Capacity of MIMO Systems With Antenna Selection

Andreas F. Molisch, *Fellow, IEEE*, Moe Z. Win, *Fellow, IEEE*,
Yang-Seok Choi, *Member, IEEE*, and Jack H. Winters, *Fellow, IEEE*

***Abstract*—We consider the capacity of multiple-input multiple-output systems with reduced complexity. One link-end uses all available antennas, while the other chooses the L out of N antennas that maximize capacity. We derive an upper bound on the capacity that can be expressed as the sum of the logarithms of ordered chi-square-distributed variables. This bound is then evaluated analytically and compared to the results obtained by Monte Carlo simulations. Our results show that the achieved capacity is close to the capacity of a full-complexity system provided that L is at least as large as the number of antennas at the other link-end. For example, for $L = 3$, $N = 8$ antennas at the receiver and three antennas at the transmitter, the capacity of the reduced-complexity scheme is 20 bits/s/Hz compared to 23 bits/s/Hz of a full-complexity scheme. We also present a suboptimum antenna subset selection algorithm that has a complexity of N^2 compared to the optimum algorithm with a complexity of $\binom{N}{L}$.**

***Index Terms*—Antenna arrays, information rates, MIMO systems.**

I. Introduction

MULTIPLE-INPUT MULTIPLE-OUTPUT (MIMO) wireless systems are those that have antenna arrays at both transmitter and receiver. Early simulation studies that revealed the potentially large capacities of those systems were done in the 1980s [1], and subsequent papers explored the capacity analytically [2], [3]. Since that time, interest in MIMO systems has exploded. Layered space–time (ST) receiver structures [4]–[6] and ST codes [7] make it possible to approach the capacity limits revealed in [2]. Commercial products based on such codes are under development [8]. Most importantly, the standard for third-generation cellular phones [3rd Generation Partnership Project (3GPP)] foresees the use of a simple ST code [9] with two transmit antennas and one or more receive antennas for circuit-switched communications and spatial multiplexing (multiple transmit data streams) for high-speed downlink packet data access [10].

In an earlier work, it was shown that the incremental gain of additional receive antennas is negligible if the total number of receive antennas N_{r} is far larger than the number of transmit antennas N_{t} [4].[1] This can be explained by the fact that additional antennas do not provide independent communication channels but just increase the diversity order. This motivates researchers to explore the possibility of replacing the maximal ratio diversity that is normally achieved in a such a MIMO system with selection diversity (SD). Thus, in this paper, we propose a reduced-complexity MIMO scheme that selects the L_{r} "best" of the available N_{r} antennas. Such a scheme can provide the full number of independent communication channels, and additionally an SD gain. Compared to the use of all antennas, the antenna selection has the advantage that only L_{r} instead of N_{r} receiver RF chains are required. We still require the full number of antenna elements, but these are usually inexpensive, as they are patch or dipole antennas that can be easily produced and placed.

Antenna selection, or more precisely, the principle of using L out of N antennas, was first studied in the context of antenna selection at one link-end, while only a single antenna is present at the other link-end [11]–[14]. This is referred to as "hybrid selection/maximum ratio combining (MRC)" in the literature. Therefore, we will employ the term "hybrid selection/MIMO" (H-S/MIMO) for the more general case studied in this paper, namely antenna selection at one link-end, and multiple antennas, all of which are used, at the other link-end.

There has been considerable interest in H-S/MIMO in recent years. The case of antenna selection at the transmitter is treated in [15] using Monte Carlo simulations; this paper also develops a criterion for optimal antenna set selection for high signal-to-noise ratios (SNRs); [16] extended this to the low-SNR case. It has been shown that antenna selection is beneficial in a low-rank environment [17] and in interference-limited systems [18]. A selection algorithm for minimizing the bit error probability of linear MIMO receivers is given in [19]. The use of ST codes in combination with antenna selection was investigated in [20] and [21]; the use of antenna selection in transmit–receive diversity systems with channel knowledge at both link-ends was

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A. F. Molisch was with AT&T Laboratories-Research, Middletown, NJ 07748 USA. He is now with Mitsubishi Electric Research Laboratory (MERL), Cambridge, MA 02139 USA and also at the Department of Electroscience, Lund University, Lund, Sweden (e-mail: Andreas.Molisch@ieee.org).

M. Z. Win was with AT&T Laboratories-Research, Middletown, NJ 07748 USA. He is now with the Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, Cambridge, MA 02139 USA (e-mail: win@ieee.org; moewin@mit.edu).

Y.-S. Choi was with AT&T Laboratories-Research, Middletown, NJ 07748 USA. He is now with Intel, Inc., Hillsboro, OR 97229 USA (e-mail: yschoi@ieee.org).

J. H. Winters was with AT&T Labs-Research, Middletown, NJ 07748 USA. He is now with Motia Inc., Middletown, NJ 07748 USA (email: jack.winters@ieee.org).

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[1]Under certain circumstances, increasing that number can even lead to performance degradation, as the channel estimation becomes more difficult and introduces estimation errors.

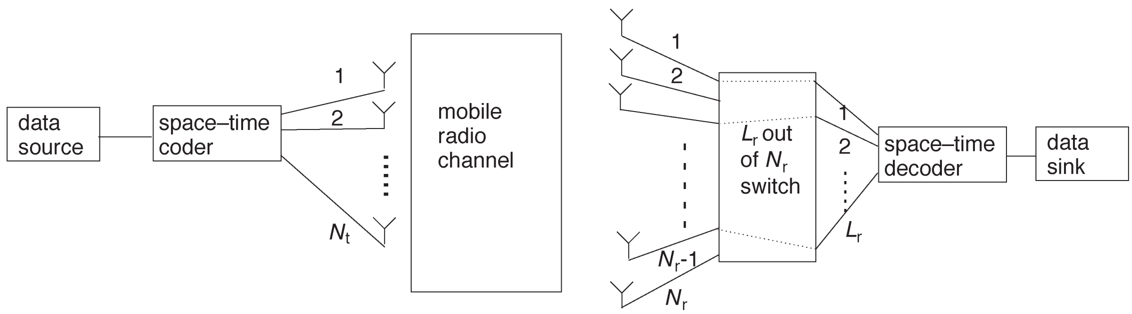

Fig. 1. Block diagram of the considered system.

treated in [22].[2] A more detailed overview of the literature is given in [28].

In this paper, we derive analytical bounds for the capacity distribution function of an H-S/MIMO system at one link-end. We show that an exact antenna selection algorithm requires high computational complexity and propose several alternative methods that have much lower complexity while performing almost as well as the exact selection criteria. The rest of the paper is organized as follows: In Section II, we set up the system model. Analytical bounds for the capacity are derived in Section III. Next, we present a fast antenna selection algorithm in Section IV. Section V gives evaluations for the analytical bounds of H-S/MIMO and compares them to numerical simulation results. Conclusions and system design considerations are given in Section VI.

II. System Model

We consider the case where the transmitter uses all available antennas while the receiver uses antenna selection. Fig. 1 exhibits a block diagram. At the transmitter, the data stream enters an ST encoder, whose outputs are forwarded to the N_{t} transmit antennas. The signals are subsequently upconverted to passband, amplified by a power amplifier, and filtered. For our model, we omit these stages, as well as their equivalents at the receiver, which allows us to treat the whole problem in equivalent baseband. Note, however, that it is exactly these parts that are most expensive and make the use of reduced-complexity systems desirable.

From the antennas, the signal is sent through the mobile radio channel, which is assumed to be flat fading and quasi-static. By quasi-static, we mean that the coherence time of the channel is so long that "a large number" of bits can be transmitted within this time. More specifically, we assume that the data are encoded with near Shannon limit achieving codes.[3] It has been shown that LDPC codes with a block length of 10 000 approach the Shannon limit within less than 1 dB [30]. For a data rate of 10 Mbits/s, such a block can be transmitted within 1 ms, which is shorter than the typical 10 ms coherence time of wireless channels. Thus, each channel realization can be associated with a (Shannon-AWGN) capacity value. The capacity thus becomes a random variable (RV), rendering the concept of "capacity cumulative distribution function" and "outage capacity" meaningful performance measures [2].

We denote the $N_{\mathrm{r}} \times N_{\mathrm{t}}$ matrix of the channel as

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1N_{\mathrm{t}}} \\ h_{21} & h_{22} & \cdots & h_{2N_{\mathrm{t}}} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_{\mathrm{r}}1} & h_{N_{\mathrm{r}}2} & \cdots & h_{N_{\mathrm{r}}N_{\mathrm{t}}} \end{pmatrix}. \tag{1}$$

If the channel is Rayleigh fading, the h_{ij} are independent identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian RVs with unit variance, i.e., the real and imaginary parts have a variance of 1/2 each. Consequently, the power carried by each transmission channel h_{ij} is chi-square distributed with 2 degrees of freedom. The channel also adds white Gaussian noise, which is assumed to be independent among the N_{r} receiver antenna elements. Following [2], we consider the case in which the h_{ij} are independently fading, as this simplifies the theoretical analysis. More involved channel models are discussed, e.g., in [31]–[33].

The received signal, which is written as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = \mathbf{x} + \mathbf{n} \tag{2}$$

is received by N_{r} antenna elements, where $\mathbf{s}$ is the transmit signal vector and $\mathbf{n}$ is the noise vector. A control algorithm (to be discussed in Sections III and IV) selects the best L_{r} of the available N_{r} antenna elements and downconverts their signals for further processing (note that only L_{r} receiver chains are required). ST encoder and decoder are assumed to be ideal so that the capacity can be achieved. We assume ideal knowledge of the channel at the receiver so that it is always possible to select the best antennas. However, we do not assume any knowledge of the channel at the transmitter. This implies that no waterfilling can be used and that the available transmitter power is equally distributed among the transmit antennas.

III. Theory

Let us first explore the scenarios that are suited for H-S/MIMO. As shown in [2], the capacity is linearly proportional to $\min(N_{\mathrm{r}}, N_{\mathrm{t}})$. Any further increase of either N_{r} or N_{t} while keeping the other fixed only increases the diversity

[2]Parallel to our work (see also [23] and [24]), an alternative algorithm for the selection of antenna subsets was presented and a lower bound of the capacity was derived in [25]–[27]; this algorithm will also be discussed in Section IV.

[3]Such a code could be, e.g., the combination of ST processing [6] with a low-density parity check code [29].

order and possibly the mean SNR, possibly. Thus, if the number of antennas at one link-end is limited, e.g., due to space restrictions, a further increase in the antenna number at the other link-end does not allow us to add statistically independent transmission channels (which would imply linear increase in system capacity), but only provides additional diversity. Since it is well known that SD has the same diversity order as that of MRC [34], we can anticipate that a hybrid scheme with $N_\mathrm{r} > L_\mathrm{r} = N_\mathrm{t}$ will give a good performance. In the next subsections, we will give a quantitative confirmations of this conjecture.

A. Exact Expression for the Capacity

The capacity of MIMO system using all antenna elements is given by [2]

$$C_\mathrm{full} = \log_2\left[\det\left(\mathbf{I}_{N_\mathrm{r}} + \frac{\overline{\Gamma}}{N_\mathrm{t}}\mathbf{H}\mathbf{H}^\dagger\right)\right] \tag{3}$$

where $\mathbf{I}_{N_\mathrm{r}}$ is the $N_\mathrm{r} \times N_\mathrm{r}$ identity matrix, $\overline{\Gamma}$ is the mean SNR per receiver branch, and superscript † denotes the Hermitian transpose. The receiver now selects those antennas that allow a maximization of the capacity, so that

$$C_\mathrm{select} = \max_{S(\widetilde{\mathbf{H}})}\left\{\log_2\left[\det\left(\mathbf{I}_{L_\mathrm{r}} + \frac{\overline{\Gamma}}{N_\mathrm{t}}\widetilde{\mathbf{H}}\widetilde{\mathbf{H}}^\dagger\right)\right]\right\} \tag{4}$$

where $\widetilde{\mathbf{H}}$ is created by deleting $N_\mathrm{r} - L_\mathrm{r}$ rows from $\mathbf{H}$, and $S(\widetilde{\mathbf{H}})$ denotes the set of all possible $\widetilde{\mathbf{H}}$, whose cardinality is $\binom{N_\mathrm{r}}{L_\mathrm{r}}$.

The optimum choice of antennas requires the knowledge of the complete channel matrix. This may seem to necessitate the use of N_r RF chains, which is in contrast with a low-complexity system. However, in a sufficiently slowly changing environment, the L_r RF chains can be cycled through the N_r antennas during the training bits. In other words, RF chains are connected to the first L_r antennas during the first part of the training sequence, then to the second L_r antenna during the next part, and so on. At the end of the training sequence, we pick the best L_r antennas. Thus, we only need a few more training bits instead of additional RF chains and the decrease in the spectral efficiency due to those additional training bits is negligible, especially in high-data-rate systems.

B. Capacity Bound for $L_\mathrm{r} \leq N_\mathrm{t}$

An exact analytical solution for C_select seems difficult. Thus, we derive analytical bounds in this subsection and verify them with Monte Carlo simulations in Section V. Our starting point is the upper capacity bound for the full-complexity system with $N_\mathrm{t} \leq N_\mathrm{r}$ [2]

$$C_\mathrm{full} \leq \sum_{i=1}^{N_\mathrm{t}} \log_2\left(1 + \frac{\overline{\Gamma}}{N_\mathrm{t}}\gamma_i\right) \tag{5}$$

where the γ_i are independent chi-square-distributed RVs with $2N_\mathrm{r}$ degrees of freedom. The equality applies in the "unrealistic case when each of the N_t transmitted components is received by a separate set of N_r antennas in a manner where each signal component is received with no interference from the others" [2].

In our case, we select the best L_r out of N_r receive antennas, where $L_\mathrm{r} \leq N_\mathrm{t}$. The upper bound can be obtained similar to (5), except for exchanging the role of transmitter and receiver, and selecting those antennas whose instantaneous realizations of γ_i are the largest. Since this equation is a crucial starting point, let us elaborate on its physical interpretation. We consider a system where each of the N_r receive antennas has its own set (of size N_t) of transmit antennas. Naturally, this case is not feasible in practice but must result in an upper bound of the capacity. Each set of transmit antennas corresponding to each of the N_r receive antennas can carry one data stream. The maximum SNR (which also achieves maximum capacity) for this data stream can be obtained with maximal ratio transmission, which in turn results in chi-square-distributed SNR with $2N_\mathrm{t}$ degrees of freedom at the receiver output. Finally, we select those L_r (out of N_r) receive antennas that give the best SNR, and thus highest capacity. The capacity bound with antenna selection is thus

$$C_\mathrm{bound} = \sum_{i=1}^{L_\mathrm{r}} \log_2\left(1 + \rho\gamma_{(i)}\right) \tag{6}$$

where $\rho = \overline{\Gamma}/N_\mathrm{t}$, and the $\gamma_{(i)}$ are ordered chi-square-distributed variables with $2N_\mathrm{t}$ degrees of freedom, out of a set of N_r.[4]

The joint statistics of the ordered SNRs $\gamma_{(i)}$ is shown in (7) at the bottom of the page [14], where $\Gamma(\cdot)$ is Euler's Gamma function [35].

Thus, the characteristic function of the capacity bound is

$$\Phi(j\nu) = \frac{N_\mathrm{r}!}{\Gamma(N_\mathrm{t})^{N_\mathrm{r}}} \int_0^\infty \mathrm{d}\gamma_{(1)} \int_0^{\gamma_{(1)}} \mathrm{d}\gamma_{(2)} \cdots \int_0^{\gamma_{(N_\mathrm{r}-1)}} \mathrm{d}\gamma_{(N_\mathrm{r})} \times \exp\left[-j\nu \sum_{i=1}^{L_\mathrm{r}} \log_2\left(1 + \rho\gamma_{(i)}\right)\right] \prod_{i=1}^{N_\mathrm{r}} \gamma_{(i)}^{N_\mathrm{t}-1} \exp\left(-\gamma_{(i)}\right). \tag{8}$$

[4]We use $\{\gamma_{(i)}\}$ to denote the order set of $\{\gamma_i\}$, i.e., $\gamma_{(1)} > \gamma_{(2)} > \cdots \gamma_{[N_\mathrm{r}]}$. Note that the possibility of at least two equal $\gamma_{(i)}$'s is excluded as $\gamma_{(i)} \neq \gamma_{(j)}$ almost surely for continuous RVs γ_i.

$$p_{\gamma_{(i)}}\left(\gamma_{(1)}, \gamma_{(2)}, \ldots, \gamma_{(N_\mathrm{r})}\right) = \begin{cases} N_\mathrm{r}! \prod_{i=1}^{N_\mathrm{r}} \frac{1}{\Gamma(N_\mathrm{t})} \gamma_{(i)}^{N_\mathrm{t}-1} \exp\left(-\gamma_{(i)}\right), & \text{for } \gamma_{(1)} > \gamma_{(2)} > \cdots > \gamma_{(N_\mathrm{r})} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

First, we perform the integrations over the $N_\mathrm{r} - L_\mathrm{r}$ discarded antennas. As shown in the Appendix, these $N_\mathrm{r} - L_\mathrm{r}$ result in an expression of the form

$$d^{(N_\mathrm{r}-L_\mathrm{r})} + \left[\sum_{p=1}^{N_\mathrm{r}-L_\mathrm{r}} \exp\left(-b_p^{(N_\mathrm{r}-L_\mathrm{r})}\gamma_{(L_\mathrm{r})}\right) \times \sum_{k=0}^{(N_\mathrm{r}-L_\mathrm{r}-p+1)(N_\mathrm{t}-1)} c_{p,k}^{(N_\mathrm{r}-L_\mathrm{r})}\gamma_{(L_\mathrm{r})}^{k}\right]. \quad (9)$$

The values of the coefficients b, d, and c are computed via an iteration. We initialize with

$$\begin{aligned} b_p^{(0)} &= 0 \quad \text{for all } p \\ c_{p,k}^{(0)} &= 0 \quad \text{for all } p, k \\ d^{(0)} &= 1 \end{aligned} \quad (10)$$

and then perform $N_\mathrm{r} - L_\mathrm{r}$ iterations

$$b_p^{(q+1)} = b_p^{(q)} + 1, \quad \text{for } 1 \le p \le q \quad (11)$$

$$b_{q+1}^{(q+1)} = 1 \quad (12)$$

[see (13)–(15) at the bottom of the page].

For the next step of the iteration, it is advantageous to rewrite (9) as

$$\sum_{p=0}^{N_\mathrm{r}-L_\mathrm{r}} \exp\left(-b_p^{(N_\mathrm{r}-L_\mathrm{r})}\gamma_{(L_\mathrm{r})}\right) \times \sum_{k=0}^{(N_\mathrm{r}-L_\mathrm{r}-p+1)(N_\mathrm{t}-1)} c_{p,k}^{(N_\mathrm{r}-L_\mathrm{r})}\gamma_{(L_\mathrm{r})}^{k+\alpha^{(N_\mathrm{r}-L_\mathrm{r})}}$$

so that

$$\begin{aligned} c_{0,0}^{(N_\mathrm{r}-L_\mathrm{r})} &= d^{(N_\mathrm{r}-L_\mathrm{r})} \\ c_{0,k}^{(N_\mathrm{r}-L_\mathrm{r})} &= 0 \\ b_0^{(N_\mathrm{r}-L_\mathrm{r})} &= 0 \\ \alpha^{(N_\mathrm{r}-L_\mathrm{r})} &= 0. \end{aligned} \quad (16)$$

We then perform the next $L_\mathrm{r} - 1$ integrations, which yield (see Appendix) an expression of the form

$$\sum_{p=0}^{N_\mathrm{r}-L_\mathrm{r}} \exp\left(-b_p^{(N_\mathrm{r}-1)}\gamma_{(1)}\right) \sum_{k=0}^{M} c_{p,k}^{(N_\mathrm{r}-1)}\gamma_{(1)}^{k+\alpha^{(N_\mathrm{r}-1)}} \quad (17)$$

where the parameters $c_{p,r}^{(N_\mathrm{r})}$, $\alpha^{(N_\mathrm{r})}$, and $b_p^{(N_\mathrm{r})}$ are again computed via a recursion. In each step, we first compute

$$\widehat{b}_p^{(q)} = b_p^{(q)} + 1 \quad (18)$$

$$\widehat{\alpha}^{(q)} = \alpha^{(q)} + \frac{j\nu}{\ln(2)} \quad (19)$$

$$\widehat{c}_{p,k}^{(q)} = \begin{cases} c_{p,k-(N_\mathrm{t}-1)}^{(q)}, & k = M \\ c_{p,k-(N_\mathrm{t}-1)}^{(q)} + \frac{j\nu}{\rho\ln(2)} c_{p,k-N_\mathrm{t}}^{(q)}, & N_\mathrm{t} - 1 \le k < M \\ \frac{j\nu}{\rho\ln(2)} c_{p,k-N_\mathrm{t}}^{(q)}, & k = N_\mathrm{t} - 2 \\ 0, & \text{otherwise} \end{cases}. \quad (20)$$

Then we can perform the second step, which is obtaining coefficients for the next iteration step

$$\alpha^{(q+1)} = \widehat{\alpha}^{(q)} \quad (21)$$

$$b_p^{(q+1)} = \widehat{b}_p^{(q)} \quad (22)$$

$$c_{p,r}^{(q+1)} = \sum_{k=0}^{r-1} \widehat{c}_{p,k}^{(q)} f_{p,r-1-k}^{(q)} \quad (23)$$

with

$$f_{p,n}^{(q)} = \frac{\left[\widehat{b}_p^{(q)}\right]^n}{\prod_{i=0}^{n}\left(k + \alpha^{(q)} + 1 + i\right)}. \quad (24)$$

The final integration and incorporation of constant multiplicative factors yields

$$\Phi(j\nu) = \frac{\rho^{\frac{j\nu L_\mathrm{r}}{\ln(2)}} N_\mathrm{r}!}{\Gamma(N_\mathrm{t})^{N_\mathrm{r}}} \sum_{p=0}^{N_\mathrm{r}-L_\mathrm{r}} \sum_{r=0}^{M} \widehat{c}_{p,r}^{(N_\mathrm{r}-1)} \frac{\Gamma\left(r + \widehat{\alpha}^{(N_\mathrm{r}-1)} + 1\right)}{\left[\widehat{b}_p^{(N_\mathrm{r}-1)}\right]^{r+\widehat{\alpha}^{(N_\mathrm{r}-1)}+1}}. \quad (25)$$

$$\widehat{c}_{p,k}^{(q)} = \begin{cases} c_{p,k-(N_\mathrm{t}-1)}^{(q)}, & \text{for } (q-p+2)(N_\mathrm{t}-1) \ge k \ge (N_\mathrm{t}-1) \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

$$d^{(q+1)} = d^{(q)}(N_\mathrm{t}-1)! + \sum_{p=1}^{q} \sum_{t=0}^{(q-p+2)(N_\mathrm{t}-1)} \widehat{c}_{p,t}^{(q)} \frac{t!}{\left(b_p^{(q+1)}\right)^{t+1}} \quad (14)$$

$$c_{p,k}^{(q+1)} = \begin{cases} -\sum_{t=0}^{(q-p+2)(N_\mathrm{t}-1)-k} \frac{\widehat{c}_{p,k+t}^{(q)}}{\left(b_p^{(q+1)}\right)^{t+1}} \frac{(k+t)!}{k!}, & \text{for } 1 \le p \le q \\ -d^{(q)} \frac{(N_\mathrm{t}-1)!}{k!}, & \text{for } p = q+1 \end{cases} \quad (15)$$

The upper summation limit M is theoretically infinite, but the sum converges reasonably fast. In our computations, $M = 50$ proved to be sufficient for $N_{\rm r} = 8$. Details about the derivation of the recursion relations for the coefficients can be found in the Appendix.

The above equation yields the characteristic function of the capacity bound (note that we have omitted the functional dependence of the parameters on ν for notational convenience). The probability density function (pdf) of the capacity bound is obtained by performing an inverse Fourier transformation, which can be accomplished by a fast Fourier transform.

C. Capacity Bound for $L_{\rm r} > N_{\rm t}$

The bound derived above is quite tight for $L_{\rm r} \leq N_{\rm t}$ but tends to become rather loose for $L_{\rm r} > N_{\rm t}$. Especially, this bound suggests an "almost" linear increase of the capacity with $L_{\rm r}$.[5] However, we have shown in Section III-A that we can only anticipate a logarithmic increase. We thus derive an alternative bound that reflects this fact.

We consider the situation where each of the $N_{\rm t}$ transmit antennas transmits an independent data stream. Furthermore, we assume the (practically impossible) situation where none of the data streams interferes with each other. This is equivalent to having $N_{\rm t}$ single input multiple output (SIMO) systems each with separate $N_{\rm r}$ receive antenna elements dedicated to the reception of one such data stream. In each of the SIMO systems, we perform H-S/MRC, so that the (normalized) SNR of the jth SIMO system is given by

$$\sum_{i=1}^{L_{\rm r}} \tilde{\gamma}_{(i)}. \tag{26}$$

Assuming that none of the data streams interferes with any other, the total capacity is then

$$C_{\rm select} \leq \sum_{j=1}^{N_{\rm t}} \log_2 \left[1 + \rho \sum_{i=1}^{L_{\rm r}} \tilde{\gamma}_{(i)} \right] = \sum_{j=1}^{N_{\rm t}} \xi_j = \Psi \tag{27}$$

where the $\tilde{\gamma}_{(i)}$ are ordered chi-square-distributed variables with 2 degrees of freedom, taken from a set of $N_{\rm r}$ available ones. Since the ξ_j are i.i.d., the characteristic function of Ψ is finally

$$\prod_{j=1}^{N_{\rm t}} \widetilde{C}_j(\nu) = \widetilde{C}_1(\nu)^{N_{\rm t}}. \tag{28}$$

The computation of $\widetilde{C}_j(\nu)$, i.e., the characteristic function of ξ_j, is similar to the method described in [22] and [36].

IV. Fast Antenna Selection Algorithms

The optimum selection of the antennas requires $\binom{N_{\rm r}}{L_{\rm r}}$ computations of determinants and is thus computationally intensive. It seems thus worthwhile to investigate suboptimum algorithms with lower computational complexity. In this section, we present a family of such algorithms that result in a small SNR penalty while drastically reducing computation time.

The determinant in (4) can be written as

$$\det\left(\mathbf{I}_{L_{\rm r}} + \frac{\overline{\Gamma}}{N_{\rm t}} \widetilde{\mathbf{H}}\widetilde{\mathbf{H}}^\dagger\right) = \prod_{k=1}^{r} \left(1 + \frac{\overline{\Gamma}}{N_{\rm t}} \tilde{\lambda}_k^2\right) \tag{29}$$

where r is the rank of the channel matrix and $\tilde{\lambda}_k$ is the singular value of $\widetilde{\mathbf{H}}$. Note that the rank and the singular values should be maximized for the maximum capacity. Suppose there are two rows of $\mathbf{H}$ which are identical. Clearly, only one of these rows should be selected in $\widetilde{\mathbf{H}}$. Since these two rows carry the same information about the signal components, any one of these two rows may be deleted. In addition, if they have different powers (i.e., square of the norm of the row), we select the row with the higher power. When there are no identical rows, we choose two rows for the possible deletion whose correlation is the highest and delete the one with the lower power. In this manner, we can have the channel matrix $\widetilde{\mathbf{H}}$ whose rows are minimally correlated and have maximum powers. The above argument leads to the following algorithm.

1) The channel vector $\mathbf{h}_k$ is defined as the kth row of $\mathbf{H}$, with k being an element of the set $X = \{1, \ldots N_{\rm r}\}$.
2) For all k and l, $k > l$, in X, compute the correlation $\Xi(k,l)$ defined as $\Xi(k,l) = |\langle \mathbf{h}_k, \mathbf{h}_l \rangle|$, where $\langle \mathbf{a}, \mathbf{b} \rangle$ represents an inner product between vector $\mathbf{a}$ and $\mathbf{b}$.
3) Loop
   a) Choose the k and l (with k, $l \in X, k > l$) that give the largest $\Xi(k,l)$. If $\|\mathbf{h}_k\|^2 \geq \|\mathbf{h}_l\|^2$, eliminate $\mathbf{h}_l$, otherwise, eliminate $\mathbf{h}_k$.
   b) Delete l (or k) from X.
   c) Go to Loop until $N_{\rm r} - L_{\rm r}$ rows are eliminated.

The method defined above shall be called the correlation based method (CBM). It does not require the SNR value and it is based on the correlation of the rows of the channel matrix $\langle \mathbf{h}_k, \mathbf{h}_l \rangle$, which can be approximated by the correlation of the noisy estimates $E\{y_k y_l^*\}$.

As an alternative method when the SNR is available, we suggest to use the mutual information between y_k and y_l. The zero-valued mutual information means that the kth receive antenna output y_k and the lth output y_l carry totally different information. This occurs when the corresponding channel vectors $\mathbf{h}_k$ and $\mathbf{h}_l$ are orthogonal. On the other hand, when the mutual information between y_k and y_l has a maximum value, y_k and y_l carry the same information so that we can delete one of them. The mutual information is defined as [37]

$$I(y_k; y_l) = G(y_k) + G(y_l) - G(y_k, y_l) \tag{30}$$

where $G(\cdot)$ denotes the entropy.[6]

[5]Note that the increase is only "almost" linear because we are dealing with ordered stochastic variables. Thus, including more terms in the summation tends to give terms that have a lower SNR and thus a lower capacity.

[6]We deviate from the usual entropy notation H to avoid confusion with the channel matrix $\mathbf{H}$.

In the MIMO system, the mutual information can be written as

$$I(y_k; y_l) = \log \frac{\left(\|\mathbf{h}_k\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right)\left(\|\mathbf{h}_l\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right)}{\left(\|\mathbf{h}_k\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right)\left(\|\mathbf{h}_l\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right) - |\langle \mathbf{h}_k, \mathbf{h}_l \rangle|^2 \frac{\overline{\Gamma}^2}{N_\mathrm{t}^2}}. \tag{31}$$

The above equation can be rewritten as

$$\begin{aligned} I(y_k; y_l) &= \log\left(\|\mathbf{h}_k\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right) \\ &\quad - \log \frac{\left(\|\mathbf{h}_k\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right)\left(\|\mathbf{h}_l\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right) - |\langle \mathbf{h}_k, \mathbf{h}_l \rangle|^2 \frac{\overline{\Gamma}^2}{N_\mathrm{t}^2}}{\left(\|\mathbf{h}_l\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right)} \\ &= \log\left(\|\mathbf{h}_k\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right) \\ &\quad - \log\left(1 + \frac{\|\mathbf{h}_k\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + \|\mathbf{h}_k\|^2 \|\mathbf{h}_l\|^2 \frac{\overline{\Gamma}^2}{N_\mathrm{t}^2} - |\langle \mathbf{h}_k, \mathbf{h}_l \rangle|^2 \frac{\overline{\Gamma}^2}{N_\mathrm{t}^2}}{\|\mathbf{h}_l\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1}\right). \end{aligned} \tag{32}$$

Since $\|\mathbf{h}_k\|^2 \|\mathbf{h}_l\|^2 \geq |\langle \mathbf{h}_k, \mathbf{h}_l \rangle|^2$, the mutual information is upper bounded as follows:

$$I(y_k; y_l) \leq \log\left(\|\mathbf{h}_k\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right). \tag{33}$$

Similarly, we have

$$I(y_k; y_l) \leq \log\left(\|\mathbf{h}_l\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right). \tag{34}$$

Finally, the mutual information is upper bounded by

$$I(y_k; y_l) \leq \min\left\{\log\left(\|\mathbf{h}_k\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right), \log\left(\|\mathbf{h}_l\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right)\right\}. \tag{35}$$

We therefore define the normalized mutual information

$$I_0(y_k; y_l) = \frac{I(y_k; y_l)}{\min\left\{\log\left(\|\mathbf{h}_k\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right), \log\left(\|\mathbf{h}_l\|^2 \frac{\overline{\Gamma}}{N_\mathrm{t}} + 1\right)\right\}} \tag{36}$$

as a measure of how close the two RVs are.

We can also apply the mutual-information-based technique to x_k, which is the signal component of y_k, in order to avoid requiring the SNR value. Then the mutual information between the data components x_k and x_l is

$$I(x_k; x_l) = \log \frac{\|\mathbf{h}_k\|^2 \|\mathbf{h}_l\|^2}{\|\mathbf{h}_k\|^2 \|\mathbf{h}_l\|^2 - |\langle \mathbf{h}_k, \mathbf{h}_l \rangle|^2}. \tag{37}$$

Similarly, we define the normalized mutual information as

$$I_0(x_k; x_l) = \frac{I(x_k; x_l)}{\min\{|\log\|\mathbf{h}_k\|^2|, |\log\|\mathbf{h}_l\|^2|\}}. \tag{38}$$

The antenna selection algorithms based on mutual information then have a similar program structure as the one based on correlation (CBM). All that is required is to replace Ξ by I_0 as defined in (36) (henceforth referred to as MIBM) or (38) (MIBM2).

V. Results

In this section, we evaluate the bounds derived in previous sections and compare them to Monte Carlo simulations. We first generate random realizations of mobile radio channels with transfer function h_{ij}, which is an i.i.d. circularly complex Gaussian RV with zero mean and a variance of 1/2 for the real and imaginary parts. From each realization of the matrix $\mathbf{H}$, a complete set $S(\widetilde{\mathbf{H}})$ of $\binom{N_\mathrm{r}}{L_\mathrm{r}}$ possible matrices $\widetilde{H}$ are obtained by eliminating all possible permutations of $N_\mathrm{r} - L_\mathrm{r}$ rows from the matrix $\mathbf{H}$. For each of the $\widetilde{H}$, we computed the capacity by (4), and selected the largest capacity from the set.

Fig. 2 shows the cumulative distribution function of capacity for $N_\mathrm{r} = 8$, $N_\mathrm{t} = 3$, and various L_r. The SNR is 20 dB, and in the following, we consider the 10% outage capacity. With full exploitation of all available elements, 21.8 bits/s/Hz can be transmitted over the channel. This number decreases gradually as the number of selected elements L_r is decreased, reaching 18.2 bits/s/Hz at $L_\mathrm{r} = 3$. For $L_\mathrm{r} < N_\mathrm{t}$, the capacity decreases drastically, since a sufficient number of antennas to provide N_t independent transmission channels is no longer available. These trends are well reflected in the bounds: the bound for the full-complexity system is 22.7 bits/s/Hz, decreasing to 20.0 bits/s/Hz at $L_\mathrm{r} = 3$. We also find that the bounds are tight for $L_\mathrm{r} < N_\mathrm{t}$, become looser for $L_\mathrm{r} \simeq N_\mathrm{t}$, and become tighter again for $L_\mathrm{r} \gg N_\mathrm{t}$. This fact can be explained as follows. As we have noted in Section III, the bound reflects the situation that each of the received signals has its own set (of size N_t) of transmit antennas. This is fulfilled perfectly for $L_\mathrm{r} = 1$ and becomes a progressively worse approximation as L_r increases. Note that this bound is used only up to $L_\mathrm{r} \leq N_\mathrm{t}$. For larger L_r, we bound the capacity by the case where we have N_t independent data streams, none of which interferes with each other. Now it is well known [38] that N receive antennas can suppress K interfering data streams while retaining a diversity order of $N - K$ for the remaining data streams. The bound is thus approximately equivalent to a situation where we have $L_\mathrm{r} + N_\mathrm{t}$ receive chains (instead of the L_r that are actually existing). The relative error thus becomes progressively smaller as L_r increases. Finally, Fig. 2 also shows the capacity of an $N_\mathrm{t} \times L_\mathrm{r}$ full complexity system. This shows us how much performance we would lose when using (for a fixed number of RF chains) only the minimum number of antenna elements.

Fig. 3 shows the influence of the SNR on the capacity. We plot the improvement of the 10% outage capacity of an H-S/MIMO system over a single-antenna system. We see that the capacity increase is very large at low SNRs (factor of 25 at $\mathrm{SNR} = 0$ dB), while for high SNRs, it tends to a fixed

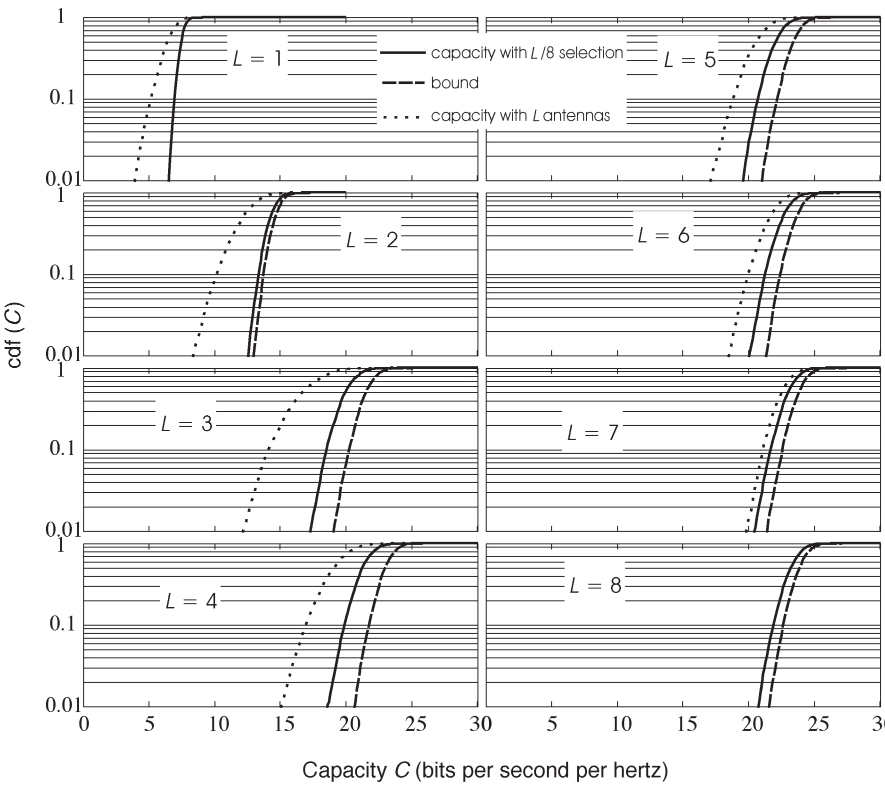

Fig. 2. Exact capacity (solid curves) and bound (dashed curves) for $N_{\mathrm{r}} = 8$, $N_{\mathrm{t}} = 3$, $\mathrm{SNR} = 20$ dB. Dotted lines show capacity of $N_{\mathrm{t}} = 3$, $N_{\mathrm{r}} = L$ system. Note that for $L = 1$, the solid and the dashed lines coincide, while for $L = 8$, the solid and the dotted lines coincide.

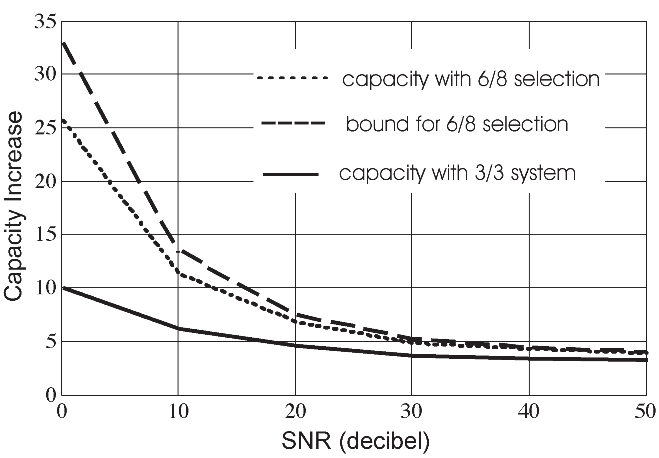

Fig. 3. Ratio of 10% outage capacity of a system with $N_{\mathrm{r}} = 8$, $L_{\mathrm{r}} = 6$, $N_{\mathrm{t}} = 3$, over that of a single-antenna system: bound (dashed); exact (dotted); and system with $N_{\mathrm{t}} = L_{\mathrm{r}} = 3$ (solid).

value of about 4. A factor of 3 in the capacity increase can be attributed to the number of independent communication channels between the transmitter and receiver. The remainder of the capacity increase is due to the diversity effect. Note also that Fig. 3 plots the improvement in 10% outage capacity. If we were to consider the mean capacity, the influence of the SNR on the relative capacity increase would be significantly reduced. For standard $N_{\mathrm{r}} = L_{\mathrm{r}} = N_{\mathrm{t}}$ systems, the relative mean capacity increase (compared to a SISO system) is practically independent of the SNR.

Another interesting point is the comparison between antenna selection criteria based on capacity and antenna selection based on the suboptimum algorithm that selects antennas with the highest powers. In our MC simulations, we also recorded for each channel realization the indices of those antennas that have the highest SNR. The indices of those antennas were then compared to those of the antennas that were chosen to maximize capacity. We found that only in about 50% of all channel realizations did the two selections agree with each other. The geometric interpretation of this behavior is that for the deterministic case (corresponding to one channel realization), the phase shifts between the antenna elements are the decisive factors for capacity, and are far more important than instantaneous SNR [39]. Fig. 4 gives the capacities that are obtained by antenna selection based on an SNR criterion. We see that for $L_{\mathrm{r}} \ll N_{\mathrm{r}}$, the 10% outage capacity decreases from 18.2 to 14.3 bits/s/Hz at 20 dB SNR when the SNR- (instead of capacity-) based criterion is used for antenna selection. This loss gets smaller as L_{r} approaches N_{r}.

The performance of our fast antenna selection algorithms is detailed in Figs. 5 and 6. Again, the number of transmit and receive antennas is 3 and 8, respectively. For comparison, the ILM technique [25] is also evaluated. Each algorithm selects three receive antennas out of eight receive antennas. Among the proposed algorithms, the MIB methods outperform the CB technique. The ILM is shown to have a performance that is very close to the exhaustive search. However, it requires Gram–Schmidt orthogonalization and thus matrix inversion/multiplications. The complexity thus goes like $N_{\mathrm{r}} L_{\mathrm{r}} N_{\mathrm{t}}^3$ or $N_{\mathrm{r}}(N_{\mathrm{r}} - L_{\mathrm{r}}) N_{\mathrm{t}}^3$, whichever is the smaller [27]. In contrast, the main computational burden of our fast algorithms comes from the calculation of vector multiplications $\langle \mathbf{h}_k, \mathbf{h}_l \rangle$.

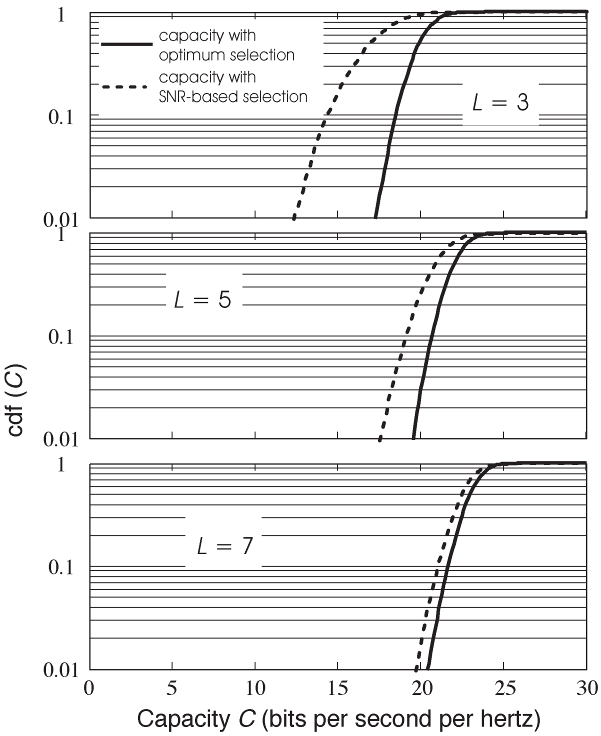

Fig. 4. CDF of the capacity of a system with $N_{\rm r} = 8$, $N_{\rm t} = 3$. Selection of antenna by capacity criterion (solid) and by SNR criterion (dotted).

Each of those has a complexity of $N_{\rm t}$, and we need $N_{\rm r}(N_{\rm r} + 1)/2$ of them. The complexity thus goes as $N_{\rm t}N_{\rm r}(N_{\rm r} + 1)/2$. The choice between ILM and MIBM2 (or a similar algorithm) is a tradeoff between performance and complexity.

Assuming that ideal coding is employed, the outage probability when the bandwidth efficiency is 15 bits/s/Hz[7] is shown in Fig. 7. The worst selection has 10 dB loss at 10^{-3} outage probability. The MIBM has about 2 dB loss while the correlation-based method exhibits around 6 dB loss. The performance of the fast algorithm MIBM2 is comparable to that of the MIBM at high outage probability. The MIBM2 has a good performance overall while (similarly to the CB methods) it does not require the SNR value. Figs. 8 and 9 illustrate the outage capacities at 1% outage rate versus the number of receive antennas, $N_{\rm r}$, under fixed $N_{\rm t} = L_{\rm r} = 3$ at 10 dB and 30 dB SNR, respectively.

VI. Summary and Conclusion

We have investigated the behavior of MIMO systems that select a subset of available antennas at one link-end. In particular, we have derived upper bounds for the capacity of antenna selection, and we have also derived several algorithms that allow the selection of the antennas without an exhaustive search over all possible antenna combinations. We compared the upper bounds to computer simulation results and also compared the reduced-complexity selection algorithms. For $L_{\rm r} \geq N_{\rm t}$, selecting the best $L_{\rm r}$ antennas gives almost the same capacity as the full-complexity system. Capacity losses are less than 3.5 bits/s/Hz for $N_{\rm r} = 8$, $N_{\rm t} = 3$, $L_{\rm r} = 3$ at 20 dB SNR. This slight performance loss is offset by a considerable reduction in hardware costs. Instead of a full $N_{\rm r}$ receiver chains, only $L_{\rm r}$ receiver chains, plus an RF switch are required. We have also derived and compared several algorithms that allow the selection of the antennas without an exhaustive search over all possible antenna combinations. These algorithms have a complexity proportional to $N_{\rm r}^2$, instead of the $\binom{N_{\rm r}}{L_{\rm r}}$ complexity of optimum algorithms, while resulting in a capacity loss of less than 1 bit/s/Hz at 10 dB and 4 bits/s/Hz at 30 dB.

Important applications for such systems are cellular and wireless local area network systems with MIMO capability. The necessity of selecting antennas at one link-end (instead of using all of them) stems from either complexity or cost considerations. For example, the number of transmit antennas foreseen for the space–time coder could be limited, as is already the case in the 3GPP standard. Furthermore, antenna selection can be especially beneficial in low-rank and interference-limited systems. Thus, the results of this paper can serve as a guideline for designing reduced-complexity MIMO cellular systems for third- and fourth-generation communications.

Appendix
Derivation of the Recursion Relation

The starting point for the derivation is (8). We first solve the $N_{\rm r} - L_{\rm r}$ innermost integrals.[8] These integrals have the form

$$\int_0^y \left[d^{(q)} + \sum_{p=1}^{q} \exp\left(-b_p^{(q)} x\right) \times \sum_{k=0}^{(q-p+1)(N_{\rm t}-1)} c_{p,k}^{(q)} x^k \right] x^{N_{\rm t}-1} \exp(-x)\,{\rm d}x \quad (39)$$

where for readability we have substituted $\gamma_{(q)} \to x$, $\gamma_{(q-1)} \to y$.

The first part of the integral can be solved as [35]

$$\int_0^y d^{(q)} x^{N_{\rm t}-1} \exp(-x)\,{\rm d}x = d^{(q)} \left[(N_{\rm t}-1)! - \exp(-y) \sum_{k=0}^{N_{\rm t}-1} \frac{(N_{\rm t}-1)!}{k!} y^k \right]. \quad (40)$$

Next, we pull out the summation over p from the integral and consider the integrals

$$J_p^{(q)} = \int_0^y \left[\exp\left(-b_p^{(q)} x\right) \sum_{k=0}^{(q-p+1)(N_{\rm t}-1)} c_{p,k}^{(q)} x^k \right] x^{N_{\rm t}-1} \times \exp(-x)\,{\rm d}x. \quad (41)$$

[7]That is, the probability that the capacity is smaller than 15 bits/s/Hz.

[8]Integrals of a similar form are also solved by the authors in [22]. For convenience of the reader, we give here a short outline of the derivation.

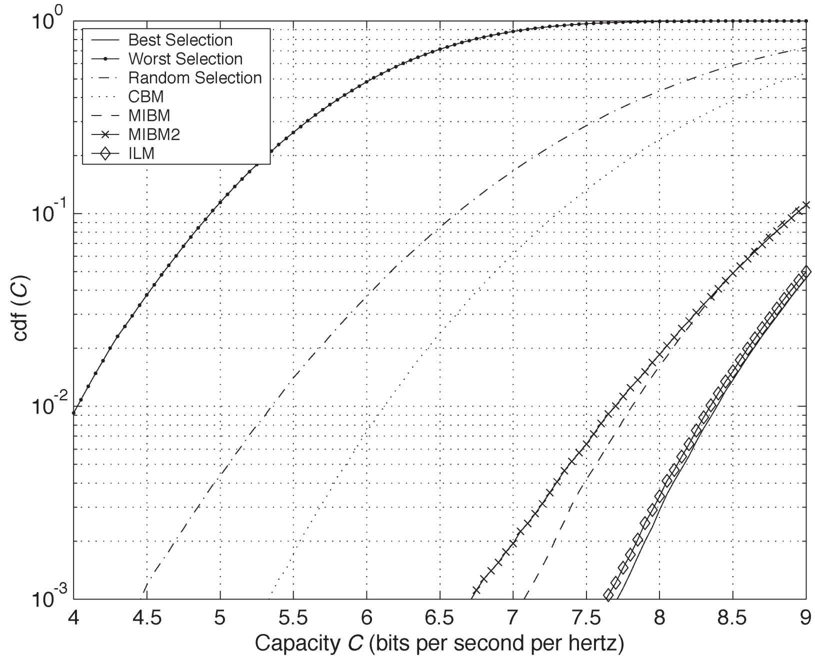

Fig. 5. Outage probabilities of fast algorithms, $N_{\mathrm{r}} = 8$, $N_{\mathrm{t}} = L_{\mathrm{r}} = 3$, $\mathrm{SNR} = 10$ dB.

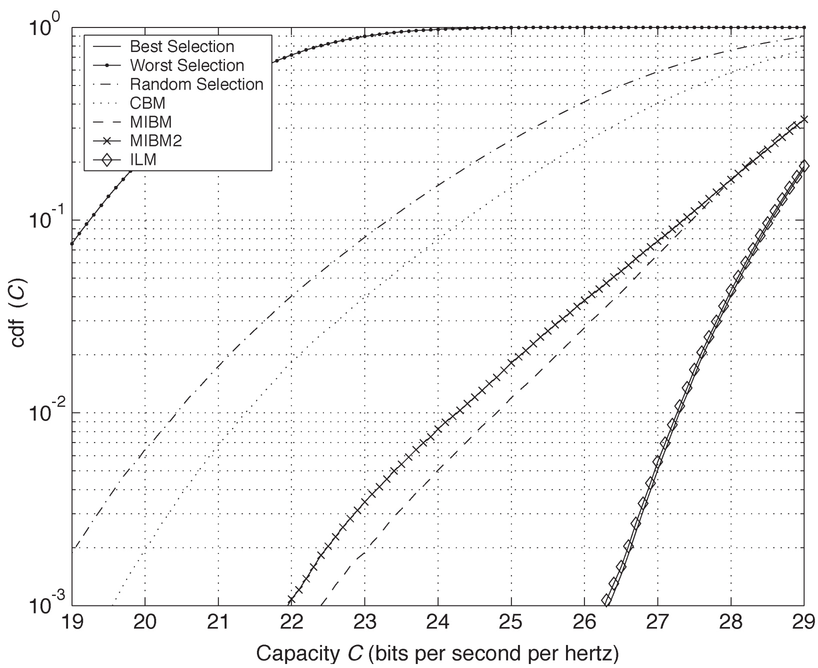

Fig. 6. Outage probabilities of fast algorithms, $N_{\mathrm{r}} = 8$, $N_{\mathrm{t}} = L_{\mathrm{r}} = 3$, $\mathrm{SNR} = 30$ dB.

By introducing

$$\widehat{b}_p^{(q)} = b_p^{(q)} + 1 \quad \text{for } 1 \le p \le q \tag{42}$$

$$M = (q - p + 2)(N_{\mathrm{t}} - 1) \tag{43}$$

$$\widehat{c}_{p,k}^{(q)} = \begin{cases} c_{p,k-(N_{\mathrm{t}}-1)}^{(q)}, & \text{for } (N_{\mathrm{t}} - 1) \le k \le M \\ 0, & \text{otherwise} \end{cases} \tag{44}$$

this integral can be written as

$$\int_0^y \exp\left(-\widehat{b}_p^{(q)} x\right) \sum_{k=0}^{M} \widehat{c}_{p,k}^{(q)} x^k \, \mathrm{d}x. \tag{45}$$

Employing [40]

$$\int \sum_{k=0}^{M} \widehat{c}_{p,k}^{(q)} x^k \mathrm{e}^{-\widehat{b}_p^{(q)} x} \, \mathrm{d}x = \frac{\mathrm{e}^{-\widehat{b}_p^{(q)} x}}{-\widehat{b}_p^{(q)}} \sum_{l=0}^{M} \frac{(-1)^l}{\left(-\widehat{b}_p^{(q)}\right)^l} \frac{d^l}{dx^l} \sum_{k=0}^{M} \widehat{c}_{p,k}^{(q)} x^k \tag{46}$$

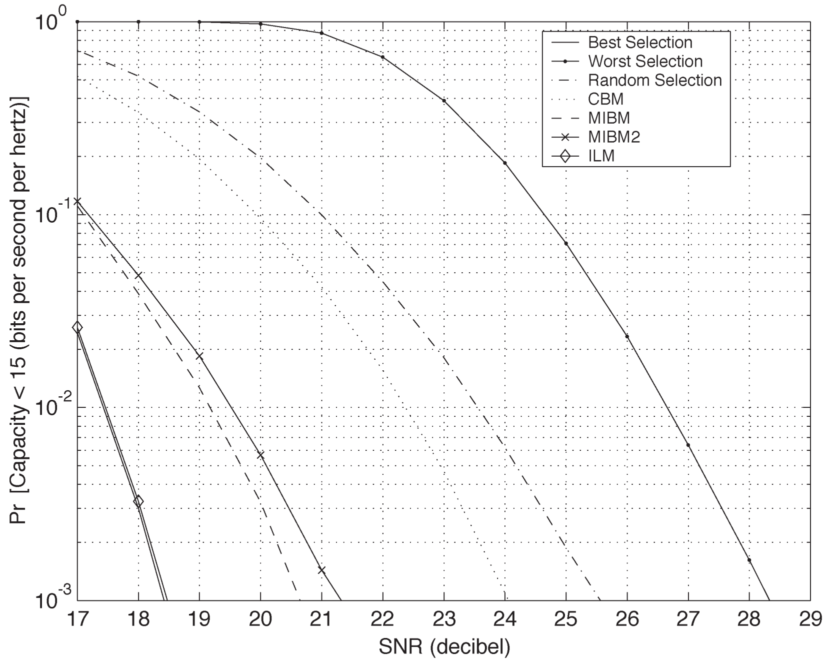

Fig. 7. Outage probability comparison, $N_{\rm r} = 8$, $N_{\rm t} = L_{\rm r} = 3$.

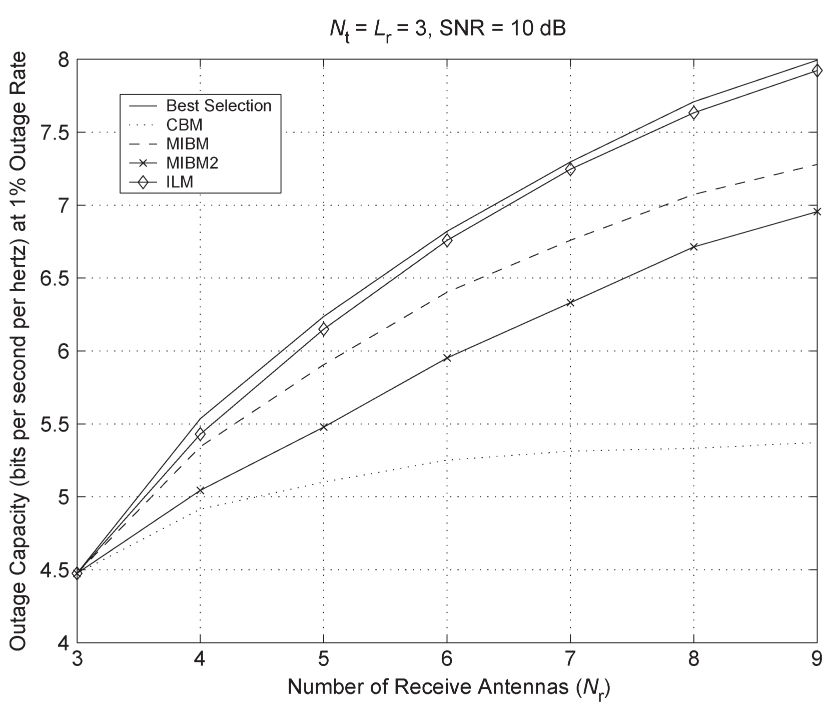

Fig. 8. Outage probability comparison (1%) as a function of the number of receive antennas $N_{\rm t} = L_{\rm r} = 3$, $\mathrm{SNR} = 10$ dB.

we get

$$J_p^{(q)} = \frac{\mathrm{e}^{-\widehat{b}_p^{(q)} x}}{-\widehat{b}_p^{(q)}} \sum_{k=0}^{M} \widehat{c}_{p,k}^{(q)} \sum_{l=0}^{k} \frac{1}{\left(\widehat{b}_p^{(q)}\right)^l} \frac{k!}{(k-l)!} x^{k-l} \Bigg|_0^y. \quad (47)$$

Introducing $r = k - l$, we can write this as

$$J_p^{(q)} = \frac{\mathrm{e}^{-\widehat{b}_p^{(q)} x}}{-\widehat{b}_p^{(q)}} \sum_{r=0}^{M} x^r \sum_{t=0}^{M-r} \widehat{c}_{p,r+t}^{(q)} \frac{1}{\left(\widehat{b}_p^{(q)}\right)^t} \frac{(r+t)!}{r!} \Bigg|_0^y. \quad (48)$$

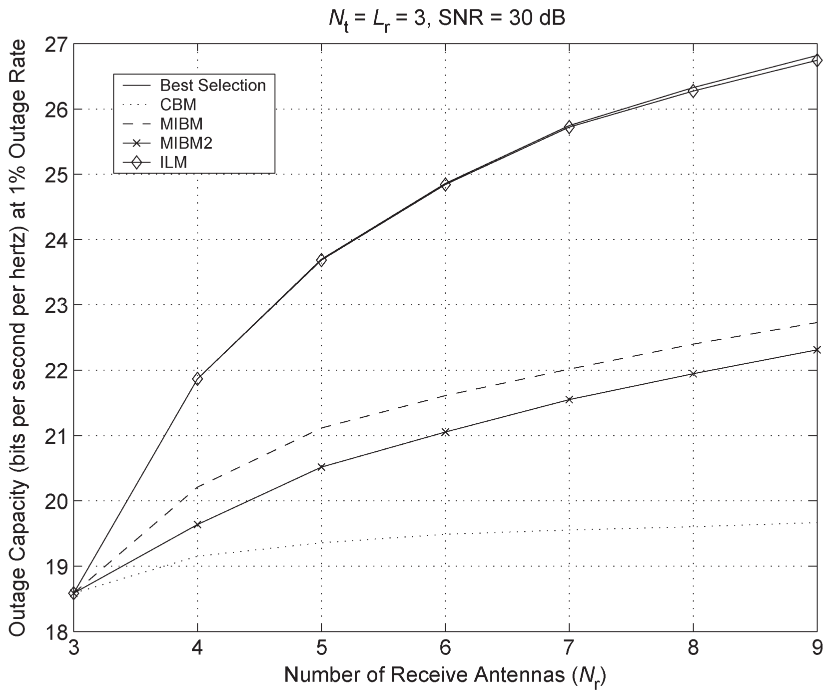

Fig. 9. Outage probability comparison (1%) as a function of the number of receive antennas $N_{\mathrm{t}} = L_{\mathrm{r}} = 3, \mathrm{SNR} = 30\ \mathrm{dB}$.

The total integral thus is

$$d^{(q)}\left[(N_{\mathrm{t}}-1)! - \exp(-y)\sum_{k=0}^{N_{\mathrm{t}}-1}\frac{(N_{\mathrm{t}}-1)!}{k!}y^k\right] + \sum_{p=1}^{q}\left[\frac{1}{\widehat{b}_p^{(q)}}\sum_{t=0}^{M}\widehat{c}_{p,t}^{(q)}\frac{t!}{\left(\widehat{b}_p^{(q)}\right)^t} - \frac{\mathrm{e}^{-\widehat{b}_p^{(q)}x}}{\widehat{b}_p^{(q)}} \times\sum_{r=0}^{M}y^r\sum_{t=0}^{M-r}\widehat{c}_{p,r+t}^{(q)}\frac{1}{\left(\widehat{b}_p^{(q)}\right)^t}\frac{(r+t)!}{r!}\right]. \quad (49)$$

Comparing this expression with the generic expression for the result of the $(q+1)$th integration

$$\left[d^{(q+1)} + \sum_{p=1}^{q+1}\exp\left(-b_p^{(q+1)}y\right)\sum_{k=0}^{(q-p+2)(N_{\mathrm{t}}-1)}c_{p,k}^{(q+1)}y^k\right] \quad (50)$$

and matching coefficients, we get the recursion relations given in (11)–(15).

As mentioned in Section III-C, we perform this iteration $N_{\mathrm{r}} - L_{\mathrm{r}}$ times and write the result in the form

$$\sum_{p=0}^{N_{\mathrm{r}}-L_{\mathrm{r}}}\exp\left(-b_p^{(N_{\mathrm{r}}-L_{\mathrm{r}})}\gamma_{(L_{\mathrm{r}})}\right) \times\sum_{k=0}^{(N_{\mathrm{r}}-L_{\mathrm{r}}-p+1)(N_{\mathrm{t}}-1)}c_{p,k}^{(N_{\mathrm{r}}-L_{\mathrm{r}})}\gamma_{(L_{\mathrm{r}})}^{k+\alpha^{(N_{\mathrm{r}}-L_{\mathrm{r}})}}. \quad (51)$$

The integrals we have to solve for the next L_{r} iteration steps are thus of the generic type

$$J^{(q)} = \int_0^y \mathrm{d}x\left[\sum_{p=0}^{N_{\mathrm{r}}-L_{\mathrm{r}}}\exp\left(-b_p^{(q)}x\right)\sum_{k=0}^{M'}c_{p,k}^{(q)}x^k\right]x^{N_{\mathrm{t}}-1} \times\exp(-x)[1+\rho x]^{\frac{j\nu}{\ln(2)}} \quad (52)$$

where $M' = (N_{\mathrm{r}} - L_{\mathrm{r}} - p + 1)(N_{\mathrm{t}} - 1) = M - (N_{\mathrm{t}} - 1)$ for the first iteration step and ∞ for the further steps (note that since the series converges well, a finite number of terms is sufficient for the numerical computations), and $x = \gamma_{(q)}$, $y = \gamma_{(q-1)}$. Since ρ usually has reasonably large values, and the behavior of the pdf is also mainly determined by the behavior of the characteristic function near $\nu = 0$ (the nth moment is the nth derivative of the characteristic function at $\nu = 0$), we approximate

$$[1+\rho x]^{\frac{j\nu}{\ln(2)}} \approx (\rho x)^{\frac{j\nu}{\ln(2)}}\left(1 + \frac{\frac{j\nu}{\ln(2)}}{\rho x}\right). \quad (53)$$

This approximation was validated (for the parameters used in Section V) by computing C_{bound} by Monte Carlo simulations and comparing it to the analytical results based on the approximation (53). Even for 1000 Monte Carlo runs, the difference between analytical and numerical results was smaller than the uncertainty of the MC results.

The integral $J^{(q)}$ can now be written as $q = N_{\mathrm{r}} - L_{\mathrm{r}}, \ldots\ldots, N_{\mathrm{r}}$

$$J^{(q)} = \rho^{\frac{j\nu}{\ln(2)}}\int_0^y \mathrm{d}x\left[\sum_{p=0}^{N_{\mathrm{r}}-L_{\mathrm{r}}}\exp\left(-\widehat{b}_p^{(q)}x\right)\sum_{k=0}^{M}\widehat{c}_{p,k}^{(q)}x^{k+\widehat{\alpha}^{(q)}}\right] \quad (54)$$

where

$$\widehat{b}_p^{(q)} = b_p^{(q)} + 1 \tag{55}$$

$$\widehat{\alpha}^{(q)} = \alpha^{(q)} + \frac{j\nu}{\ln(2)} \tag{56}$$

$$\alpha^{(N_\mathrm{r}-L_\mathrm{r})} = 0 \tag{57}$$

and

$$\widehat{c}_{p,k}^{(q)} = \begin{cases} c_{p,k-(N_\mathrm{t}-1)}^{(q)}, & \text{for } k = M \\ c_{p,k-(N_\mathrm{t}-1)}^{(q)} + \frac{j\nu}{\rho\ln(2)} c_{p,k-N_\mathrm{t}}^{(q)}, & \text{for } N_\mathrm{t}-1 \le k < M \\ \frac{j\nu}{\rho\ln(2)} c_{p,k-N_\mathrm{t}}^{(q)}, & \text{for } k = N_\mathrm{t}-2 \\ 0, & \text{otherwise} \end{cases}. \tag{58}$$

Note that $N_\mathrm{t} \ge 2$.

Now from [40]

$$\int_0^y x^{k+\widehat{\alpha}} \exp(-ax)\mathrm{d}x = a^{-(k+\widehat{\alpha}+1)} \gamma_{\mathrm{Euler}}(k+\widehat{\alpha}+1, ay) \tag{59}$$

where γ_{Euler} denotes here Euler's Gamma function of the second kind. Using its series expansion [35], the above integral becomes

$$a^{-(k+\widehat{\alpha}+1)} \exp(-ay) \sum_{n=0}^{\infty} \frac{(ay)^{k+\widehat{\alpha}+1+n}}{\prod_{i=0}^{n}(k+\widehat{\alpha}+1+i)} \tag{60}$$

$$= \exp(-ay) \sum_{n=0}^{\infty} y^{k+\widehat{\alpha}+1+n} f_n \tag{61}$$

with

$$f_n = \frac{a^n}{\prod_{i=0}^{n}(k+\widehat{\alpha}+1+i)}. \tag{62}$$

Applying this result now to (54), we get

$$J^{(q)} = \rho^{\frac{j\nu}{\ln(2)}} \left[\sum_{p=0}^{N_\mathrm{r}-L_\mathrm{r}} \exp\left(-\widehat{b}_p^{(q)} y\right) \times \sum_{k=0}^{M} \widehat{c}_{p,k}^{(q)} \sum_{n=0}^{\infty} y^{k+\widehat{\alpha}^{(q)}+1+n} f_n \right]. \tag{63}$$

Comparing this to the generic form

$$\left[\sum_{p=0}^{N_\mathrm{r}-L_\mathrm{r}} \exp\left(-b_p^{(q+1)} x\right) \sum_{k=0}^{M} c_{p,k}^{(q+1)} x^{\alpha^{(q+1)}+k} \right] \tag{64}$$

we find by comparison the coefficients in expressions (21)–(24).

By making use of the parameters $\widehat{b}$, $\widehat{c}$, the final integral is of the form

$$J^{(N_\mathrm{r})} = \int_0^\infty \left[\sum_{p=0}^{N_\mathrm{r}-L_\mathrm{r}} \exp\left(-\widehat{b}_p^{(N_\mathrm{r}-1)} x\right) \times \sum_{k=0}^{M} \widehat{c}_{p,k}^{(N_\mathrm{r}-1)} x^{k+\widehat{\alpha}^{(N_\mathrm{r}-1)}} \right] \mathrm{d}x \tag{65}$$

which yields [40]

$$\sum_{p=0}^{N_\mathrm{r}-L_\mathrm{r}} \sum_{r=0}^{\infty} \widehat{c}_{p,r}^{(N_\mathrm{r}-1)} \frac{\Gamma\left(r+\widehat{\alpha}^{(N_\mathrm{r}-1)}+1\right)}{\left(\widehat{b}_p^{(N_\mathrm{r}-1)}\right)^{r+\widehat{\alpha}^{(N_\mathrm{r}-1)}+1}}. \tag{66}$$

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Andreas F. Molisch (S'89–M'95–SM'00–F'05) received the Dipl. Ing., Dr. techn., and habilitation degrees from the Technical University (TU) Vienna, Vienna, Austria, in 1990, 1994, and 1999, respectively.

From 1991 to 2000, he was with the TU Vienna, becoming an Associate Professor there in 1999. From 2000 to 2002, he was with the Wireless Systems Research Department at AT&T Laboratories-Research, Middletown, NJ. Since then, he has been a Senior Principal Member of Technical Staff with Mitsubishi Electric Research Laboratories, Cambridge, MA. He is also Professor and Chairholder for radio systems at Lund University, Lund, Sweden. He has done research in the areas of SAW filters, radiative transfer in atomic vapors, atomic line filters, smart antennas, and wideband systems. His current research interests are MIMO systems, measurement and modeling of mobile radio channels, and ultrawide bandwidth (UWB). He has authored, coauthored, or edited four books, eight book chapters, some 85 journal papers, and numerous conference contributions.

Dr. Molisch is an editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, co-editor of a recent special issue on MIMO and smart antennas in Journal on Wireless Communications Mobile Computing, and coeditor of an upcoming IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATONS special issue on UWB. He has participated in the European research initiatives "COST 231," "COST 259," and "COST273," where he is chairman of the MIMO channel working group. He is also vice chairman of Commission C (signals and systems) of URSI (International Union of Radio Scientists), chairman of the IEEE 802.15.4a channel modeling group, and recipient of several awards.

Moe Z. Win (S'85–M'87–SM'97–F'04) received the B.S. degree (*magna cum laude*) from Texas A&M University, College Station, in 1987, and the M.S. degree from the University of Southern California (USC), Los Angeles, in 1989, both in electrical engineering. As a Presidential Fellow at USC, he received both the M.S. degree in applied mathematics and the Ph.D. degree in electrical engineering in 1998.

Dr. Win is an Associate Professor at the Laboratory for Information and Decision Systems (LIDS), Massachusetts Institute of Technology, Cambridge. Prior to joining LIDS, he spent 5 years at AT&T Research Laboratories and 7 years at the Jet Propulsion Laboratory. His main research interests are the application of mathematical and statistical theories to communication, detection, and estimation problems. Specific current research topics include measurement and modeling of time-varying channels, design and analysis of multiple antenna systems, ultrawide bandwidth (UWB) communications systems, optical communications systems, and space communications systems.

Dr. Win has been involved actively in organizing and chairing sessions, and has served as a member of the Technical Program Committee in a number of international conferences. He served as the Technical Program Chair for the IEEE Communication Theory Symposia of ICC-2004 and Globecom-2000, as well as for the IEEE Conference on Ultra Wideband Systems and Technologies in 2002, the Technical Program Vice Chair for the IEEE International Conference on Communications in 2002, and the Tutorial Chair for the IEEE Semiannual International Vehicular Technology Conference in Fall 2001. He is the current Chair and past Secretary (2002–2004) for the Radio Communications Committee of the IEEE Communications Society. He currently serves as Area Editor for Modulation and Signal Design and Editor for Wideband Wireless and Diversity, both for IEEE TRANSACTIONS ON COMMUNICATIONS. He served as the Editor for Equalization and Diversity from July 1998 to June 2003 for the IEEE TRANSACTIONS ON COMMUNICATIONS, and as a Guest-Editor for the 2002 IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS Special Issue on Ultra-Wideband Radio in Multiaccess Wireless Communications. He received the International Telecommunications Innovation Award from Korea Electronics Technology Institute in 2002, the Young Investigator Award from the Office of Naval Research in 2003, and the IEEE Antennas and Propagation Society Sergei A. Schelkunoff Transactions Prize Paper Award in 2003. In 2004, he was named Young Aerospace Engineer of the Year by the AIAA and received the Fulbright Foundation Senior Scholar Lecturing and Research Fellowship, the Institute of Advanced Study Natural Sciences and Technology Fellowship, the Outstanding International Collaboration Award from the Industrial Technology Research Institute of Taiwan, and the Presidential Early Career Award for Scientists and Engineers from the White House. He is an IEEE Distinguished Lecturer and elected Fellow of the IEEE, cited "for contributions to wideband wireless transmission."

Yang-Seok Choi (S'91–M'01) received the B.S. degree from Korea University, Seoul, South Korea, in 1990, the M.S.E.E. degree from the Korea Advanced Institute of Science and Technology, Taejon, South Korea, in 1992, and the Ph.D. degree from Polytechnic University, Brooklyn, NY, in 2000, all in electrical engineering.

From 1992 to 1996, he was with Samsung Electronics, Co., Ltd., Suwon, Korea, where he developed 32 QAM modem for HDTV and QPSK ASIC for DBS. During 2000 summer he held a Summer intern position at AT&T Labs-Research Shannon Lab, Florham Park, NJ. In 2000, he joined National Semiconductor, East Brunswick, NJ, where he was involved in the development of W-CDMA. During 2001–2002, he was a Senior Technical Staff Member at AT&T Labs-Research, Middletown, NJ where he researched on MIMO systems, OFDM systems and information theory. From 2002 to 2004 he had been with ViVATO, Inc., Spokane, WA, working on MIMO OFDM systems, smart antenna systems, and antenna/beam selection techniques. He researched on Smart antenna applications to CSMA protocol and co-invented Complementary Beamforming. In 2004, he joined Intel Corporation, Hillsboro, OR where he studies on Broadband Wireless communications systems. His research interests include MIMO, OFDM, MC-CDMA, smart antenna, blind identification/equalizer, carrier/timing recovery, space-time coding, cross-layer design and capacity of time-varying multipath channel. He holds seven U.S. patents.

Jack H. Winters (S'77–M'81–SM'88–F'96) received the Ph.D. degree in electrical engineering from The Ohio State University, Columbus, in 1981.

He is the Chief Scientist at Motia, Inc., where he is involved with smart antennas for wireless systems. In 1981, he joined AT&T and worked in the research area for more than 20 years. At AT&T, he was Division Manager of the Wireless Systems Research Division of AT&T Labs-Research, Middletown, NJ. He is an IEEE Distinguished Lecturer for both the IEEE Communications and the Vehicular Technology Societies, Area Editor for Transmission Systems for the IEEE TRANSACTIONS ON COMMUNICATIONS, and a New Jersey Inventor of the Year for 2001.