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Antenna Subset Diversity with Non-Ideal Channel Estimation

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***Abstract*— In modern wireless systems employing diversity techniques, combining all the available diversity branches may not be feasible due to complexity and resource constraints. To alleviate these issues, subset diversity (SSD) systems have been proposed. Here, we develop a framework for evaluating the symbol error probability for antenna SSD, where the signals from a subset of antenna elements are selected and combined in the presence of *channel estimation error*. We consider independent identically distributed Rayleigh fading channels and use an estimator structure based on the maximum likelihood (ML) estimate which arises naturally as the sample mean of N_p pilot symbols. The analysis is valid for arbitrary two-dimensional signaling constellations. The expressions give insight into the performance losses of non-ideal SSD when compared to ideal SSD. Due to estimation error, these losses occur in branch combining as well as in branch selection. However, our analytical results show that the practical ML channel estimator still preserves the diversity order of an ideal SSD system with N_d branches. Finally, we investigate the asymptotic signal-to-noise ratio penalty due to estimation error.**

***Index Terms*— Antenna subset diversity, non-ideal channel estimation, pilot symbol estimation, weighting error, imperfect channel knowledge.**

I. Introduction

SUBSET diversity (SSD) is a method in which a subset of the available diversity branches are selected, based on a suitable criterion, and then combined, thereby improving the performance of wireless systems [1]–[9]. Diversity presents itself in a wireless channel in several realizations – spatial, polarization, angle, frequency, time, delay, multipath, multiuser, etc. [10], [11]. Among them, spatial diversity using multiple antennas [1]–[3] plays an increasingly important role in next generation wireless systems. Regardless of the specific type of realization, it may not be feasible in many situations to utilize all of the available diversity branches.

These complexity and performance issues have motivated studies of diversity combining methods that process only a *subset* of the available N_d diversity branches, but achieve better performance than a single branch receiver. SSD arises naturally as a generalization of these other diversity methods which include selection diversity (SD), where the best branch is selected, and hybrid selection/maximal-ratio combining (H-S/MRC), where the best L_d out of N_d diversity branches are selected. Such subset based schemes are a way to reduce resource use, including power consumption and cost of RF electronics, in receiver designs while maintaining the benefits of increased diversity order [1]–[9].

Many previous studies of SSD have assumed that perfect channel knowledge is available at the receiver. For example, in [3], it was shown that ideal H-S/MRC achieves a diversity order equal to the number of available diversity branches, despite using only a subset of them. However, practical diversity receivers must estimate the channel on each diversity branch, and thereby incur a performance loss [13]–[23]. In the case of SSD, the estimation plays a dual role; it affects both the selection process as well as the combining mechanism. Indeed, in an SSD receiver the subset of diversity branches chosen is based on the receiver's knowledge of the channel, i.e., the *estimated* channel gains. Therefore, it is possible that the receiver makes an erroneous selection.

In comparison to ideal SSD, practical SSD systems will incur a loss in signal-to-noise ratio (SNR).[1] This loss occurs because completely coherent combining is not possible *and* because the selection mechanism is not perfect. A measure of such loss that is suitable for digital systems is the SNR penalty required to maintain a target symbol error probability (SEP) [3]. The SNR penalty is the increase in SNR required for a non-ideal SSD system to achieve the same target SEP as an ideal SSD system. In general the SNR penalty is a function

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[1]Throughout this paper, we use the term SNR to refer to instantaneous SNR. The term average SNR is explicitly used to describe the SNR averaged over the fading ensemble.

of the target SEP, and therefore also a function of the average SNR.

In this paper we develop a framework for evaluating the SEP of SSD with estimation error in independent identically distributed (i.i.d.) Rayleigh fading. The framework is valid for arbitrary two-dimensional signaling constellations. The analysis is framed in terms of antenna SSD, where a subset of antennas are selected, but the results are equally valid for other forms of diversity.[2] As a benchmark we also develop expressions for the bit error probability (BEP) of binary phase-shift keying (BPSK) signaling when the selection is ideal, i.e., based on perfect knowledge of the channel gains, but the combining utilizes the channel estimates. This illustrates the role that channel estimation plays in the *selection* of the diversity branches. Finally, we quantify the asymptotic SNR penalty of SSD with non-ideal channel estimation.

This paper is organized as follows. In the next section, the models for the system and estimator are presented. In Section III we evaluate the SEP of SSD systems with estimation error. Using the expressions we develop, Section IV presents expressions for SSD based on specific selection policies. Section V discusses the asymptotic SNR penalty of SSD with non-ideal channel estimates and in Section VI we give numerical results. Finally, in Section VII we present concluding remarks.

II. Model

We consider a diversity system with $N_{\rm d}$ available antenna elements utilizing an arbitrary two-dimensional M-ary signaling constellation with polygonal decision boundaries. The received signal on the kth diversity branch, after demodulation, matched filtering, and sampling, is given by

$$r_k = h_k s_i + n_k \qquad k = 1, 2, \ldots, N_{\rm d} ,$$

where s_i, $i = 1, 2, \ldots, M$, represents the complex message symbol, h_k is a complex, multiplicative gain introduced by the channel on the kth branch, and n_k represents a sample of additive noise on the kth branch. The average symbol energy is indicated by $E_{\rm s}$. The additive noise is modeled as a circularly symmetric complex Gaussian random variable (r.v.) with zero mean and variance $N_0/2$ per dimension and is assumed to be independent among the diversity branches. We consider i.i.d. Rayleigh fading channels, that is, each channel gain can be written as a circularly symmetric complex Gaussian r.v., $h_k = h_{k,{\rm r}} + jh_{k,{\rm i}}$, with $\mathbb{E}\{h_k\} = 0$ and $\mathbb{E}\left\{|h_k|^2\right\} = \mathbb{E}\left\{|h_{k,{\rm r}}|^2\right\} + \mathbb{E}\left\{|h_{k,{\rm i}}|^2\right\} = 2\sigma_h^2$.

If the channel gains, $\boldsymbol{h} = [h_1 \;\; h_2 \;\; \cdots \;\; h_{N_{\rm d}}]$, are known to the receiver, the output of the ideal SSD combiner is given by

$$D = \sum_{k\,:\,k\in\mathcal{O}} h_k^* r_k ,$$

where $\mathcal{O}$ is a set of indices indicating which subset of antenna elements to combine. Note that $\mathcal{O} \subseteq \{1, 2, \ldots, N_{\rm d}\}$. In practice, however, $\boldsymbol{h}$ must be estimated; thus the combiner output is

$$D = \sum_{k\,:\,k\in\mathcal{O}_{\rm N}} \hat{h}_k^* r_k , \tag{1}$$

where $\hat{h}_k$ is the estimate of the multiplicative gain, h_k, of the kth branch. Clearly, the performance of this combining scheme greatly depends on the quality of the estimate $\hat{h}_k$. Note also that the set of selected antenna elements is indicated by $\mathcal{O}_{\rm N}$, where, in this case, the antenna elements are chosen based on their estimated channel gains.

One way to estimate the channel gains is by using pilot symbols

$$p_{k,j} = \sqrt{E_{\rm p}} h_k + n_{k,j} ,$$

where $p_{k,j}$ represents the pilot received on the kth branch during the jth previous signaling interval and $E_{\rm p}$ is the energy of the pilot symbol. An estimate can be formed by averaging $N_{\rm p}$ pilot symbols received within the coherence time of the channel[3]

$$\hat{h}_k = \frac{\sum_{j=1}^{N_{\rm p}} p_{k,j}}{\sqrt{E_{\rm p}} N_{\rm p}} = h_k + e_k .$$

In this case e_k is the complex Gaussian estimation error with zero mean and variance $\sigma_e^2 = N_0/(2E_{\rm p}N_{\rm p})$ per dimension. The pilot energy is related to the signal energy through the quantity $\varepsilon = E_{\rm p}/E_{\rm s}$. Note that $\hat{h}_k$ is a complex Gaussian r.v. since it is the sum of two complex Gaussian r.v.'s. It is important to stress that $N_{\rm p}$ represents the number of *received* pilot symbols used in forming an estimate of each branch. Depending on the choice of transmitter and receiver architectures of a SSD system, the actual number of transmitted pilots may need to be larger to guarantee an estimate of each branch based on $N_{\rm p}$ pilots.[4]

The main goal of our analysis is to obtain insights into the effect of non-ideal selection and non-ideal combining, where the antenna elements are chosen based on the estimate of the channel state at the receiver. Before doing so, we will first consider the case where no errors are made during the selection process, that is, when the selection mechanism has perfect knowledge of the channel gains, but the combining process uses the estimated channel gains. We refer to this selection mechanism as "genie-aided selection." While this is not realistic, it allows us to isolate the impact of selection errors from performance losses due to the imperfect combining process.

A. Genie-Aided Selection

The genie-aided selection method operates on the squared magnitude of the true channel gain,

$$g_k \triangleq |h_k|^2 .$$

The vector of *ordered* magnitudes is then defined as[5]

$$\boldsymbol{g}_{[N_{\rm d}]} \triangleq \left[g_{[1]} \;\; g_{[2]} \;\; \cdots \;\; g_{[N_{\rm d}]}\right]^t ,$$

[2]For example see [24], where subset diversity arises as multipath components in a Rake receiver.

[3]Note that the channel estimation process is carried out to track changes in the channel, and thus pilot symbols need only be transmitted with a rate suitable to track the fading.

[4]For example, in the case of antenna diversity with $L_{\rm d}$ receiver chains, if $L_{\rm d} < N_{\rm d}$ it is not possible to receive the transmitted pilots on all branches simultaneously. It can be shown that $\lceil N_{\rm d}/L_{\rm d}\rceil N_{\rm p}$ pilots need to be transmitted to ensure at least $N_{\rm p}$ pilots are received for estimation on each of the $N_{\rm d}$ branches.

[5]The notation $(\cdot)^t$ denotes transpose.

where $\{g_{[i]}\}$ is the ordered set of $\{g_i\}$ such that $g_{[1]} > g_{[2]} > \ldots > g_{[N_d]}$. Note that the possibility of at least two equal $g_{[l]}$'s is excluded, since $g_{[l]} \neq g_{[m]}$ *almost surely* for continuous r.v.'s $g_{[l]}$'s [25], [26].

When the selection is based on $\boldsymbol{g}_{[N_d]}$, the corresponding index set denoted by $\mathcal{O}_G$ is determined by a binary-valued selection vector, $\boldsymbol{a}$, with lth element $a_l \in \{0, 1\}$ that indicates which of the antenna elements are included in the combining process. For example, $a_l = 1$ indicates that $g_{[l]}$ is included in the combining process. In particular,

$$\mathcal{O}_G \triangleq \left\{k \,:\, a_{\chi(k)} = 1\right\} ,$$

where $\chi(\cdot)$ is the bijective mapping from the indices of the unordered physical branches to the indices of the ordered branches. In this case the ordering is with respect to the g_k's, derived from ideal channel state information. It is convenient to define the total contribution of the channel gains as

$$\gamma_G \triangleq \sum_{k \in \mathcal{O}_G} |h_k|^2 = \sum_{l=1}^{N_d} a_l g_{[l]} = \langle \boldsymbol{a}, \boldsymbol{g}_{[N_d]} \rangle .$$

Note that γ_G is the norm-square of the vector whose elements are from the selected subset of fading gains. The performance analysis for genie-aided selection is given in Section III.

B. Non-Ideal Selection

The non-ideal selection mechanism operates on the squared magnitude of the channel gain estimates, defined as

$$\widetilde{g}_k \triangleq \left|\hat{h}_k\right|^2 .$$

The vector of *ordered* magnitudes is defined as

$$\widetilde{\boldsymbol{g}}_{[N_d]} \triangleq \left[\widetilde{g}_{[1]} \;\; \widetilde{g}_{[2]} \;\; \cdots \;\; \widetilde{g}_{[N_d]}\right]^t ,$$

such that $\widetilde{g}_{[1]} > \widetilde{g}_{[2]} > \ldots > \widetilde{g}_{[N_d]}$. Again, note that the possibility of at least two equal $\widetilde{g}_{[l]}$'s is excluded, since $\widetilde{g}_{[l]} \neq \widetilde{g}_{[m]}$ *almost surely* for continuous r.v.'s $\widetilde{g}_{[l]}$'s.

When the selection is based on $\widetilde{\boldsymbol{g}}_{[N_d]}$, the corresponding index set denoted by $\mathcal{O}_N$ is determined by a binary-valued selection vector, $\boldsymbol{a}$, with lth element $a_l \in \{0, 1\}$ that indicates which of the antenna elements are included in the combining process. For example, $a_l = 1$ indicates that $\widetilde{g}_{[l]}$ is included in the combining process. In particular,

$$\mathcal{O}_N \triangleq \left\{k \,:\, a_{\widetilde{\chi}(k)} = 1\right\} ,$$

where $\widetilde{\chi}(\cdot)$ is the bijective mapping from the indices of the unordered physical branches to the indices of the ordered branches. In this case the ordering is done with respect to the $\widetilde{g}_k$'s, derived from the estimated channel state information. The total contribution of the channel estimates is defined as

$$\gamma_N \triangleq \sum_{k \in \mathcal{O}_N} \left|\hat{h}_k\right|^2 = \sum_{l=1}^{N_d} a_l \widetilde{g}_{[l]} = \langle \boldsymbol{a}, \widetilde{\boldsymbol{g}}_{[N_d]} \rangle .$$

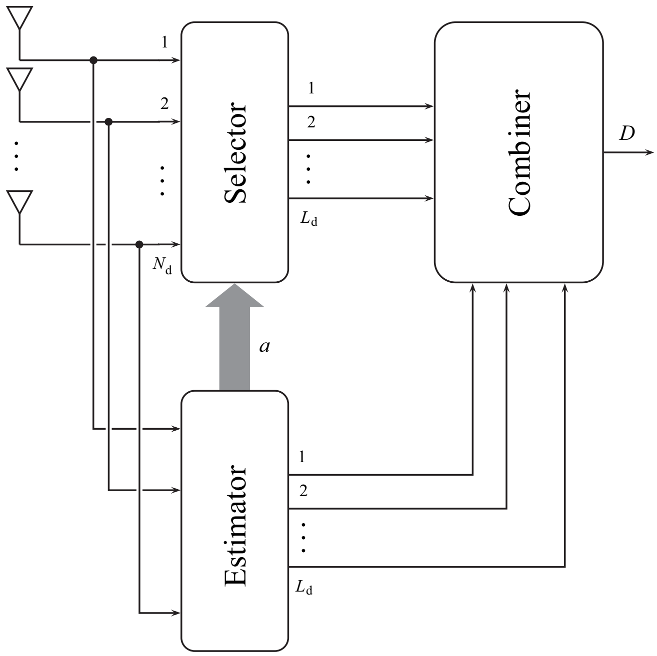

Fig. 1. The basic structure of an antenna SSD receiver.

Note that γ_N is the norm-square of the vector whose elements consist of the selected subset of *estimated* fading gains.

Later, for both genie-aided and non-ideal selection, it will be shown that particular diversity combining schemes, such as H-S/MRC with non-ideal channel estimation, reduce to special cases of the selection vector $\boldsymbol{a}$. Figure 1 provides a conceptual view of a SSD system utilizing non-ideal channel estimates.

III. Analysis for Subset Diversity

In this section we give expressions for the exact SEP for SSD based on both the genie-aided and non-ideal selection schemes outlined above. These expressions require only the evaluation of a single integral with finite limits.

A. Performance Analysis for Genie-Aided Selection with Non-Ideal Combining

Using a similar procedure as in [14], [17], the conditional bit error probability of BPSK, conditioned on the set of channel gains involved in the combining process, can be derived as

$$\Pr\{e \,|\, \mathcal{S}_G\} = \frac{1}{4\pi} \int_{-\pi}^{\pi} \left\{ \exp\left[-\frac{\gamma_G \varsigma \Gamma}{8\sigma_h^2} \frac{(1-\zeta^2)^2}{g(\theta;\zeta)}\right] + f(\theta;\zeta, L_d) \exp\left[-\frac{\gamma_G \varsigma \Gamma}{8\sigma_h^2} g(\theta;\zeta)\right] \right\} d\theta , \quad (2)$$

where

$$\zeta = \frac{\left|\sqrt{N_\mathrm{p}\,\varepsilon} - 1\right|}{\sqrt{N_\mathrm{p}\,\varepsilon} + 1}$$

$$\varsigma = \left(\sqrt{N_\mathrm{p}\,\varepsilon} + 1\right)^2$$

$$f(\theta;\zeta,L_\mathrm{d}) = \frac{1}{2^{(2L_\mathrm{d}-2)}} \sum_{n=1}^{L_\mathrm{d}-1} \cos\left[n\left(\theta + \frac{\pi}{2}\right)\right]\left(\zeta^{-n} - \zeta^{n}\right) \times \sum_{k=0}^{L_\mathrm{d}-1-n} \binom{2L_\mathrm{d}-1}{k}$$

$$g(\theta;\zeta) = 1 + 2\zeta\sin\theta + \zeta^2 .$$

Here, $\Gamma = \mathbb{E}\left\{|h_k|^2\right\}\frac{E_\mathrm{s}}{N_0}$ is the average SNR per branch. The set $\mathcal{S}_\mathrm{G}$ consists of the channel gains included in the combining process and is defined as

$$\mathcal{S}_\mathrm{G} \triangleq \{h_k : k \in \mathcal{O}_\mathrm{G}\} .$$

Noting that the conditional BEP depends on $\mathcal{S}_\mathrm{G}$ only through γ_G, the exact BEP for BPSK using genie-aided selection is given by

$$P_{e,\,\mathrm{GSNC}}^{\mathrm{BPSK}}(\Gamma) = \frac{1}{4\pi}\int_{-\pi}^{\pi}\left\{M_{\gamma_\mathrm{G}}\left[-\frac{\varsigma\Gamma}{8\sigma_h^2}\frac{(1-\zeta^2)^2}{g(\theta;\zeta)}\right] + f(\theta;\zeta,L_\mathrm{d})M_{\gamma_\mathrm{G}}\left[-\frac{\varsigma\Gamma}{8\sigma_h^2}g(\theta;\zeta)\right]\right\}d\theta\,, \quad (3)$$

where $M_{\gamma_\mathrm{G}}(s) \triangleq \mathbb{E}\{e^{s\,\gamma_\mathrm{G}}\}$ is the moment generating function (m.g.f.) of γ_G. The subscript GSNC denotes "genie-aided selection with non-ideal combining" and indicates that the antenna selection is based on perfect knowledge of the channel, while the combining process uses estimated values.[6] Note that the expression in (3) is valid for SSD in i.i.d. Rayleigh fading with BPSK signaling: we simply need to characterize the m.g.f. of γ_G.

B. Performance Analysis for Non-Ideal Selection and Combining

We begin by determining the SEP conditioned on the set of channel estimates as determined by the index set $\mathcal{O}_\mathrm{N}$ through the selection vector $\boldsymbol{a}$. Then we average over the distribution of the channel estimates to obtain the unconditional SEP. This analysis is valid when the receiver makes decisions based on a minimum distance criteria.

1) Conditional SEP: The decision variable, D, is given by

$$D = \sum_{k:\,k\in\mathcal{O}_\mathrm{N}} \hat{h}_k^* r_k = \sum_{k:\,k\in\mathcal{O}_\mathrm{N}} \hat{h}_k^*\left[\left(\hat{h}_k - e_k\right)s_i + n_k\right], \quad (4)$$

where we have used the fact that $\hat{h}_k = h_k + e_k$. It is convenient to define the set $\mathcal{S}_\mathrm{N}$ which consists of the channel estimates for the branches included in the combining process as

$$\mathcal{S}_\mathrm{N} \triangleq \left\{\hat{h}_k : k \in \mathcal{O}_\mathrm{N}\right\} .$$

[6]While this may not be practically feasible in a real system, it allows us to isolate the impact of selection errors from performance losses due to the imperfect combining process.

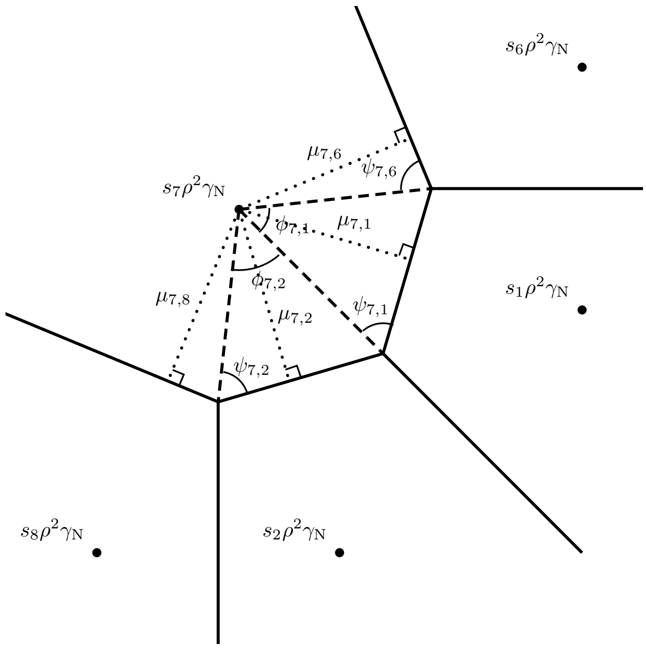

Fig. 2. A portion of the received signal constellation, and its associated decision regions.

Theorem 1: The decision variable, D, when conditioned on the set $\mathcal{S}_\mathrm{N}$ and the transmitted symbol s_i, is a circularly symmetric complex Gaussian r.v. with conditional mean and variance given by

$$\mathbb{E}\left\{D\,\middle|\,\mathcal{S}_\mathrm{N},\,s_i\right\} = s_i\rho^2\gamma_\mathrm{N} \quad (5)$$

$$\mathrm{var}\left\{D\,\middle|\,\mathcal{S}_\mathrm{N},\,s_i\right\} = \mathrm{var}\left\{\Re\{D\}\,\middle|\,\mathcal{S}_\mathrm{N},\,s_i\right\} + \mathrm{var}\left\{\Im\{D\}\,\middle|\,\mathcal{S}_\mathrm{N},\,s_i\right\} = 2\left(\frac{N_0}{2} + |s_i|^2\,\sigma_e^2\rho^2\right)\gamma_\mathrm{N}\,, \quad (6)$$

where $\gamma_\mathrm{N} = \sum_{k\in\mathcal{O}_\mathrm{N}}\left|\hat{h}_k\right|^2$ and we have defined $\rho^2 \triangleq \frac{\sigma_h^2}{\sigma_e^2+\sigma_h^2}$.

Proof: See the Appendix. □

Note that the set $\mathcal{S}_\mathrm{N}$ consists of the channel *estimates* included in the combining process; that is, the choice of which branches are combined is solely based on the receiver's knowledge of the channel. This differs from conventional analyses where it is often assumed that the receiver has perfect knowledge of the channel. In general, a set of antennas chosen based on the noisy estimates may be different from a set chosen based on the true channel gains. Such a difference is important to include in the analysis of a practical diversity system.

For an arbitrary two-dimensional signaling constellation, the decision variable can be represented in polar form as $D = R\,e^{j\Theta}$ and we can write the conditional SEP as [27], [28]

$$\Pr\{e|\mathcal{S}_\mathrm{N},\,s_i\} = \sum_{j\in\mathcal{B}_i}\int_0^{\phi_{i,j}}\int_{r_{i,j}(\theta)}^{\infty} f_{R,\Theta|\mathcal{S}_\mathrm{N}}(r,\theta)\,dr\,d\theta\,, \quad (7)$$

where $\mathcal{B}_i$ is the set consisting of the indices for the signaling points that share a decision boundary with s_i and $f_{R,\Theta|\mathcal{S}_\mathrm{N}}(r,\theta)$ is the joint distribution of the magnitude and phase of the decision variable D given $\mathcal{S}_\mathrm{N}$. Since $D|\mathcal{S}_\mathrm{N}$ is a complex, circularly symmetric Gaussian r.v., R and Θ are conditionally independent, and

$$f_{R,\Theta|\mathcal{S}_\mathrm{N}}(r,\theta) = \frac{r}{2\pi\sigma^2}\exp\left[-\frac{r^2}{2\sigma^2}\right], \qquad 0 \le \theta < 2\pi\,, \quad (8)$$

where $\sigma^2 = \left(\frac{N_0}{2} + |s_i|^2\,\sigma_e^2\rho^2\right)\gamma_\mathrm{N}$. Substituting (8) into (7) gives

$$\Pr\{e|\mathcal{S}_\mathrm{N},\, s_i\} = \frac{1}{2\pi}\sum_{j\in\mathcal{B}_i}\int_0^{\phi_{i,j}}\exp\left[-\frac{\mu_{i,j}^2}{2\sigma^2\sin^2(\theta+\psi_{i,j})}\right]d\theta\,, \quad (9)$$

where we have used $r_{i,j}(\theta) = \frac{\mu_{i,j}}{\sin(\theta+\psi_{i,j})}$ and $\phi_{i,j},\,\psi_{i,j}$ are angles that describe the decision region corresponding to s_i (see Fig. 2 for an example[7]). Using the law of cosines, the quantity $\mu_{i,j}$ is related to the transmitted signal points after reception as

$$\mu_{i,j} = \frac{1}{2}\sqrt{E_\mathrm{s}w_{i,j}}\,\rho^2\gamma_\mathrm{N}\,,$$

where $w_{i,j} = \xi_i + \xi_j - 2\sqrt{\xi_i\,\xi_j}\cos(\theta_i - \theta_j)$ and the signal points are represented in polar form as $s_i = \sqrt{\xi_i E_\mathrm{s}}\,e^{j\theta_i}$ with $\xi_i \triangleq \frac{E_i}{E_\mathrm{s}}$. Simplifying gives

$$\Pr\{e|\mathcal{S}_\mathrm{N},\, s_i\} = \frac{1}{2\pi}\sum_{j\in\mathcal{B}_i}\int_0^{\phi_{i,j}}\exp\left[-\gamma_\mathrm{N}\frac{w_{i,j}E_\mathrm{s}\rho^4}{8\sigma^2\sin^2(\theta+\psi_{i,j})}\right]d\theta\,. \quad (10)$$

2) Exact SEP: Since the conditional SEP in (10) depends on $\mathcal{S}_\mathrm{N}$ only through γ_N, the SEP is found by averaging (10) over the distribution of γ_N:

$$\begin{aligned}\Pr\{e|s_i\} &= \frac{1}{2\pi}\sum_{j\in\mathcal{B}_i}\int_0^{\infty}\int_0^{\phi_{i,j}}\exp\left[-x\frac{w_{i,j}E_\mathrm{s}\rho^4}{8\sigma^2\sin^2(\theta+\psi_{i,j})}\right] \\ &\qquad\qquad \times f_{\gamma_\mathrm{N}}(x)\,d\theta\,dx \\ &= \frac{1}{2\pi}\sum_{j\in\mathcal{B}_i}\int_0^{\phi_{i,j}}M_{\gamma_\mathrm{N}}\left[-\frac{w_{i,j}E_\mathrm{s}\rho^4}{8\sigma^2\sin^2(\theta+\psi_{i,j})}\right]d\theta\,, \quad (11)\end{aligned}$$

where $M_{\gamma_\mathrm{N}}(s) \triangleq \mathbb{E}\{e^{s\,\gamma_\mathrm{N}}\}$ is the m.g.f. of γ_N. The overall error probability can be determined by averaging $\Pr\{e|s_i\}$ over the *a priori* probability that symbol s_i is transmitted:

$$P_{e,\,\mathrm{NSNC}}(\Gamma) = \sum_{i=1}^{M}p_i\sum_{j\in\mathcal{B}_i}\frac{1}{2\pi}\int_0^{\phi_{i,j}}M_{\gamma_\mathrm{N}}\left[-\frac{w_{i,j}\Upsilon_i(\Gamma)\frac{\rho^2}{\sigma_h^2}}{8\sin^2(\theta+\psi_{i,j})}\right]d\theta\,, \quad (12)$$

where

$$\Upsilon_i(\Gamma) \triangleq \frac{\Gamma N_\mathrm{p}\,\varepsilon}{\left(\frac{1}{\Gamma} + N_\mathrm{p}\,\varepsilon + \xi_i\right)}\,.$$

The subscript NSNC denotes "non-ideal selection with non-ideal combining" and indicates that both the antenna selection process and combining mechanism are based solely on the estimated channel knowledge. Note that the expression in (12) is valid for SSD in i.i.d. Rayleigh fading with an arbitrary two-dimensional signaling constellation: we simply need to characterize the m.g.f. of γ_N.

C. Expressions for the m.g.f. of γ_G and γ_N

Regardless of the selection mechanism, whether genie-aided or non-ideal, we are interested in finding the m.g.f. of γ, $M_\gamma(\cdot)$, where

$$\begin{aligned}\gamma &= \langle \boldsymbol{a},\, \boldsymbol{z}_{[N]}\rangle \\ \boldsymbol{z}_{[N]} &\triangleq [z_{[1]}\quad z_{[2]}\quad\cdots\quad z_{[N]}]^t\end{aligned}$$

and $\{z_{[i]}\}$ is the ordered set of $\{z_i\}$, denoting the squared magnitudes of i.i.d. complex Gaussian r.v.'s, such that $z_{[1]} > z_{[2]} > \ldots > z_{[N]}$.

In general direct analysis of γ is prohibitively complicated, involving N nested integrals. However, this can be alleviated by applying the results of the Virtual Branch technique [2]. The ordered set of squared magnitudes, $\boldsymbol{z}_{[N]}$, can be related to a new set of *virtual branch* squared magnitudes, V_n's, using

$$\boldsymbol{z}_{[N]} = \boldsymbol{T}_\mathrm{VB}\boldsymbol{V}_N,$$

where $\boldsymbol{T}_\mathrm{VB} : \mathbb{R}^N \to \mathbb{R}^N$ is the upper triangular virtual branch transformation matrix given by

$$\boldsymbol{T}_\mathrm{VB} = \begin{bmatrix}1 & \frac{1}{2} & \cdots & \frac{1}{N} \\ & \frac{1}{2} & \cdots & \frac{1}{N} \\ & & \ddots & \vdots \\ & & & \frac{1}{N}\end{bmatrix} \quad (13)$$

and $\boldsymbol{V}_N \triangleq [V_1,\, V_2,\, \ldots,\, V_N]^t$. Since $\boldsymbol{T}_\mathrm{VB}$ is upper triangular, the Jacobian of the transformation is easily obtained, and one can then show that the virtual branch variables are i.i.d. with m.g.f. given by

$$M_{V_n}(s) \triangleq \mathbb{E}\{e^{s\,V_n}\} = \frac{1}{1-\mathbb{E}\{z_n\}\,s}\,.$$

[7]Note that the decision region for s_7 is semi-infinite, that is, there are no boundaries to the upper left. For the purposes of calculation of (9) in this special case, we use $\phi_{7,6} = \pi - \psi_{7,6}$, $\phi_{7,8} = \pi - \sin^{-1}\left(\frac{\mu_{7,8}}{\mu_{7,2}}\sin\psi_{7,2}\right)$, and $\psi_{7,8} = 0$.

$$P_{e,\,\mathrm{GSNC}}^{\mathrm{BPSK}}(\Gamma) = \frac{1}{4\pi}\int_{-\pi}^{\pi}\left\{\prod_{n=1}^{N_\mathrm{d}}\left(1 + b_n\frac{\varsigma\Gamma}{4}\left[\frac{(1-\zeta^2)^2}{g(\theta;\zeta)}\right]\right)^{-1} + f(\theta;\zeta,L_\mathrm{d})\prod_{n=1}^{N_\mathrm{d}}\left(1 + b_n\frac{\varsigma\Gamma}{4}g(\theta;\zeta)\right)^{-1}\right\}d\theta \quad (16)$$

$$P_{e,\,\mathrm{NSNC}}(\Gamma) = \sum_{i=1}^{M}p_i\sum_{j\in\mathcal{B}_i}\frac{1}{2\pi}\int_0^{\phi_{i,j}}\prod_{n=1}^{N_\mathrm{d}}\left(1 + b_n\frac{w_{i,j}\Upsilon_i(\Gamma)}{4\sin^2(\theta+\psi_{i,j})}\right)^{-1}d\theta \quad (17)$$

Noting that $\gamma = \langle \boldsymbol{a}, \boldsymbol{T}_{\text{VB}} \boldsymbol{V}_N \rangle$ and using the fact that the virtual branch variables are independent, it can be shown that

$$M_{\gamma}(s) = \prod_{n=1}^{N} M_{\text{V}_n}(b_n s),$$

where b_n is the nth element of $\boldsymbol{b} = \boldsymbol{T}_{\text{VB}}^t \boldsymbol{a}$. Applying the above result to the case of genie-aided and non-ideal selection, respectively, gives:

$$M_{\gamma_{\text{G}}}(s) = \prod_{n=1}^{N_{\text{d}}} \frac{1}{1 - \mathbb{E}\left\{|h|^2\right\} b_n s} = \prod_{n=1}^{N_{\text{d}}} \frac{1}{1 - 2\sigma_h^2 b_n s} \quad (14)$$

$$M_{\gamma_{\text{N}}}(s) = \prod_{n=1}^{N_{\text{d}}} \frac{1}{1 - \mathbb{E}\left\{\left|\hat{h}\right|^2\right\} b_n s} = \prod_{n=1}^{N_{\text{d}}} \frac{1}{1 - 2\frac{\sigma_h^2}{\rho^2} b_n s}. \quad (15)$$

D. Error Probability Expressions

Combining (3) and (14) gives the complete expression for the SEP of SSD with GSNC as shown in (16) at the bottom of the previous page. Similarly, combining (12) and (15) gives the complete expression for the SEP of SSD with NSNC as shown in (17) at the bottom of the previous page.

We can further specialize (17) for the cases of MPSK and MQAM. For the case of MPSK, where the symbols have equal energy, (17) reduces to[8]

$$P_{e,\,\text{NSNC}}^{\text{MPSK}}(\Gamma) = \frac{1}{\pi} \int_0^{\frac{M-1}{M}\pi} \prod_{n=1}^{N_{\text{d}}} \left(1 + b_n \frac{\Upsilon(\Gamma)\, c_{\text{MPSK}}}{\sin^2\theta}\right)^{-1} d\theta, \quad (18)$$

where $c_{\text{MPSK}} = \sin^2\left(\frac{\pi}{M}\right)$. Note that in (18) we have used the fact that $w_{i,j} = 2 - 2\cos\left(\frac{2\pi}{M}\right) = 4\sin^2\left(\frac{\pi}{M}\right)$. For the case of MQAM,

$$P_{e,\,\text{NSNC}}^{\text{MQAM}}(\Gamma) = \frac{1}{M} \sum_i \frac{\omega_i^{\text{a}}}{2\pi} \int_0^{\frac{\pi}{2}} \prod_{n=1}^{N_{\text{d}}} \left(1 + b_n \frac{\Upsilon_i(\Gamma)\, c_{\text{MQAM}}}{\sin^2\left(\theta + \frac{\pi}{4}\right)}\right)^{-1} d\theta + \frac{\omega_i^{\text{b}}}{2\pi} \int_0^{\frac{3\pi}{4}} \prod_{n=1}^{N_{\text{d}}} \left(1 + b_n \frac{\Upsilon_i(\Gamma)\, c_{\text{MQAM}}}{\sin^2\theta}\right)^{-1} d\theta, \quad (19)$$

where $c_{\text{MQAM}} = 3/(2(M-1))$ and ω_i^{a}, ω_i^{b}, and ξ_i are given in Table I for $M = \{4, 16, 64, 256\}$. Note that the summation in (19) is performed over the nonzero terms given in Table I in the column corresponding to a particular value of M.

Using standard techniques (see, for example [2], [8], and [27]), the exact SEP for SSD with an arbitrary two-dimensional signaling constellation and perfect channel state information can be derived as

$$P_{e,\text{ISIC}}(\Gamma) = \sum_{i=1}^{M} p_i \sum_{j \in \mathcal{B}_i} \frac{1}{2\pi} \int_0^{\phi_{i,j}} \prod_{n=1}^{N_{\text{d}}} \left(1 + b_n \frac{w_{i,j}\Gamma}{4\sin^2(\theta + \psi_{i,j})}\right)^{-1} d\theta. \quad (20)$$

[8]For MPSK $\xi_i = 1\ \forall i$, thus $\Upsilon_i(\Gamma)$ does not depend on i, in which case we simply write $\Upsilon(\Gamma)$.

TABLE I

PARAMETERS FOR MQAM SIGNALING CONSTELLATIONS

i	ξ_i $M=4$	ξ_i $M=16$	ξ_i $M=64$	ξ_i $M=256$	ω_i^{a}	ω_i^{b}
1	1	9/5	7/3	45/17	0	8
2		1	37/21	197/85	8	16
3		1/5	25/21	169/85	16	0
4			29/21	173/85	8	16
5			17/21	29/17	32	0
6			3/7	121/85	16	0
7			25/21	9/5	8	16
8			13/21	25/17	32	0
9			5/21	101/85	32	0
10			1/21	81/85	16	0
11				137/85	8	16
12				109/85	32	0
13				1	32	0
14				13/17	32	0
15				49/85	16	0
16				25/17	8	16
17				97/85	32	0
18				73/85	32	0
19				53/85	32	0
20				37/85	32	0
21				5/17	16	0
22				117/85	8	16
23				89/85	32	0
24				13/17	32	0
25				9/17	32	0
26				29/85	32	0
27				1/5	32	0
28				9/85	16	0
29				113/85	8	16
30				1	32	0
31				61/85	32	0
32				41/85	32	0
33				5/17	32	0
34				13/85	32	0
35				1/17	32	0
36				1/85	16	0

The subscript ISIC means "ideal selection with ideal combining" and is used to indicate that both the selection process and combining mechanism are based on perfect knowledge of the channel.

E. Asymptotic Expressions

For large SNR, we can consider the behavior of $P_{e,\,\text{GSNC}}(\Gamma)$. From (16) we have,

$$P_{Ae,\,\text{GSNC}}^{\text{BPSK}}(\Gamma) = \frac{K}{\Gamma^{N_{\text{d}}}} \left(\frac{4}{\varsigma}\right)^{N_{\text{d}}} \frac{1}{4\pi} \int_{-\pi}^{\pi} \left\{ \left[\frac{g(\theta;\zeta)}{(1-\zeta^2)^2}\right]^{N_{\text{d}}} + \frac{f(\theta;\zeta, L_{\text{d}})}{[g(\theta;\zeta)]^{N_{\text{d}}}} \right\} d\theta, \quad (21)$$

where $P_{Ae}(\Gamma)$ indicates the probability of error for asymptotically large Γ and $K = \left(\prod_{n=1}^{N_{\text{d}}} \frac{1}{b_n}\right)$. Similarly, for the case

TABLE II
EXPRESSIONS FOR THE ASYMPTOTIC SEP, $P_{Ae}(\Gamma)$

	NSNC	ISIC
MPSK	$\frac{K}{\Gamma^{N_\text{d}}}\left(\frac{N_\text{p}\,\varepsilon+1}{N_\text{p}\,\varepsilon}\right)^{N_\text{d}}\frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\left(\frac{\sin^2\theta}{c_\text{MPSK}}\right)^{N_\text{d}}d\theta$	$\frac{K}{\Gamma^{N_\text{d}}}\frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\left(\frac{\sin^2\theta}{c_\text{MPSK}}\right)^{N_\text{d}}d\theta$
MQAM	$\frac{K}{\Gamma^{N_\text{d}}}\frac{1}{M}\sum_i\left(\frac{N_\text{p}\,\varepsilon+\xi_i}{N_\text{p}\,\varepsilon}\right)^{N_\text{d}}\left[\frac{\omega_i^\text{a}}{2\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\left(\theta+\frac{\pi}{4}\right)}{c_\text{MQAM}}\right)^{N_\text{d}}d\theta+\frac{\omega_i^\text{b}}{2\pi}\int_0^{\frac{3\pi}{4}}\left(\frac{\sin^2\theta}{c_\text{MQAM}}\right)^{N_\text{d}}d\theta\right]$	$\frac{K}{\Gamma^{N_\text{d}}}\frac{1}{M}\sum_i\left[\frac{\omega_i^\text{a}}{2\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\left(\theta+\frac{\pi}{4}\right)}{c_\text{MQAM}}\right)^{N_\text{d}}d\theta+\frac{\omega_i^\text{b}}{2\pi}\int_0^{\frac{3\pi}{4}}\left(\frac{\sin^2\theta}{c_\text{MQAM}}\right)^{N_\text{d}}d\theta\right]$

of NSNC, (17) gives

$$P_{Ae,\,\text{NSNC}}(\Gamma) = \frac{K}{\Gamma^{N_\text{d}}}\sum_{i=1}^{M}p_i\underbrace{\left(\frac{N_\text{p}\,\varepsilon+\xi_i}{N_\text{p}\,\varepsilon}\right)^{N_\text{d}}}_{\triangleq C_i} \times\underbrace{\sum_{j\in\mathcal{B}_i}\frac{1}{2\pi}\int_0^{\phi_{i,j}}\left(\frac{4\sin^2(\theta+\psi_{i,j})}{w_{i,j}}\right)^{N_\text{d}}d\theta}_{\triangleq I_i}. \quad (22)$$

Equation (22) has several important implications: 1) SSD with non-ideal channel estimation preserves the diversity of the overall system (N_d) despite the fact that only a subset of antenna elements are *selected and combined* (L_d out of N_d) in the presence of estimation error, 2) the quantity C_i completely captures the degradation due to the channel estimation, and 3) the quantity I_i captures the geometry of the signaling scheme.

Similar to (22), the asymptotic behavior of ISIC for large Γ can be obtained from (20) as

$$P_{Ae,\,\text{ISIC}}(\Gamma) = \frac{K}{\Gamma^{N_\text{d}}}\sum_{i=1}^{M}p_i\sum_{j\in\mathcal{B}_i}\frac{1}{2\pi}\int_0^{\phi_{i,j}}\left(\frac{4\sin^2(\theta+\psi_{i,j})}{w_{i,j}}\right)^{N_\text{d}}d\theta = \frac{K}{\Gamma^{N_\text{d}}}\sum_{i=1}^{M}p_iI_i\,. \quad (23)$$

Note that for the case of BPSK (23) reduces to

$$P_{Ae,\,\text{ISIC}}^{\text{BPSK}}(\Gamma) = \frac{K}{\Gamma^{N_\text{d}}}\frac{\Gamma\left(\frac{1}{2}+N_\text{d}\right)}{2\sqrt{\pi}\Gamma(1+N_\text{d})}\,, \quad (24)$$

where $\Gamma(\cdot)$ denotes the Gamma function.

As noted in Section III-B our NSNC results are applicable for arbitrary two-dimensional signaling schemes. Table II summarizes the asymptotic behavior of $P_e(\Gamma)$ for MPSK and MQAM for the cases of non-ideal and ideal selection and combining. One can easily generalize these results to other two-dimensional signaling constellations.

IV. SPECIAL CASES

In this section we characterize the SEP for SSD with non-ideal selection and combining for a few specific selection policies. Specifically, we examine the case of H-S/MRC, MRC, and SD with non-ideal channel estimation.

A. H-S/MRC

In H-S/MRC the receiver combines the signals from the L_d antennas with the largest estimated channel gain magnitude out of N_d available antennas. For this selection policy

$$\boldsymbol{a} = [\underbrace{1\ 1\ \cdots\ 1}_{L_\text{d}\text{ terms}}\ 0\ 0\ \cdots\ 0]^t$$

and

$$b_n = \begin{cases}1\,, & n\le L_\text{d}\\ \frac{L_\text{d}}{n}\,, & \text{otherwise.}\end{cases}$$

Using the resulting b_n gives

$$P_{e,\,\text{H-S/MRC}}(\Gamma) = \sum_{i=1}^{M}p_i\sum_{j\in\mathcal{B}_i}\frac{1}{2\pi}\int_0^{\phi_{i,j}}\left(1+\frac{w_{i,j}\Upsilon_i(\Gamma)}{4\sin^2(\theta+\psi_{i,j})}\right)^{-L_\text{d}} \times\prod_{n=L_\text{d}+1}^{N_\text{d}}\left(1+\frac{L_\text{d}}{n}\frac{w_{i,j}\Upsilon_i(\Gamma)}{4\sin^2(\theta+\psi_{i,j})}\right)^{-1}d\theta\,. \quad (25)$$

For MPSK and MQAM signaling, (25) reduces to the expressions shown in Table III.

B. MRC

In the case of MRC, the receiver combines the signals from all the available antennas. This selection policy amounts to setting $\boldsymbol{a}$ to a vector of ones;

$$\boldsymbol{a} = [\underbrace{1\ 1\ \cdots\ 1}_{L_\text{d}\,=\,N_\text{d}\text{ terms}}]^t\,,$$

giving $b_n = 1\ \forall n$. Using this selection policy gives

$$P_{e,\,\text{MRC}}(\Gamma) = \sum_{i=1}^{M}p_i\sum_{j\in\mathcal{B}_i}\frac{1}{2\pi}\int_0^{\phi_{i,j}}\left(1+\frac{w_{i,j}\Upsilon_i(\Gamma)}{4\sin^2(\theta+\psi_{i,j})}\right)^{-N_\text{d}}d\theta\,. \quad (26)$$

For MPSK and MQAM signaling, (26) reduces to the expressions shown in Table III. These results can also be obtained by setting $L_\text{d} = N_\text{d}$ in the equations given for H-S/MRC.

TABLE III
EXPRESSIONS FOR $P_{e,\text{NSNC}}(\Gamma)$ UNDER SPECIFIC SELECTION POLICIES AND MODULATION FORMATS

	MPSK	MQAM
H-S/MRC	$\frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\left(1+\frac{\Upsilon(\Gamma)\,c_{\text{MPSK}}}{\sin^2\theta}\right)^{-L_\text{d}} \times \prod_{n=L_\text{d}+1}^{N_\text{d}}\left(1+\frac{L_\text{d}}{n}\frac{\Upsilon(\Gamma)\,c_{\text{MPSK}}}{\sin^2\theta}\right)^{-1} d\theta$	$\frac{1}{M}\sum_i \frac{\omega_i^{\text{a}}}{2\pi}\int_0^{\frac{\pi}{2}}\left(1+\frac{\Upsilon_i(\Gamma)\,c_{\text{MQAM}}}{\sin^2\left(\theta+\frac{\pi}{4}\right)}\right)^{-L_\text{d}} \times \prod_{n=L_\text{d}+1}^{N_\text{d}}\left(1+\frac{L_\text{d}}{n}\frac{\Upsilon_i(\Gamma)\,c_{\text{MQAM}}}{\sin^2\left(\theta+\frac{\pi}{4}\right)}\right)^{-1} d\theta + \frac{\omega_i^{\text{b}}}{2\pi}\int_0^{\frac{3\pi}{4}}\left(1+\frac{\Upsilon_i(\Gamma)\,c_{\text{MQAM}}}{\sin^2\theta}\right)^{-L_\text{d}} \times \prod_{n=L_\text{d}+1}^{N_\text{d}}\left(1+\frac{L_\text{d}}{n}\frac{\Upsilon_i(\Gamma)\,c_{\text{MQAM}}}{\sin^2\theta}\right)^{-1} d\theta$
MRC	$\frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\left(1+\frac{\Upsilon(\Gamma)\,c_{\text{MPSK}}}{\sin^2\theta}\right)^{-N_\text{d}} d\theta$	$\frac{1}{M}\sum_i \frac{\omega_i^{\text{a}}}{2\pi}\int_0^{\frac{\pi}{2}}\left(1+\frac{\Upsilon_i(\Gamma)\,c_{\text{MQAM}}}{\sin^2\left(\theta+\frac{\pi}{4}\right)}\right)^{-N_\text{d}} d\theta + \frac{\omega_i^{\text{b}}}{2\pi}\int_0^{\frac{3\pi}{4}}\left(1+\frac{\Upsilon_i(\Gamma)\,c_{\text{MQAM}}}{\sin^2\theta}\right)^{-N_\text{d}} d\theta$
SD	$\frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\left(1+\frac{\Upsilon(\Gamma)\,c_{\text{MPSK}}}{\sin^2\theta}\right)^{-1} \times \prod_{n=2}^{N_\text{d}}\left(1+\frac{1}{n}\frac{\Upsilon(\Gamma)\,c_{\text{MPSK}}}{\sin^2\theta}\right)^{-1} d\theta$	$\frac{1}{M}\sum_i \frac{\omega_i^{\text{a}}}{2\pi}\int_0^{\frac{\pi}{2}}\left(1+\frac{\Upsilon_i(\Gamma)\,c_{\text{MQAM}}}{\sin^2\left(\theta+\frac{\pi}{4}\right)}\right)^{-1} \times \prod_{n=2}^{N_\text{d}}\left(1+\frac{1}{n}\frac{\Upsilon_i(\Gamma)\,c_{\text{MQAM}}}{\sin^2\left(\theta+\frac{\pi}{4}\right)}\right)^{-1} d\theta + \frac{\omega_i^{\text{b}}}{2\pi}\int_0^{\frac{3\pi}{4}}\left(1+\frac{\Upsilon_i(\Gamma)\,c_{\text{MQAM}}}{\sin^2\theta}\right)^{-1} \times \prod_{n=2}^{N_\text{d}}\left(1+\frac{1}{n}\frac{\Upsilon_i(\Gamma)\,c_{\text{MQAM}}}{\sin^2\theta}\right)^{-1} d\theta$

C. SD

For SD, the receiver uses only the signal from the antenna with the largest estimated channel gain magnitude. This amounts to using the following selection vector

$$\boldsymbol{a} = [1 \;\; \underbrace{0 \;\; 0 \;\; \cdots \;\; 0}_{N_\text{d}-1 \text{ terms}}]^t \,,$$

and

$$b_n = \begin{cases} 1\,, & n = 1 \\ \frac{1}{n}\,, & \text{otherwise.} \end{cases}$$

Using this policy, the SEP is given by

$$P_{e,\,\text{SD}}(\Gamma) = \sum_{i=1}^{M} p_i \sum_{j\in\mathcal{B}_i} \frac{1}{2\pi}\int_0^{\phi_{i,j}}\left(1+\frac{w_{i,j}\Upsilon_i(\Gamma)}{4\sin^2\left(\theta+\psi_{i,j}\right)}\right)^{-1} \times \prod_{n=2}^{N_\text{d}}\left(1+\frac{1}{n}\frac{w_{i,j}\Upsilon_i(\Gamma)}{4\sin^2\left(\theta+\psi_{i,j}\right)}\right)^{-1} d\theta\,. \quad (27)$$

For MPSK and MQAM signaling, (27) reduces to the expressions shown in Table III. These results can also be obtained by setting $L_\text{d} = 1$ in the equations given for H-S/MRC.

V. SNR PENALTY

We can examine the penalty that a non-ideal antenna subset diversity system suffers in relation to an ideal system. In comparison to ideal SSD, SSD with non-ideal channel estimation will incur a loss in SNR, due to the fact that the selection mechanism is imperfect and completely coherent combining is not possible. For analog systems, the SNR penalty is defined in terms of the degradation in the SNR. Instead, as in [3], [15], we consider a measure that is more suitable for digital systems; the SNR penalty required to maintain a target SEP.

For a digital communication system, we define the SNR penalty, β, as the increase in SNR required for the SSD system under consideration to achieve the same target SEP as ideal SSD.[9] Implicitly, we have

$$P_{e,\,\text{NSNC}}(\beta\Gamma) = P_{e,\,\text{ISIC}}(\Gamma)\,,$$

where $P_{e,\,\text{NSNC}}(\cdot)$, $P_{e,\,\text{ISIC}}(\cdot)$, β, and Γ are the SEP for antenna subset diversity with non-ideal selection and combining, the SEP for SSD with ideal selection and combining, the SNR penalty, and the average branch SNR, respectively.

[9]Several other penalties involving these systems can also be defined. The interested reader is directed to [29] where the authors investigate them in detail.

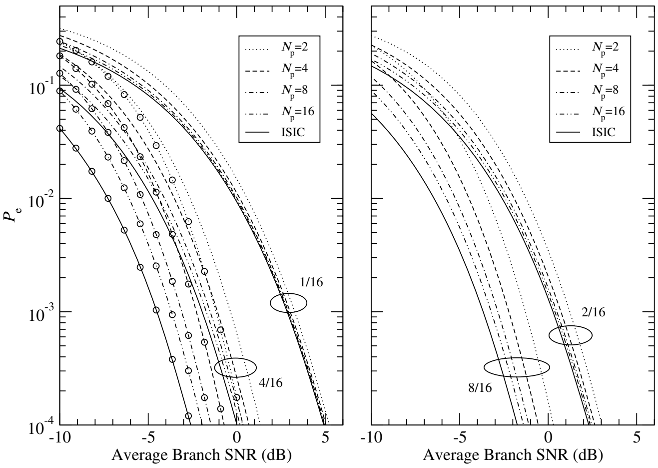

Fig. 3. Performance of GSNC with $N_\mathrm{d} = 16$, $\varepsilon = 1$, for various L_d and N_p. Combining all 16 branches is indicated with circles.

Note that the SNR penalty is a function of the target SEP, and therefore a function of the average SNR; that is, $\beta = \beta(\Gamma)$. A closed form expression for β is difficult to obtain, if at all possible. However, using (22) and (23) we can derive the asymptotic SNR penalty, β_A, for large SNR, such that

$$P_{Ae,\,\mathrm{NSNC}}(\beta_{A,\mathrm{NSNC}}\Gamma) = P_{Ae,\,\mathrm{ISIC}}(\Gamma)\,.$$

From (22) and (23) the SNR penalty of NSNC can be written as

$$\beta_{A,\mathrm{NSNC}} = \left(\frac{\sum_{i=1}^{M} p_i \left(\frac{N_\mathrm{p}\,\varepsilon+\xi_i}{N_\mathrm{p}\,\varepsilon}\right)^{N_\mathrm{d}} I_i}{\sum_{i=1}^{M} p_i I_i} \right)^{\frac{1}{N_\mathrm{d}}} . \tag{28}$$

It is important to note that (28) has no dependence on L_d or the particular selection policy (determined by the binary valued selection vector $\boldsymbol{a}$). This means that for a particular system, regardless of signaling constellation, changing the selection policy will have no effect on the asymptotic SNR penalty. Furthermore, for an arbitrary signaling constellation, it can be shown that

$$\frac{N_\mathrm{p}\,\varepsilon + \xi_{\min}}{N_\mathrm{p}\,\varepsilon} \le \beta_{A,\mathrm{NSNC}} \le \frac{N_\mathrm{p}\,\varepsilon + \xi_{\max}}{N_\mathrm{p}\,\varepsilon}\,, \tag{29}$$

where $\xi_{\min} \triangleq \min_{i=1,\dots,M}\, \xi_i$ and $\xi_{\max} \triangleq \max_{i=1,\dots,M}\, \xi_i$.

For the specific case of MPSK it can be shown, using results from Table II, that the asymptotic SNR penalty reduces to

$$\beta_{A,\mathrm{NSNC}}^{\mathrm{MPSK}} = \frac{N_\mathrm{p}\,\varepsilon + 1}{N_\mathrm{p}\,\varepsilon}\,. \tag{30}$$

The asymptotic SNR penalty given in (30) has interesting implications. It states that for asymptotically large SNR, the penalty for an MPSK SSD system with non-ideal channel estimation as compared to an ideal SSD system is the same regardless of the number of antennas used in the combining process. That is, asymptotically, only the estimation accuracy ($N_\mathrm{p}\,\varepsilon$) plays a role in determining the SNR penalty. Figures 7 – 8 show β_A as a function of $N_\mathrm{p}\,\varepsilon$ for several values of L_d and N_d.

Similarly, for the case of GSNC, we can derive the asymptotic SNR penalty, such that

$$P_{Ae,\,\mathrm{GSNC}}^{\mathrm{BPSK}}(\beta_{A,\mathrm{GSNC}}^{\mathrm{BPSK}}\Gamma) = P_{Ae,\,\mathrm{ISIC}}^{\mathrm{BPSK}}(\Gamma)\,.$$

Then, the SNR penalty, $\beta_{A,\mathrm{GSNC}}^{\mathrm{BPSK}}$, can be computed, using (21) and (24) as

$$\beta_{A,\mathrm{GSNC}}^{\mathrm{BPSK}} = \frac{4}{\varsigma} \left\{ \frac{\Gamma(1+N_\mathrm{d})}{2\sqrt{\pi}\,\Gamma\!\left(\frac{1}{2}+N_\mathrm{d}\right)} \times \int_{-\pi}^{\pi} \left(\left[\frac{g(\theta;\zeta)}{(1-\zeta^2)^2}\right]^{N_\mathrm{d}} + \frac{f(\theta;\zeta,L_\mathrm{d})}{[g(\theta;\zeta)]^{N_\mathrm{d}}} \right) d\theta \right\}^{\frac{1}{N_\mathrm{d}}} . \tag{31}$$

Unlike the case of NSNC, from (31) we see that $\beta_{A,\mathrm{GSNC}}^{\mathrm{BPSK}}$ has a dependence on both L_d and N_d.

VI. Numerical Results and Discussion

The analytical framework developed in the previous section can be used to evaluate the performance of arbitrary two-dimensional signaling constellations for SSD systems.[10] For brevity, and as a point of comparison, we restrict our discussion to BPSK signaling for H-S/MRC systems employing GSNC or NSNC, where the channel estimation process for each branch is based on the reception of N_p pilot symbols. As pointed out earlier, the actual number of transmitted pilots may need to be larger, depending on the specific architecture of the SSD system, to guarantee that each channel estimate is based on N_p pilots.

Figures 3 – 6 show the performance of BPSK signaling for SSD systems employing GSNC or NSNC operating in Rayleigh fading. In all figures, performance of non-ideal systems is shown with dashed or dotted lines, while solid lines indicate the performance of systems with ISIC. The curves are labeled with $L_\mathrm{d}/N_\mathrm{d}$ indicating that L_d out of N_d available antennas are combined. The ISIC curves serve as a lower bound on the performance of SSD systems employing GSNC and NSNC. It is immediately apparent from the figures that the diversity order of non-ideal systems, whether GSNC or NSNC, is preserved and matches that of a theoretical system employing ISIC as shown analytically in Sec. III-E.

Specifically, Figs. 3 and 4 indicate that, for a fixed number of pilot symbols and available antennas (i.e., fixed N_p and N_d), performance improves as the number of combined antennas increases. In addition, one can see that there is a significant difference in performance between GSNC and NSNC for low L_d and few pilot symbols. For example, with $L_\mathrm{d} = 1$ and $N_\mathrm{p} = 2$ the required SNR at a BEP of 10^{-3} is about 3 dB in GSNC and about 5 dB in NSNC. The curves also indicate that this difference becomes less significant as either L_d or N_p increases.

Figure 3 shows that the performance loss of GSNC compared to ISIC, due solely to non-ideal combining, increases with the number of combined branches. This behavior is further verified by Fig. 7. In this figure, β_A can be viewed

[10]Extensive simulations were also performed and the results are in agreement with the theoretical analysis.

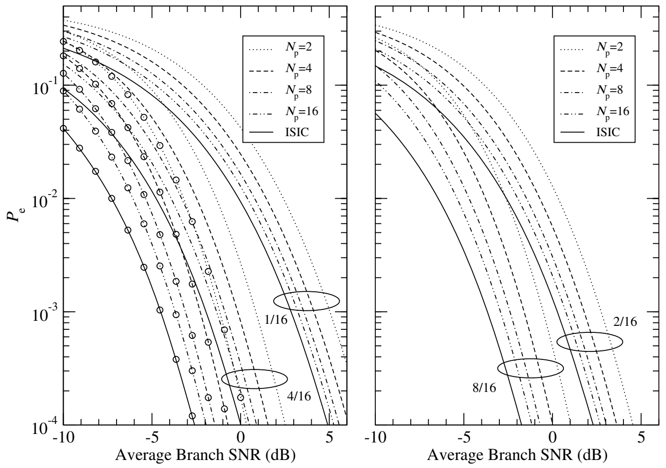

Fig. 4. Performance of NSNC with $N_\mathrm{d} = 16$, $\varepsilon = 1$, for various L_d and N_p. Combining all 16 branches is indicated with circles.

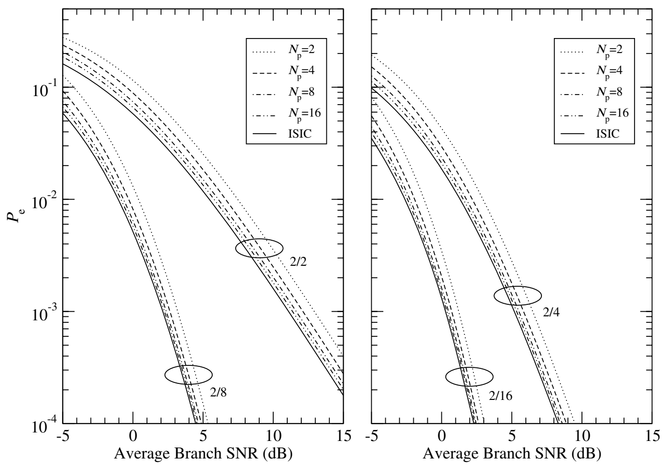

Fig. 5. Performance of GSNC with $L_\mathrm{d} = 2$, $\varepsilon = 1$, for various N_d and N_p.

as the performance loss between GSNC or NSNC and ISIC.[11] From this figure, we see that when N_d is fixed and L_d increases the SNR penalty of GSNC increases. Intuitively, this is the case because under GSNC the branches are selected perfectly, thus for $L_\mathrm{d} = 1$ the branch chosen will have the best channel quality out of all branches, yielding the smallest penalty. When $L_\mathrm{d} = 2$, the next branch that is chosen will have lower quality than the first, causing an increase in the penalty. The effects are similar as more branches are added, until $L_\mathrm{d} = N_\mathrm{d}$ when the penalty matches that of NSNC.

This is in contrast to Fig. 4, which shows that increasing the number of combined diversity branches has little effect on the performance loss of NSNC compared to ISIC. This is further verified by Fig. 7, where all the SNR penalty curves for NSNC are identical regardless of L_d.

Figures 5 and 6 show the performance of SSD systems with GSNC or NSNC for fixed L_d and various values of N_d. Again, one can see that there is a significant difference in performance between GSNC and NSNC when few pilot symbols are used. For example, for $2/8$ and $N_\mathrm{p} = 2$, the required SNR at a BEP of 10^{-3} is about 3 dB in GSNC and about 4.5 dB in NSNC.

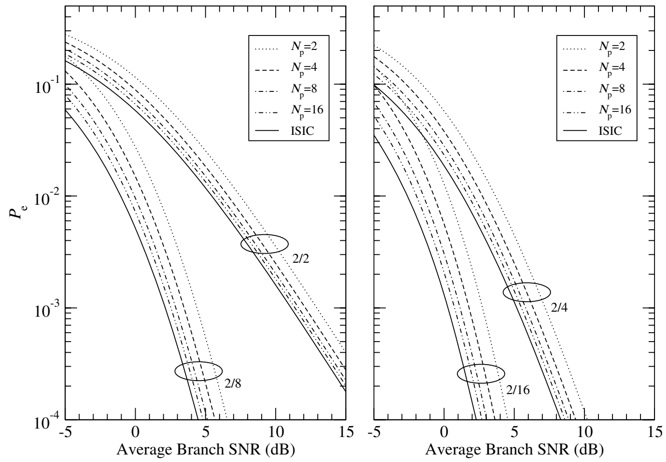

Fig. 6. Performance of NSNC with $L_\mathrm{d} = 2$, $\varepsilon = 1$, for various N_d and N_p.

These figures also indicate that the difference increases with increasing N_d or decreasing N_p.

Figure 5 indicates that the performance loss of GSNC compared to ISIC, decreases as the number of available diversity branches increases. This effect is further verified by Fig. 8. The figure indicates that, for fixed L_d, the performance loss suffered by GSNC decreases as N_d increases. This can be justified because under GSNC branches are perfectly selected, thereby yielding higher quality estimates for the combining process. In other words, as the number of available diversity branches increases, while the number of combined branches remains fixed, the quality of the estimates improves, thus yielding higher performance.

This is in contrast to Fig. 6 which shows that increasing the number of available diversity branches has little effect on the performance loss of NSNC compared to ISIC. This is further confirmed by Fig. 8, where all the SNR penalty curves for NSNC are identical regardless of N_d.

Finally, note that for the case of MRC ($L_\mathrm{d} = N_\mathrm{d}$) the curves for GSNC and NSNC are identical for a given N_p, as expected. This can be seen by comparing the $16/16$ curves denoted by circles in Figs. 3 and 4, or the $2/2$ curves in Figs. 5 and 6. This is because the selection mechanism plays no role in the performance of the system when combining all of the available diversity branches, making GSNC and NSNC equivalent.

VII. Conclusion

We developed an analytical framework for evaluating the SEP for SSD, where the signals from a subset of diversity branches are selected and combined in the presence of channel estimation error. We considered i.i.d. Rayleigh fading channels and used an estimator structure based on the ML estimate which arises naturally as the sample mean of N_p pilot symbols. The framework is valid for arbitrary two-dimensional signaling constellations. The analysis and numerical results indicated that SSD systems employing GSNC or NSNC suffer performance degradation, due to the effects of channel estimation, compared to SSD systems employing ISIC. Despite this, our analytical results showed that the practical ML channel estimator still preserves the diversity order of an ideal SSD system with N_d branches. As a benchmark, we

[11] This loss, or SNR penalty, is defined in terms of the average branch SNR in (28) and (31).

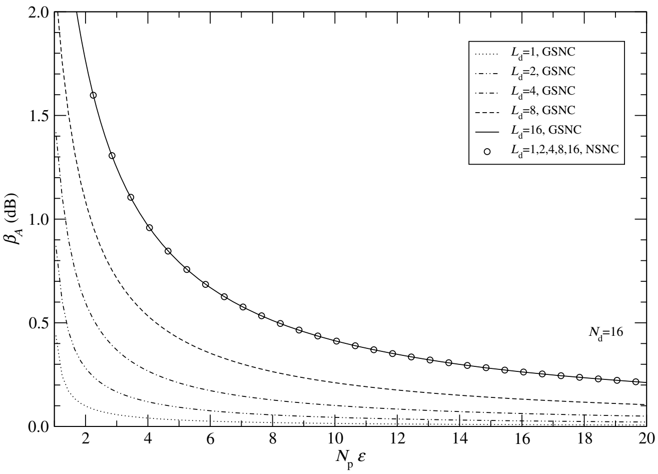

Fig. 7. SNR penalty as a function of $N_{\mathrm{p}}\,\varepsilon$, for $N_{\mathrm{d}} = 16$ various L_{d}.

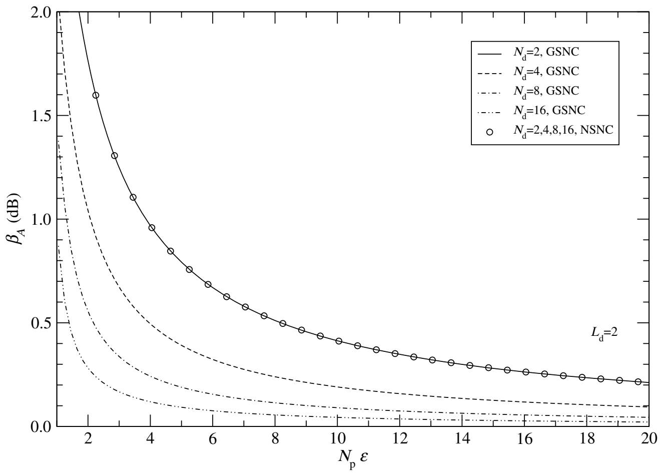

Fig. 8. SNR penalty as a function of $N_{\mathrm{p}}\,\varepsilon$, for $L_{\mathrm{d}} = 2$ and various N_{d}.

analyzed the performance of GSNC, where the estimation error only impacts the combining process, not the selection process, and showed that the SNR penalty depends on both N_{d} and L_{d}. However, this was not the case for NSNC, where channel estimation impacts both branch selection and branch combining. In this case, the asymptotic SNR penalty has a surprising lack of dependence on both the number of available diversity branches and the number of utilized branches.

APPENDIX
DISTRIBUTION OF THE DECISION VARIABLE

In this appendix we prove that D conditioned on $\mathcal{S}_{\mathrm{N}}$ is a nonzero mean, circularly symmetric complex Gaussian r.v.

Proof: To facilitate analysis, D may be rewritten as

$$D = \sum_{k:\, k \in \mathcal{O}_{\mathrm{N}}} d_k\,, \tag{32}$$

where we have defined the auxiliary variables

$$d_k \triangleq \hat{h}_k^* \left[\left(\hat{h}_k - e_k \right) s_i + n_k \right] .$$

Expanding d_k gives (33), shown at the bottom of the page, where the subscripts $(\cdot)_{k,\mathrm{r}}$, $(\cdot)_{k,\mathrm{i}}$ indicate the real and imaginary parts of the kth term, respectively.

Since each d_k depends on the channel only through $\hat{h}_k$, conditioning D on the set $\mathcal{S}_{\mathrm{N}}$ is equivalent to conditioning each d_k (for $k \in \mathcal{O}_{\mathrm{N}}$) on the particular $\hat{h}_k$ on which it depends. Thus, we proceed by examining the conditional distributions of the real and imaginary parts of each d_k, when conditioned on $\hat{h}_k$. From (33) we have,

$$\begin{aligned} \Re\{d_k\} = s_{i,\mathrm{r}} \left|\hat{h}_k\right|^2 &+ \left[\hat{h}_{k,\mathrm{r}} n_{k,\mathrm{r}} + \hat{h}_{k,\mathrm{i}} n_{k,\mathrm{i}}\right] \\ &- \left[s_{i,\mathrm{r}} \hat{h}_{k,\mathrm{r}} + s_{i,\mathrm{i}} \hat{h}_{k,\mathrm{i}}\right] e_{k,\mathrm{r}} \\ &- \left[s_{i,\mathrm{r}} \hat{h}_{k,\mathrm{i}} - s_{i,\mathrm{i}} \hat{h}_{k,\mathrm{r}}\right] e_{k,\mathrm{i}} \end{aligned} \tag{34}$$

$$\begin{aligned} \Im\{d_k\} = s_{i,\mathrm{i}} \left|\hat{h}_k\right|^2 &+ \left[\hat{h}_{k,\mathrm{r}} n_{k,\mathrm{i}} - \hat{h}_{k,\mathrm{i}} n_{k,\mathrm{r}}\right] \\ &+ \left[s_{i,\mathrm{r}} \hat{h}_{k,\mathrm{i}} - s_{i,\mathrm{i}} \hat{h}_{k,\mathrm{r}}\right] e_{k,\mathrm{r}} \\ &- \left[s_{i,\mathrm{r}} \hat{h}_{k,\mathrm{r}} + s_{i,\mathrm{i}} \hat{h}_{k,\mathrm{i}}\right] e_{k,\mathrm{i}} \end{aligned} \tag{35}$$

The first term in (34), conditioned on $\hat{h}_k$, is a constant. The second term is distributed as a zero mean Gaussian r.v. with variance $(N_0/2)|\hat{h}_k|^2$. For the third and fourth terms, we first need to find the conditional distribution of $e_{k,\mathrm{r}}$ and $e_{k,\mathrm{i}}$ given $\hat{h}_k$. Using Bayes' rule we have

$$\begin{aligned} f_{e_{\mathrm{i}}|\hat{h}_{\mathrm{i}}}(x|y) = f_{e_{\mathrm{r}}|\hat{h}_{\mathrm{r}}}(x|y) &= \frac{f_{\hat{h}_{\mathrm{r}}|e_{\mathrm{r}}}(y|x) f_{e_{\mathrm{r}}}(x)}{f_{\hat{h}_{\mathrm{r}}}(y)} \\ &= \frac{\frac{1}{\sqrt{2\pi\sigma_h^2}} \exp\left\{-\frac{(y-x)^2}{2\sigma_h^2}\right\} \frac{1}{\sqrt{2\pi\sigma_e^2}} \exp\left\{-\frac{x^2}{2\sigma_e^2}\right\}}{\frac{1}{\sqrt{2\pi\left(\sigma_h^2+\sigma_e^2\right)}} \exp\left\{-\frac{y^2}{2\left(\sigma_h^2+\sigma_e^2\right)}\right\}} \\ &= \frac{1}{\sqrt{2\pi}} \sqrt{\frac{\sigma_h^2+\sigma_e^2}{\sigma_h^2\sigma_e^2}} \exp\left\{-\frac{\left(x - \frac{\sigma_e^2}{\sigma_e^2+\sigma_h^2} y\right)^2}{2\frac{\sigma_h^2\sigma_e^2}{\sigma_h^2+\sigma_e^2}}\right\}. \end{aligned} \tag{36}$$

Thus, conditioned on $\hat{h}_k$, e_k is a complex Gaussian r.v. with mean $\hat{h}_k\,\sigma_e^2/(\sigma_e^2 + \sigma_h^2)$ and variance per dimension $\sigma_h^2\sigma_e^2/(\sigma_h^2 + \sigma_e^2)$. Using these results, we see that $\Re\{d_k\}$ is a conditionally Gaussian random variable with mean and

$$d_k = s_i \left|\hat{h}_k\right|^2 + \left[\hat{h}_{k,\mathrm{r}} n_{k,\mathrm{r}} + \hat{h}_{k,\mathrm{i}} n_{k,\mathrm{i}}\right] + j\left[\hat{h}_{k,\mathrm{r}} n_{k,\mathrm{i}} - \hat{h}_{k,\mathrm{i}} n_{k,\mathrm{r}}\right] - s_i\left[\hat{h}_{k,\mathrm{r}} e_{k,\mathrm{r}} + \hat{h}_{k,\mathrm{i}} e_{k,\mathrm{i}}\right] + j s_i\left[\hat{h}_{k,\mathrm{i}} e_{k,\mathrm{r}} - \hat{h}_{k,\mathrm{r}} e_{k,\mathrm{i}}\right] \tag{33}$$

variance given by

$$\mathbb{E}\Big\{\Re\{d_k\}\,\Big|\,\hat{h}_k\Big\} = s_{i,\mathrm{r}}\left|\hat{h}_k\right|^2 \frac{\sigma_h^2}{\sigma_e^2+\sigma_h^2} \quad (37)$$

$$\mathrm{var}\Big\{\Re\{d_k\}\,\Big|\,\hat{h}_k\Big\} = \left|\hat{h}_k\right|^2\left(\frac{N_0}{2}+|s_i|^2\,\frac{\sigma_h^2\sigma_e^2}{\sigma_h^2+\sigma_e^2}\right). \quad (38)$$

Using a similar argument it can be shown that the random variable $\Im\{d_k\}$ in (35) is also conditionally Gaussian with

$$\mathbb{E}\Big\{\Im\{d_k\}\,\Big|\,\hat{h}_k\Big\} = s_{i,\mathrm{i}}\left|\hat{h}_k\right|^2 \frac{\sigma_h^2}{\sigma_e^2+\sigma_h^2} \quad (39)$$

$$\mathrm{var}\Big\{\Im\{d_k\}\,\Big|\,\hat{h}_k\Big\} = \left|\hat{h}_k\right|^2\left(\frac{N_0}{2}+|s_i|^2\,\frac{\sigma_h^2\sigma_e^2}{\sigma_h^2+\sigma_e^2}\right). \quad (40)$$

Furthermore, it can be shown that

$$\mathbb{E}\Big\{\Re\{d_k\}\,\Im\{d_k\}\,\Big|\,\hat{h}_k\Big\} - \mathbb{E}\Big\{\Re\{d_k\}\,\Big|\,\hat{h}_k\Big\}\,\mathbb{E}\Big\{\Im\{d_k\}\,\Big|\,\hat{h}_k\Big\} = 0\,.$$

Therefore, when conditioned on $\hat{h}_k$, $\Re\{d_k\}$ and $\Im\{d_k\}$ are uncorrelated Gaussian r.v.'s and hence they are conditionally independent. Thus, d_k is a conditionally circularly symmetric complex Gaussian r.v. with nonzero mean. Since D is the sum of conditionally independent circulalry symmetric r.v.'s, d_k, it is also a conditionally circularly symmetric complex Gaussian r.v. with nonzero mean. □

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