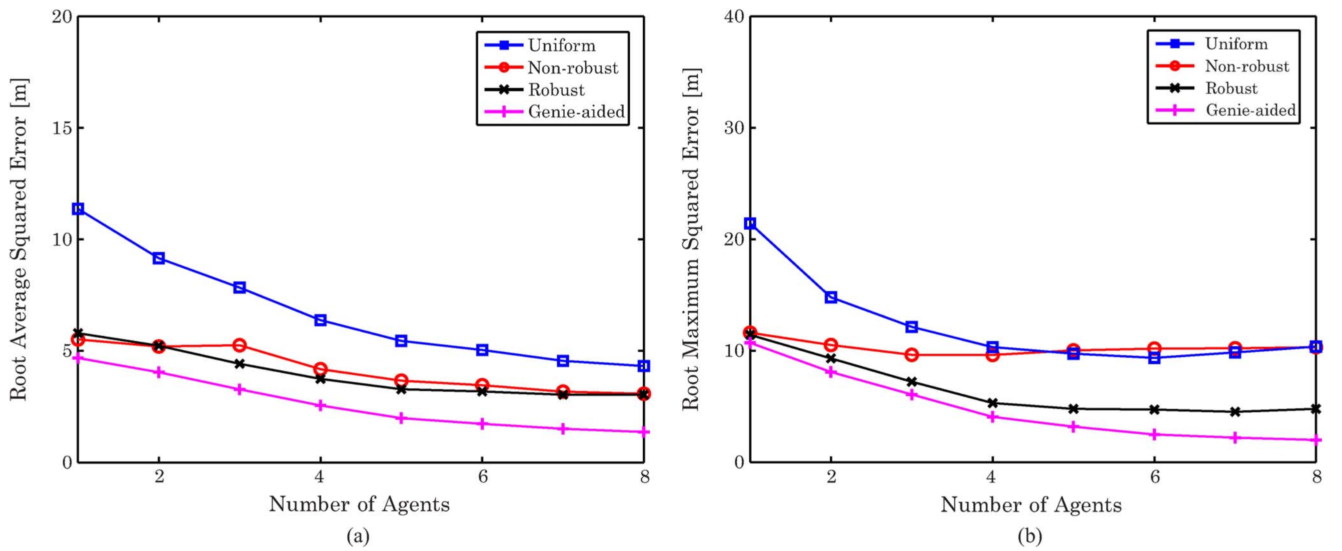


Fig. 4. Position error as a function of the number of agents in the centralized setting: (a) root average squared error of all the navigation trajectories; (b) root maximum squared error among all the navigation trajectories.

transmitting power to each link equally. In specific, for the centralized setting,

$$x_{jk}^{(n)} = \frac{X_{\mathrm{tot}}}{N_{\mathrm{b}}N_{\mathrm{a}} + N_{\mathrm{a}}(N_{\mathrm{a}}-1)}, \qquad \forall\, k, j$$

and for distributed setting,

$$\begin{aligned} x_{jk}^{(n)} &= X_{\mathrm{anc}}^{k}/N_{\mathrm{b}}, && j \in \mathcal{N}_{\mathrm{b}}, \quad k \in \mathcal{N}_{\mathrm{a}} \\ x_{kj}^{(n)} &= x_{jk}^{(n)} = X_{\mathrm{agt}}^{k}/(N_{\mathrm{a}}-1), && k, j \in \mathcal{N}_{\mathrm{a}}. \end{aligned}$$

- Non-robust Algorithm: This algorithm adopts the non-robust energy allocation strategy, which uses the agents' positions in the last time slot to compute the network parameters (e.g., angles, distances, and ERCs among agents), and assumes the uncertainties to be zeros. In specific, assume $d_{jk}^{(n+1)} = \hat{d}_{jk}^{(n)}$ and $\phi_{jk}^{(n+1)} = \hat{\phi}_{jk}^{(n)}$, and compute the energy allocation strategies accordingly.
- Genie-aided Algorithm: This algorithm adopts a genie-aided energy allocation strategy, which uses the agents' true position (provided by a genie) and the precise network parameters. This genie-aided algorithm provides a benchmark for evaluating the performance of the proposed algorithm.

B. Cooperative Network Navigation

We first consider a navigation network where agents move randomly. For any k and n, $\mathbf{p}_k^{(n)}$ is modeled as a 2-D RV that is uniformly distributed in a circular area with radius of $\Delta = 25$ m and the center $\mathbf{p}_k^{(n-1)}$. For each navigation process, the total number of time slots is set to be $N = 200$.

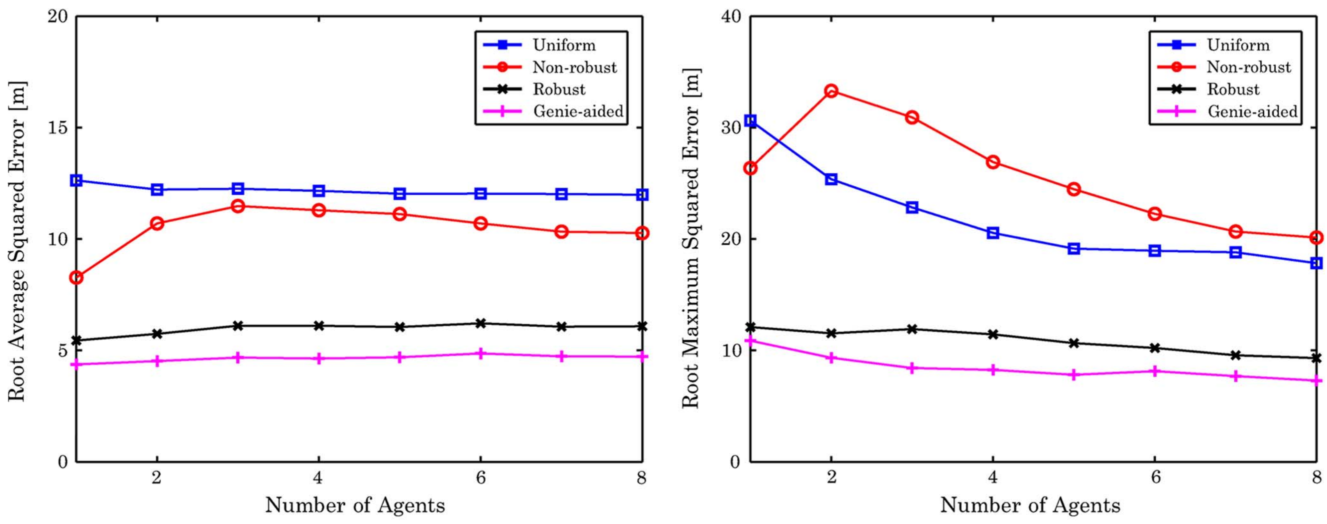
Fig. 4 plots the localization performance in the centralized setting. Fig. 4(a) and (b) show the root average and root maximum squared errors (among all the navigation trajectories) as a function of the number of agents, respectively. First, the robust algorithm significantly outperforms the uniform algorithm, reducing the root average squared errors by at least 30% and the root maximum squared errors by at least 40%. Second, the non-robust algorithm performs poorly in terms of the root maximum squared error (more than 10 meters). This shows the necessity of the robust energy allocation design that can guarantee the worst-case performance. Third, the root average squared errors typically decrease with the number of agent for every algorithm while the root maximum squared errors of the uniform algorithm and the non-robust algorithm do not show this trend. These different behaviors show that increasing the number of agents in a navigation network may lead to a better average localization performance though the robustness of the network may not be improved.

Fig. 5 plots the localization performance in the distributed setting. Fig. 5(a) and (b) show the root average and root maximum squared errors (among all the navigation trajectories) as a function of the number of the agents. First, the robust algorithm outperforms both the uniform and non-robust algorithms and the performance of the robust algorithm is close to that of the genie-aided algorithm. Second, the root average squared errors and root maximum squared errors are greater than those in the centralized setting, due to the restriction of the algorithm that each node has only estimates of local network parameters.

C. Effect of Step Size

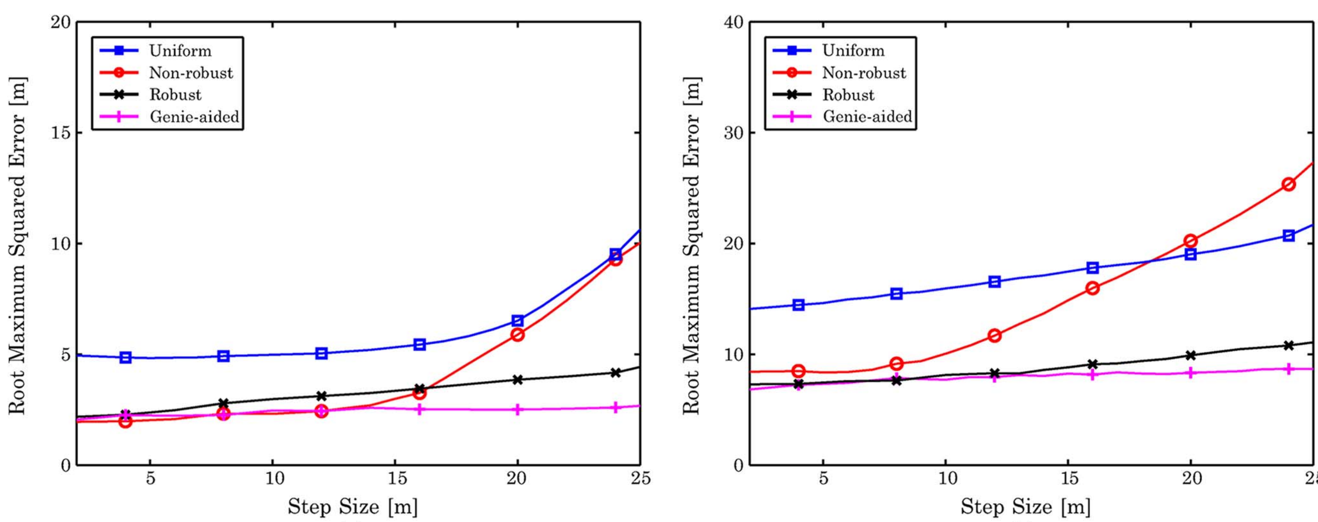
We then investigate the effect of the step size in network localization. For any k, $\mathbf{p}_k^{(1)}$ is modeled as a 2-D random variable that is uniformly distributed in a circular area with radius of Δ and the center is $\mathbf{p}_k^{(0)}$.

Fig. 6 plots the root maximum squared error averaged over all the sample points of $\mathbf{p}_k^{(1)}$. Fig. 6(a) and (b) plots the root maximum squared error as a function of the step size in the centralized and distributed setting, respectively. First, the robust algorithm outperforms the non-robust and the uniform algorithms. Compared to the genie-aided algorithm, the performance gap is not significant in the centralized setting for $\Delta \leq 16$ m and in the distributed setting for all Δ. This demonstrates the robustness of the proposed algorithm. Second, for all the



(a) (b)

Fig. 5. Position error as a function of the number of agents in the distributed setting: (a) root average squared error of all the navigation trajectories; (b) root maximum squared error among all the navigation trajectories.



(a) (b)

Fig. 6. Root maximum squared errors as a function of step size Δ for cooperative networks: (a) centralized setting; (b) distributed setting.

algorithms, the root average squared position errors increase with the step size, since the uncertainty in the parameters affects the performance of the energy allocation strategy and the deviation of the initial value from the true agent position influences the accuracy of the localization method. Third, algorithms in the centralized setting have better performance than their distributed counterparts, because the measurements among all the agents are available in the centralized setting.

VI. Conclusion

In this paper, we developed energy-efficient network navigation algorithms that jointly consider location inference and power control. For both centralized and distributed settings, the cooperative navigation algorithms with energy allocation are proposed. In particular, we first determine the confidence regions of agents' positions and then design robust energy allocation strategies and position estimation methods. The upper bounds for the worst-case CRLB are derived as the performance metric for energy allocation. Simulation results show that the proposed algorithm significantly outperform the navigation algorithms with uniform or non-robust power control. The outcome of this paper provides guidelines for the operations in energy-efficient network navigation.

Appendix A
Proof of Proposition 1

Let $\phi^{+}_{jk} = \phi^{(n)}_{jk} + \hat{\phi}^{(n)}_{jk}$ and $\phi^{-}_{jk} = \phi^{(n)}_{jk} - \hat{\phi}^{(n)}_{jk}$. Note that

$$\begin{aligned}
&\mathbf{J}_{\mathrm{e}}\big(\mathbf{p}^{(n)}\big) - \mathbf{Q}\big(\hat{\mathbf{p}}^{(n)}\big)\\
&\quad= \sum_{k\in\mathcal{N}_{\mathrm{a}}}\sum_{j\in\mathcal{N}_{\mathrm{b}}} x^{(n)}_{jk}\,\xi^{(n)}_{jk}\,\mathbf{e}_k\,\mathbf{e}_k^{\mathrm{T}}\otimes\mathbf{J}_{jk}\\
&\qquad+\sum_{k\in\mathcal{N}_{\mathrm{a}}}\sum_{j\in\mathcal{N}_{\mathrm{a}}\backslash\{k\}} x^{(n)}_{jk}\,\xi^{(n)}_{jk}\,(\mathbf{e}_k-\mathbf{e}_j)(\mathbf{e}_k-\mathbf{e}_j)^{\mathrm{T}}\otimes\mathbf{J}_{jk}
\end{aligned}$$

where

$$\mathbf{J}_{jk} = \begin{bmatrix} \delta_{jk}^{(n)} - \sin\phi_{jk}^{+}\sin\phi_{jk}^{-} & \cos\phi_{jk}^{+}\sin\phi_{jk}^{-} \\ \cos\phi_{jk}^{+}\sin\phi_{jk}^{-} & \delta_{jk}^{(n)} + \sin\phi_{jk}^{+}\sin\phi_{jk}^{-} \end{bmatrix}.$$

In [40], it is shown that $\mathbf{J}_{kj} \succeq \mathbf{0}$. For any $\mathbf{y} \in \mathbb{R}^{2N_\mathrm{a}}$, let $\mathbf{y}_k = [\, y_{2k-1} \;\; y_{2k}\,]^\mathrm{T}$ and $\mathbf{y}_j = [\, y_{2j-1} \;\; y_{2j}\,]^\mathrm{T}$, then we have

$$\mathbf{y}^\mathrm{T}\big[(\mathbf{e}_k - \mathbf{e}_j)(\mathbf{e}_k - \mathbf{e}_j)^\mathrm{T} \otimes \mathbf{J}_{jk}\big]\mathbf{y} = (\mathbf{y}_k - \mathbf{y}_j)^\mathrm{T}\mathbf{J}_{jk}\,(\mathbf{y}_k - \mathbf{y}_j) \geq 0$$

where the last inequality is due to the fact that $\mathbf{J}_{jk} \succeq \mathbf{0}$. This shows that $(\mathbf{e}_k - \mathbf{e}_j)(\mathbf{e}_k - \mathbf{e}_j)^\mathrm{T} \otimes \mathbf{J}_{jk} \succeq \mathbf{0}$. One can verify that $\mathbf{e}_k\,\mathbf{e}_k^\mathrm{T} \otimes \mathbf{J}_{jk} \succeq \mathbf{0}$ in an analogous way. Consequently,

$$\mathbf{J}_\mathrm{e}\big(\mathbf{p}^{(n)}\big) \succeq \mathbf{Q}\big(\hat{\mathbf{p}}^{(n)}\big)$$

for $\phi_{jk}^{(n)}$ in (7). Since $\mathrm{tr}\{(\cdot)^{-1}\}$ is a monotonically decreasing function, we have

$$\max_{\{\mathbf{p}_k^{(n)} \in \mathcal{U}_k^{(n)}\}_{k\in\mathcal{A}}} \sum_{k\in\mathcal{N}_\mathrm{a}} \mathcal{P}\big(\mathbf{p}_k^{(n)}\big) \leq \mathrm{tr}\left\{\mathbf{Q}^{-1}\big(\hat{\mathbf{p}}^{(n)}\big)\right\}$$

when $\mathbf{Q}(\hat{\mathbf{p}}^{(n)}) \succeq 0$.

Appendix B
Proof of Proposition 4

Note that $\mathrm{tr}\{[\cdot]^{-1}\}$ is an decreasing function. Thus, for any $\mathbf{p}_k^{(n)}$, $\mathcal{P}_\mathrm{d}\big(\mathbf{p}_k^{(n)}\big)$ is upper bounded by $\overline{\mathcal{P}}\big(\mathbf{p}_k^{(n)}\big)$, which is obtained by replacing $\mu_{jk}^{(n)}$ with $\overline{\mu}_{jk}^{(n)}$ in $\mathcal{P}_\mathrm{d}\big(\mathbf{p}_k^{(n)}\big)$. Therefore, after some algebra, one can show that

$$\begin{aligned}\max_{\{\mathbf{p}_j \in \widetilde{\mathcal{U}}_{\mathrm{A},j}\}_{j\in\mathcal{N}_\mathrm{a}}} \mathcal{P}_\mathrm{d}\big(\mathbf{p}_k^{(n)}\big) &\leq \max_{\{\mathbf{p}_j \in \widetilde{\mathcal{U}}_{\mathrm{A},j}\}_{j\in\mathcal{N}_\mathrm{a}}} \overline{\mathcal{P}}\big(\mathbf{p}_k^{(n)}\big) \\ &= \frac{4\cdot f_k^{(n)}}{\big(f_k^{(n)}\big)^2 - (S_k)^2}\end{aligned} \tag{23}$$

where

$$S_k = \left\| \sum_{j\in\mathcal{N}_\mathrm{b}} \xi_{jk}^{(n)} x_{jk}^{(n)} \begin{bmatrix}\cos\phi_{jk}^{*} \\ \sin\phi_{jk}^{*}\end{bmatrix} + \sum_{j\in\mathcal{N}_\mathrm{a}\backslash\{k\}} \xi_{jk}^{(n)} y_{jk}^{(n)} \begin{bmatrix}\cos\phi_{jk}^{*} \\ \sin\phi_{jk}^{*}\end{bmatrix} \right\|$$

in which $\{\phi_{jk}^{*}\}$ are the optimal angles that achieve the maximum of $\overline{\mathcal{P}}(\mathbf{p}_k^{(n)})$ in (23). Then, to prove Proposition 4, we only need to show that

$$S_k \leq g_{k,m}^{(n)} \leq f_k^{(n)}$$

for sufficiently large M, which can be proved in the same way shown in [41, Proposition 4].

Appendix C
Proof of Proposition 2

It suffices to show that there exists an optimal solution $\{x_{jk}^{*}\}$ such that

$$\sum_{j\in\mathcal{N}_\mathrm{b}} \mathbb{I}\big\{x_{jk}^{*}\big\} \leq 4, \qquad \forall\, k \in \mathcal{N}_\mathrm{a}. \tag{24}$$

Suppose $\{x_{jk}\}$ is an optimal solution for $\mathscr{P}_{\text{R-C-SDP}}^{(n)}$, we next construct an optimal solution $\{x_{jk}^{*}\}$ satisfying (24) based on $\{x_{jk}\}$.

Consider the following solution $\{x_{jk}^{*}\}$ such that

$$x_{jk}^{*} = \begin{cases} y_{jk}, & j \in \mathcal{N}_\mathrm{b},\, k \in \mathcal{N}_\mathrm{a} \\ x_{jk}, & \text{otherwise} \end{cases}$$

where $\{y_{jk} \in \mathbb{R}^{+}\}_{j\in\mathcal{N}_\mathrm{b},\,k\in\mathcal{N}_\mathrm{a}}$ is the set with the minimum number of nonzero elements that satisfies that $\forall j \in \mathcal{N}_\mathrm{a}$

$$\sum_{j\in\mathcal{N}_\mathrm{b}} (y_{jk} - x_{jk})\xi_{jk}^{(n)}\Big(\mathbf{J}_\mathrm{r}\big(\hat{\phi}_{jk}^{(n)}\big) - \delta_{jk}^{(n)}\,\mathbf{I}\Big) = \mathbf{0}$$

$$\sum_{j\in\mathcal{N}_\mathrm{b}} (y_{jk} - x_{jk}) = 0.$$

Using the same method developed in [52], one can verify that

$$\sum_{j\in\mathcal{N}_\mathrm{b}} \mathbb{I}\{y_{jk}\} \leq 4, \qquad \forall\, k \in \mathcal{N}_\mathrm{a}$$

which completes the proof.

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