

Unified Analysis of UWB Transmitted-Reference Schemes in the Presence of Narrowband Interference

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Abstract—Transmitted-Reference (TR) signaling, in conjunction with an autocorrelation receiver (AcR), offers a low-complexity alternative to Rake reception in ultrawide bandwidth systems. This paper provides a unified performance analysis of various TR schemes by developing an analytical framework based on the sampling expansion approach. Specifically, we derive the uncoded bit error probability (BEP) of different TR signaling schemes, including TR and differential TR (DTR) signaling valid for a broad class of fading channels. We consider both AcRs and modified AcRs with noise averaging. We further develop a quasi-analytical method as well as an approximate analytical method to extend the BEP analysis to include the effect of narrowband interference (NBI). We show that the approximate analytical method is particularly useful in obtaining BEP expressions that provide insight into the effect of NBI. We quantify the effects of NBI and channel power dispersion profile on the optimum integration interval of an AcR. Finally, we compare TR and DTR signaling in terms of their sensitivity to NBI.

Index Terms—Ultrawide bandwidth systems, transmitted-reference, differential transmitted-reference, sampling expansion, narrowband interference.

I. INTRODUCTION

RECENTLY, there has been a renewed interest in utilizing ultrawide bandwidth (UWB) spread-spectrum communications for future military, homeland security, and commercial applications [1]–[4]. UWB systems involve transmission of a train of extremely narrow pulses by employing either time-hopping (TH) or direct sequence (DS) techniques for multiple-access and pulse position modulation (PPM) or pulse amplitude modulation (PAM) for data transmission [1], [2]. The key motivation for using UWB systems is the ability to highly resolve multipath, as well as the availability of technology

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to implement and generate UWB signals with relatively low complexity. The fine delay resolution properties make UWB radio a viable candidate for communications [5]–[8], as well as for ranging and localization in dense multipath environments [9]–[14].

Due to its large transmission bandwidth, UWB systems need to coexist and contend with many narrowband communication systems. Therefore, a thorough performance analysis of such systems in the presence of narrowband systems is essential for successful deployment of UWB systems. Previous work in this area includes the study of the effect of UWB signals on some commercial narrowband communication systems [15]–[18], the analysis of UWB systems in the presence of narrowband interference (NBI) with conventional receiver structures involving a locally generated reference (LGR) [19]–[25], and the development of techniques to suppress NBI in UWB systems [26]–[30].

As an alternative to LGR systems, a reference signal can be transmitted along with the data. Such a signaling scheme, referred to as transmitted-reference (TR) signaling, was first considered in the early 1950s [31]. TR signaling involves the transmission of a reference and data signal pair, separated either in time [32], [33] or in frequency [34], [35].¹ Due to its simplicity, there is renewed interest in TR signaling for UWB systems [36]–[40], which can exploit multipath diversity inherent in the environment without the need for channel estimation and stringent acquisition [41]–[44]. The receiver can simply be an autocorrelation receiver (AcR), which can be modified to include noise averaging for better performance. Since TR signaling allocates a significant part of the symbol energy to transmitting reference pulses, differential encoding over consecutive symbols can also be used to alleviate inefficient resource usage. This alternative TR signaling is referred to as differential transmitted-reference (DTR) signaling [45], [46]. However, only a few results are available for the performance of TR and DTR signaling schemes in the presence of NBI [47]–[49].

In this paper, we analyze the performance of TR and DTR signaling schemes in the presence of NBI. We adopt the sampling expansion approach, originally proposed in [38], [39] to analyze TR schemes. First, we derive the uncoded BEP for TR and DTR signaling when an AcR or a modified AcR is used in a broad class of dense multipath channels

¹In order for this pair of separated signals to experience the same channel, either the time separation must be less than the channel coherence time, or the frequency separation must be less than the channel coherence bandwidth.

[45]. We then extend our BEP analysis to include NBI [50]. As in [23], we model NBI as a single-tone interferer with Rayleigh distributed amplitude.² We develop a quasi-analytical method as well as an approximate analytical method to evaluate the BEP of TR and DTR signaling in the presence of NBI. We show that the approximate analytical method is particularly useful in obtaining BEP expressions that provide insight into the effect of NBI. Numerical results reveal that the approximation agrees well with quasi-analytical results. We quantify the effects of NBI and channel power dispersion profile on the optimum integration interval of an AcR. We then compare TR and DTR signaling in terms of their sensitivity to NBI in both noise-limited and interference-limited regimes.

The paper is organized as follows. Section II presents the system and channel models for our signaling schemes. In Section III, a unified BEP analysis of TR and DTR signaling based on the sampling expansion approach is developed. The effect of NBI on the performance of TR and DTR signaling is studied in Section IV. To illustrate our proposed methodology, we consider Nakagami- m fading channels and present numerical examples in Section V. Finally, conclusions are given in the last section.

II. SYSTEM AND CHANNEL MODELS

A. Transmitted-Reference

The transmitted signal of TR signaling for user k can be decomposed into a reference signal block $b_r^{(k)}(t)$ and a data modulated signal block $b_d^{(k)}(t)$ as given by

$$s_{\text{TR}}^{(k)}(t) = \sum_i b_r^{(k)}(t - iN_s T_f) + d_i^{(k)} b_d^{(k)}(t - iN_s T_f), \quad (1)$$

where T_f is the average pulse repetition period, $d_i^{(k)} \in \{-1, 1\}$ is the i th data symbol, and $N_s T_f$ is the symbol duration [39]. The reference signal and modulated signal blocks, each consisting of $N_s/2$ transmitted signal pulses, can be written as³

$$b_r^{(k)}(t) = \sum_{j=0}^{\frac{N_s}{2}-1} \sqrt{E_p} a_j^{(k)} p(t - j2T_f - c_j^{(k)} T_p),$$

$$b_d^{(k)}(t) = \sum_{j=0}^{\frac{N_s}{2}-1} \sqrt{E_p} a_j^{(k)} p(t - j2T_f - c_j^{(k)} T_p - T_r), \quad (2)$$

where $b_d^{(k)}(t)$ is equal to a version of $b_r^{(k)}(t)$, delayed by T_r , and $p(t)$ is a unit energy bandpass signal pulse with duration T_p and center frequency f_c . The energy of the transmitted pulse is then $E_p = E_s/N_s$, where E_s is the symbol energy. In our case of binary signaling, the symbol energy equals the energy per bit, E_b . To enhance the robustness of TR systems against interference and to allow multiple access, DS and/or TH spread spectrum techniques can be used as shown in (2). In DS signaling, $\{a_j^{(k)}\}$ is the bipolar pseudo-random sequence

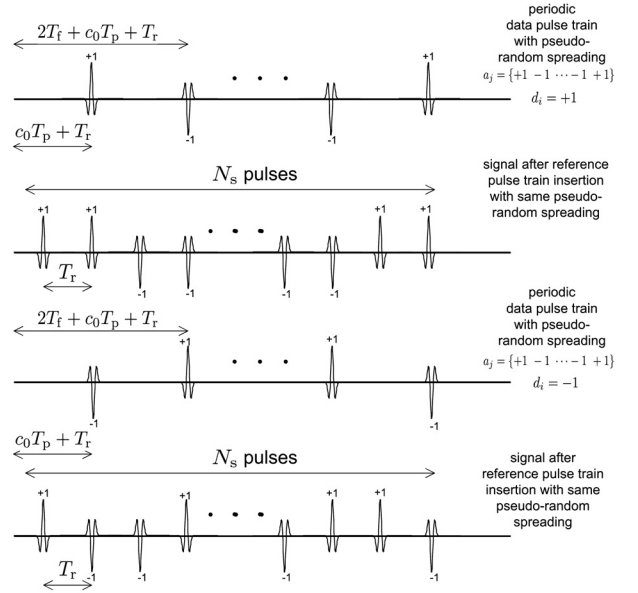


Fig. 1. Illustration of TR signaling scheme.

of the k th user.⁴ In TH signaling, $\{c_j^{(k)}\}$ is the pseudo-random sequence of the k th user, where $c_j^{(k)}$ is an integer in the range $0 \leq c_j^{(k)} < N_h$, and N_h is the maximum allowable integer shift. A simplified example that illustrates TR signaling is shown in Fig. 1. The duration of the received UWB pulse is $T_g = T_p + T_d$, where T_d is the maximum excess delay of the channel. To preclude inter-symbol interference (ISI) and intra-symbol interference (isi)⁵, we assume that $T_r \geq T_g$ and $(N_h - 1)T_p + T_r + T_g \leq 2T_f$, where T_r is the time separation between each pair of data and reference pulses. If the symbol interval is less than the channel coherence time, all these pairs of separated signals will experience the same channel.⁶ Note that T_r is constant for our case, as shown in Fig. 1, in contrast to [36], where the inter-pulse delays vary for different pairs of data-modulated and reference pulses.

B. Differential Transmitted-Reference

The transmitted signal of DTR signaling for user k is given by

$$s_{\text{DTR}}^{(k)}(t) = \sum_i e_i^{(k)} b^{(k)}(t - iN_s T_f), \quad (3)$$

where $b^{(k)}(t)$ is the k th user's block-modulated signal with symbol interval $N_s T_f$, and N_s is the number of transmitted signal pulses in each block. The data symbol $d_i^{(k)}$ is now differentially encoded such that $e_i^{(k)} = e_{i-1}^{(k)} d_i^{(k)}$, where $d_i^{(k)} \in \{-1, 1\}$. The $b^{(k)}(t)$ -shaped signal block can be written as

$$b^{(k)}(t) = \sum_{j=0}^{N_s-1} \sqrt{E_p} a_j^{(k)} p(t - jT_f - c_j^{(k)} T_p), \quad (4)$$

⁴Walsh-Hadamard sequences are used in [51].

⁵ISI and isi may not always be negligible due to constraints on T_f and data-rate requirements. In this case, our results will serve as a lower bound.

⁶For TR signaling with an AcR, only adjacent data and reference pulses need to be within the channel coherence time. The condition that all pulses within a symbol experience the same channel, however, will enable us to extend our analysis to TR signaling with a modified AcR, where the channel is assumed to be constant over two symbols.

²Results in [23] show that NBI can be reasonably well approximated by a tone interferer.

³Note that other combinations of data and reference pulses are also possible [51]. For simplicity and without loss of generality, we have adopted conventional TR signaling, in which the number of reference and data pulses are equal and N_s is even [37], [39], [46].

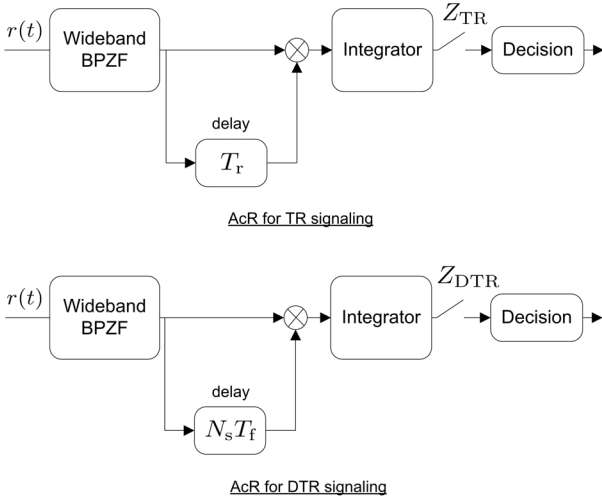


Fig. 2. AcR for TR and DTR signaling

where $\{a_j^{(k)}\}$ and $\{c_j^{(k)}\}$ are the DS and TH sequences that provide robustness to interference and multiple-access capability for DTR systems. Note that a DTR signal looks similar to a short-code CDMA signal, except that pulses are separated by at least T_g and TH is also present. The TH sequence is pseudo-random with the range $0 \leq c_j^{(k)} < N_h$, where N_h satisfies $(N_h - 1)T_p + T_g \leq T_f$ to preclude ISI and isi. The channel must be constant over two symbols in order to use differential encoding over two adjacent symbols.

C. Channel model

The received signal for TR signaling can be written as $r(t) = (h * s_{\text{TR}})(t) + n(t)$, where $h(t)$ is the impulse response of the channel and $n(t)$ is zero-mean, white Gaussian noise with two-sided power spectral density $N_0/2$. Note that a similar equation also applies to DTR signaling by replacing $s_{\text{TR}}(t)$ with $s_{\text{DTR}}(t)$. The channel impulse response, modeled as linear time-invariant, can be written as $h(t) = \sum_{l=1}^L \alpha_l \delta(t - \tau_l)$, where L is the number of resolvable multipath components, and α_l and τ_l respectively denote the attenuation and delay of l th path. We can express $\alpha_l = |\alpha_l| \exp(j\phi_l)$, where $\phi_l = 0$ or π with equal probability. As in [7], [8], we consider the resolvable dense multipath channel,⁷ i.e., $|\tau_l - \tau_j| \geq T_p, \forall l \neq j$, where $\tau_l = \tau_1 + (l - 1)T_p$, and $\{\alpha_l\}$ are assumed to be statistically independent random variables (r.v.'s).

III. RECEIVER MODELS

This paper focuses on a single user system. As a result, we will suppress the index k for notational simplicity in the rest of the paper. We assume perfect synchronization at the receiver.⁸

⁷Such an assumption may not be always true [52], [53]. However, the dense resolvable multipath channel serves as a reasonable approximation to realistic UWB channels. Therefore, our BEP analysis still provides insight into the performance of TR signaling.

⁸It has been shown that we can relax the assumption of perfect synchronization in TR signaling due to its robustness against synchronization errors [54], [55]. However, exactly how this synchronization is achieved [54], [56] and the sensitivity analysis of synchronization errors [55] are beyond the scope of this paper.

Without loss of generality, we consider the detection of the data symbol at $i = 0$.

A. Autocorrelation Receiver

As shown in Fig. 2, the AcR first passes the received signal through an ideal bandpass zonal filter (BPZF) with center frequency f_c to eliminate out-of-band noise. If the bandwidth W of the BPZF is large enough, then the signal spectrum will pass through undistorted. Consequently, the ISI and isi caused by filtering will be negligible. In this case, the output of the BPZF for TR and DTR signaling can be expressed respectively as

$$\begin{aligned} \tilde{r}_{\text{TR}}(t) &= \sum_i \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{l=1}^L \left[\sqrt{E_p} \alpha_l a_j p(t - iN_s T_f - j2T_f - c_j T_p - \tau_l) \right. \\ &\quad \left. + \sqrt{E_p} \alpha_l a_j d_i p(t - iN_s T_f - j2T_f - c_j T_p - T_r - \tau_l) \right] \\ &\quad + \tilde{n}(t), \end{aligned} \quad (5)$$

and

$$\begin{aligned} \tilde{r}_{\text{DTR}}(t) &= \sum_i \sum_{j=0}^{N_s-1} \sum_{l=1}^L \sqrt{E_p} \alpha_l a_j e_i p(t - iN_s T_f - jT_f - c_j T_p - \tau_l) \\ &\quad + \tilde{n}(t), \end{aligned} \quad (6)$$

where $\tilde{n}(t)$ is a zero-mean, Gaussian random process with autocorrelation function

$$R_{\tilde{n}}(\tau) = W N_0 \text{sinc}(W\tau) \cos(2\pi f_c \tau). \quad (7)$$

Note that when $W \gg 1/T_g$, $R_{\tilde{n}}(\tau)$ in (7) is approximately equal to zero for $|\tau| \geq T_g$. This implies that the noise samples separated by more than T_g or at a multiple of $1/W$ are statistically independent.

The filtered received signal is passed through a correlator with integration interval T ($T_p \leq T \leq T_g$), as shown in Fig. 2, to collect the received signal energy. The integration interval T determines the number of multipath components (or equivalently, the amount of energy) captured by the receiver, as well as the amount of noise and interference accumulation. As will be shown in later sections, the optimum T depends on various channel conditions, such as the decay factor of the channel power dispersion profile (PDP), and on the signal-to-interference ratios (SIR's). The decision statistics generated at the integrator output of the AcR can be written respectively as

$$Z_{\text{TR}} = \sum_{j=0}^{\frac{N_s}{2}-1} \int_{j2T_f + T_r + c_j T_p}^{j2T_f + T_r + c_j T_p + T} \tilde{r}_{\text{TR}}(t) \tilde{r}_{\text{TR}}(t - T_r) dt, \quad (8)$$

and

$$Z_{\text{DTR}} = \sum_{j=0}^{N_s-1} \int_{jT_f + c_j T_p}^{jT_f + c_j T_p + T} \tilde{r}_{\text{DTR}}(t) \tilde{r}_{\text{DTR}}(t - N_s T_f) dt, \quad (9)$$

for TR and DTR signaling.

$$Z_{\text{ATR}} = \sum_{j=0}^{\frac{N_s}{2}-1} a_j \int_{j2T_f+T_r+c_jT_p+T}^{j2T_f+T_r+c_jT_p+T} \tilde{r}_{\text{TR}}(t) \left[\frac{2}{N_s} \sum_{k=-j}^{\frac{N_s}{2}-1-j} a_{j+k} \tilde{r}_{\text{TR}}(t - (N_s - 2k)T_f - (c_j - c_{j+k})T_p - T_r) \right] dt, \quad (10)$$

$$Z_{\text{ADTR}} = \sum_{j=0}^{N_s-1} a_j \int_{jT_f+c_jT_p}^{jT_f+c_jT_p+T} \tilde{r}_{\text{DTR}}(t) \left[\frac{1}{N_s} \sum_{k=-j}^{N_s-1-j} a_{j+k} \tilde{r}_{\text{DTR}}(t - (N_s - k)T_f - (c_j - c_{j+k})T_p) \right] dt. \quad (11)$$

B. Modified Autocorrelation Receiver

The AcR performance for both TR and DTR signaling can be improved by averaging respectively over $N_s/2$ and N_s received reference pulses from the previous symbol [37]–[39], [45].⁹ This, however, requires the channel to remain constant over two symbols. The decision statistics of this modified AcR for TR and DTR signaling are given respectively by (10) and (11), shown at the top of this page. Next, we develop an analytical framework based on the sampling expansion approach to provide a unified performance analysis of TR and DTR systems in dense multipath channels.

IV. PERFORMANCE ANALYSIS

A. Transmitted-Reference

We begin by deriving the BEP for TR signaling with an AcR in the absence of NBI. It can be shown that Z_{TR} in (8) can be written as [38], [39]

$$\begin{aligned} Z_{\text{TR}} &= \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T \left[\check{b}_r(t + j2T_f + c_jT_p) + \check{n}(t + j2T_f + c_jT_p) \right] \\ &\quad \times \left[d_0 \check{b}_d(t + j2T_f + c_jT_p + T_r) \right. \\ &\quad \left. + \check{n}(t + j2T_f + c_jT_p + T_r) \right] dt, \end{aligned} \quad (12)$$

where $\check{b}_r(t) \triangleq (b_r * h * h_{\text{ZF}})(t)$, $\check{b}_d(t) \triangleq (b_d * h * h_{\text{ZF}})(t)$, and $h_{\text{ZF}}(t)$ is the impulse response of the BPZF. Note that if the symbol interval is less than the coherence time, all pairs of separated pulses will experience the same channel; hence $\check{b}_r(t + j2T_f + c_jT_p) = \check{b}_d(t + j2T_f + c_jT_p + T_r)$ for all $t \in (0, T)$ and j . In this case, we can significantly simplify the expression in (12) as follows:

$$\begin{aligned} Z_{\text{TR}} &= \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T \left[w_j(t) + \eta_{1,j}(t) \right] \left[d_0 w_j(t) + \eta_{2,j}(t) \right] dt \\ &= \sum_{j=0}^{\frac{N_s}{2}-1} U_j, \end{aligned} \quad (13)$$

where we have used $w_j(t) \triangleq \check{b}_r(t + j2T_f + c_jT_p) = \sqrt{E_p} a_j \sum_{l=1}^L \alpha_l p(t - \tau_l)$, $\eta_{1,j}(t) \triangleq \check{n}(t + j2T_f + c_jT_p)$, and $\eta_{2,j}(t) \triangleq \check{n}(t + j2T_f + c_jT_p + T_r)$, all defined over the interval $[0, T]$. Note that because the noise samples are taken at least T_g apart, they are essentially independent,

regardless of c_j .¹⁰ We further observe that U_j is simply the integrator output corresponding to the j th received modulated monocycle. Following the sampling expansion approach in [38], [39], we can represent U_j as

$$U_j = \frac{1}{2W} \sum_{m=1}^{2WT} \left(d_0 w_{j,m}^2 + w_{j,m} \eta_{2,j,m} + d_0 w_{j,m} \eta_{1,j,m} + \eta_{1,j,m} \eta_{2,j,m} \right), \quad (14)$$

where $w_{j,m}$, $\eta_{1,j,m}$, and $\eta_{2,j,m}$, for odd m (even m) are the real (imaginary) parts of the samples of the equivalent low-pass version of $w_j(t)$, $\eta_{1,j}(t)$, and $\eta_{2,j}(t)$, respectively, sampled at Nyquist rate W over the interval $[0, T]$.

Conditioned on d_0 and $a_j = +1$, we can express (14) in the form of a summation of squares:

$$U_j|_{d_0=+1} = \sum_{m=1}^{2WT} \left[\left(\frac{1}{\sqrt{2W}} w_{j,m} + \beta_{1,j,m} \right)^2 - \beta_{2,j,m}^2 \right], \quad (15)$$

$$U_j|_{d_0=-1} = \sum_{m=1}^{2WT} \left[- \left(\frac{1}{\sqrt{2W}} w_{j,m} - \beta_{2,j,m} \right)^2 + \beta_{1,j,m}^2 \right], \quad (16)$$

where $\beta_{1,j,m} = \frac{1}{2\sqrt{2W}}(\eta_{2,j,m} + \eta_{1,j,m})$ and $\beta_{2,j,m} = \frac{1}{2\sqrt{2W}}(\eta_{2,j,m} - \eta_{1,j,m})$ are statistically independent Gaussian r.v.'s with variance $\sigma_{\text{TR}}^2 = \frac{N_0}{4}$. Due to the statistical symmetry of U_j with respect to d_0 and $\{a_j\}$, we simply need to calculate the BEP conditioned on $d_0 = +1$ and $a_j = +1$. For notational simplicity, we define the normalized r.v.'s Y_1 , Y_2 , Y_3 , and Y_4 as

$$\begin{aligned} Y_1 &\triangleq \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} \left(\frac{1}{\sqrt{2W}} w_{j,m} + \beta_{1,j,m} \right)^2, \\ Y_2 &\triangleq \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} \beta_{2,j,m}^2, \\ Y_3 &\triangleq \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} \left(\frac{1}{\sqrt{2W}} w_{j,m} - \beta_{2,j,m} \right)^2, \\ Y_4 &\triangleq \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} \beta_{1,j,m}^2. \end{aligned} \quad (17)$$

Conditioned on the channel, Y_1 and Y_3 are noncentral chi-squared r.v.'s, whereas Y_2 and Y_4 are central chi-squared r.v.'s each having $N_s WT$ degrees of freedom. Both Y_1 and Y_3 have

⁹This averaging can be thought of as forming an estimate of the channel. In fact, when the observation noise is Gaussian, this is equivalent to forming a maximum likelihood estimate of the channel [57].

¹⁰As a result, no assumption on c_j is required since the above analysis is independent of $\{c_j\}$.

TABLE I
TR SIGNALING SCHEMES

Signaling	Non-centrality parameter of Y_1	Variance of $\beta_{1,j,m}$	Degrees of freedom of Y_1
TR	$\frac{E_s}{N_0} \sum_{l=1}^{L_{CAP}} \alpha_l^2$	$\frac{N_0}{4}$	$\frac{N_s WT}{2}$
ATR	$\frac{2N_s}{(N_s+2)} \left(\frac{E_s}{N_0} \sum_{l=1}^{L_{CAP}} \alpha_l^2 \right)$	$\frac{N_0(N_s+2)}{8N_s}$	$\frac{N_s WT}{2}$
DTR	$\frac{2E_s}{N_0} \sum_{l=1}^{L_{CAP}} \alpha_l^2$	$\frac{N_0}{4}$	$N_s WT$
ADTR	$\frac{4N_s}{(N_s+1)} \left(\frac{E_s}{N_0} \sum_{l=1}^{L_{CAP}} \alpha_l^2 \right)$	$\frac{N_0(N_s+1)}{8N_s}$	$N_s WT$

the same non-centrality parameter, given by

$$\mu_{TR} = \frac{1}{2\sigma_{TR}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T w_j^2(t) dt = \frac{E_s}{N_0} \sum_{l=1}^{L_{CAP}} \alpha_l^2, \quad (18)$$

where $L_{CAP} \triangleq [\min\{WT, WT_g\}]$ denotes the actual number of multipath components captured by the AcR. The probability density functions (pdfs) of Y_1 and Y_2 conditioned on μ_{TR} are given by

$$f_{Y_1|\mu_{TR}}(y_1) = f_{NC}(y_1, \mu_{TR}, q_{TR}), \quad (19)$$

$$f_{Y_2|\mu_{TR}}(y_2) = f_C(y_2, q_{TR}), \quad (20)$$

where $q_{TR} = \frac{N_s WT}{2}$. We have defined the following pdfs for notational convenience

$$f_{NC}(y, \mu, n) \triangleq e^{-(y+\mu)} \left(\frac{y}{\mu} \right)^{\frac{(n-1)}{2}} I_{n-1}(2\sqrt{y\mu}), \quad y \geq 0$$

$$f_C(y, n) \triangleq \frac{y^{(n-1)}}{(n-1)!} \exp(-y), \quad y \geq 0$$

where $I_{n-1}(\cdot)$ is the $(n-1)$ th order modified Bessel function of the first kind. The functions $f_{NC}(y, \mu, n)$ and $f_C(y, n)$ respectively are the pdfs of the noncentral and central chi-squared r.v.'s with $2n$ degrees of freedom and non-centrality parameter μ [58]. Using (19) and (20), the BEP of TR signaling with an AcR is given by

$$\begin{aligned} P_{e,TR} &= \mathbb{P}\{Z_{TR} \leq 0 | d_0 = +1\} \\ &= \mathbb{E}_{\mu_{TR}} \{\mathbb{P}\{Y_1 < Y_2 | d_0 = +1\}\} \\ &= \frac{1}{2^{q_{TR}}} \left[\sum_{n=0}^{q_{TR}-1} \left(\frac{(-j)^n}{n!} \right) \frac{d^n}{dv^n} \psi_{\mu_{TR}}(jv/2) \right]_{jv=-1} \\ &\quad \times \sum_{k=n}^{q_{TR}-1} \frac{1}{2^k} \frac{(k+q_{TR}-1)!}{(k-n)!(q_{TR}+n-1)!} \\ &\triangleq P_e(\psi_{\mu_{TR}}(jv), q_{TR}), \end{aligned} \quad (21)$$

where $\psi_{\mu_{TR}}(jv) \triangleq \mathbb{E}\{\exp(jv\mu_{TR})\}$ is the characteristic function (CF) of μ_{TR} . When the channel is resolvable and multipath components are statistically independent, $\psi_{\mu_{TR}}(jv) = \prod_{l=1}^{L_{CAP}} \psi_l(\frac{jvE_s}{N_0})$, where $\psi_l(jv)$ is the CF of α_l^2 whose closed-form expression is known for a broad class of channel fading statistics [58]. The detailed derivation of (21) can be found in [38], [39], [45].

Next, we extend the above analysis to derive the BEP of TR signaling with a modified AcR [38], [39], [45]. In this case, the non-centrality parameter of Y_1 in (17) becomes

$$\mu_{ATR} \triangleq \frac{1}{2\sigma_{ATR}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T w_j^2(t) dt = \frac{2N_s}{N_s+2} \left(\frac{E_s}{N_0} \sum_{l=1}^{L_{CAP}} \alpha_l^2 \right). \quad (22)$$

As shown in Table I, the non-centrality parameter of a modified AcR (eq. (22)) is at most two times larger than that of AcR (eq. (18)). Using (21), the BEP of TR signaling with a modified AcR can be written as

$$P_{e,ATR} = P_e(\psi_{\mu_{ATR}}(jv), q_{TR}). \quad (23)$$

B. Differential Transmitted-Reference

We begin by deriving the BEP for DTR signaling with an AcR in the absence of NBI, following an approach similar to the case of TR signaling. In this case, we can represent U_j as

$$U_j = \frac{1}{2W} \sum_{m=1}^{2WT} (d_0 w_{j,m}^2 + e_{-1} w_{j,m} \eta_{2,j,m} + e_0 w_{j,m} \eta_{1,j,m} + \eta_{1,j,m} \eta_{2,j,m}), \quad (24)$$

where $w_{j,m}$, $\eta_{1,j,m}$, and $\eta_{2,j,m}$, for odd m (even m) are the real (imaginary) parts of the samples of the equivalent low-pass version of $w_j(t) \triangleq (b * h * h_{ZF})(t + jT_f + c_j T_p) = \sqrt{E_p} a_j \sum_{l=1}^L \alpha_l p(t - \tau_l)$, $\eta_{1,j}(t) \triangleq \tilde{n}(t + jT_f + c_j T_p - N_s T_f)$, and $\eta_{2,j}(t) \triangleq \tilde{n}(t + jT_f + c_j T_p)$, respectively, sampled at Nyquist rate W over the interval $[0, T]$.

Similar to TR signaling, our following analysis requires no assumption on $\{c_j\}$, and we exploit statistical symmetry of U_j with respect to d_0 and $\{a_j\}$. Conditioned on $d_0 = +1$, we can express (24) in the form of (15)¹¹ where in this case $\beta_{1,j,m} = \frac{1}{2\sqrt{2}W} (e_{-1} \eta_{2,j,m} + e_0 \eta_{1,j,m})$ and $\beta_{2,j,m} = \frac{1}{2\sqrt{2}W} (e_{-1} \eta_{2,j,m} - e_0 \eta_{1,j,m})$ are statistically independent Gaussian r.v.'s with variance $\sigma_{DTR}^2 = \frac{N_0}{4}$. Due to symmetry, we only need to

¹¹When $d_0 = +1$, the pairs of differentially encoded bits are either $(e_{-1}, e_0) = (+1, +1)$ or $(e_{-1}, e_0) = (-1, -1)$ with probability $\frac{1}{2}$ each. By symmetry, we only need to consider $(e_{-1}, e_0) = (+1, +1)$.

consider Y_1 and Y_2 in (17), where the non-centrality parameter of Y_1 is now given by

$$\mu_{\text{DTR}} \triangleq \frac{1}{2\sigma_{\text{DTR}}^2} \sum_{j=0}^{N_s-1} \int_0^T w_j^2(t) dt = \frac{2E_s}{N_0} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2, \quad (25)$$

and the pdfs of Y_1 and Y_2 conditioned on μ_{DTR} are given by

$$f_{Y_1|\mu_{\text{DTR}}}(y_1) = f_{\text{NC}}(y_1, \mu_{\text{DTR}}, q_{\text{DTR}}), \quad (26)$$

$$f_{Y_2|\mu_{\text{DTR}}}(y_2) = f_{\text{C}}(y_2, q_{\text{DTR}}), \quad (27)$$

where $q_{\text{DTR}} = N_s WT$. As shown in Table I, we can observe that the basic difference between TR and DTR signaling lies not only in a doubled non-centrality parameter, but also in double the degrees of freedom. The non-centrality parameter reflects the amount of useful energy captured by the correlator at the receiver, hence, larger values result in better performance. On the contrary, the degrees of freedom of Y_1 account for the noise accumulation in the integration interval; thus, larger values result in poorer performance. Following the derivation leading to (21), the BEP of DTR signaling with an AcR can be written as

$$P_{e,\text{DTR}} = P_e(\psi_{\mu_{\text{DTR}}}(jv), q_{\text{DTR}}), \quad (28)$$

where $\psi_{\mu_{\text{DTR}}}(jv) \triangleq \mathbb{E}\{\exp(jv\mu_{\text{DTR}})\} = \prod_{l=1}^{L_{\text{CAP}}} \psi_l(\frac{jv2E_s}{N_0})$ is the CF of μ_{DTR} .

For DTR signaling with a modified AcR, the non-centrality parameter of Y_1 in (17) becomes

$$\mu_{\text{ADTR}} \triangleq \frac{1}{2\sigma_{\text{ADTR}}^2} \sum_{j=0}^{N_s-1} \int_0^T w_j^2(t) dt = \frac{4N_s}{(N_s+1)} \left(\frac{E_s}{N_0} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2 \right), \quad (29)$$

where the variance σ_{ADTR}^2 of $\beta_{1,j,m}$ and $\beta_{2,j,m}$ is¹²

$$\sigma_{\text{ADTR}}^2 = \frac{\mathbb{V}\{e_{i-1}\eta_{2,j,m}\} + \mathbb{V}\{e_i\eta_{1,j,m}\}}{8W} = \frac{N_0(N_s+1)}{8N_s}. \quad (30)$$

The pdfs of Y_1 and Y_2 conditioned on μ_{ADTR} are now given by

$$f_{Y_1|\mu_{\text{ADTR}}}(y_1) = f_{\text{NC}}(y_1, \mu_{\text{ADTR}}, q_{\text{DTR}}), \quad (31)$$

$$f_{Y_2|\mu_{\text{ADTR}}}(y_2) = f_{\text{C}}(y_2, q_{\text{DTR}}). \quad (32)$$

Following the derivation leading to (21), the BEP of DTR signaling with a modified AcR is given by

$$P_{e,\text{ADTR}} = P_e(\psi_{\mu_{\text{ADTR}}}(jv), q_{\text{DTR}}). \quad (33)$$

In the next section, we extend our BEP analysis of TR and DTR signaling to include NBI.

V. NARROWBAND INTERFERENCE ANALYSIS

In the presence of NBI, the received signal can be written as $r(t) = (h * s)(t) + J(t) + n(t)$, where $J(t)$ denotes the NBI and $s(t)$ denotes the signal transmitted via TR or DTR signaling. The autocorrelation function of the superposition of the two independent noise processes, $n_{\text{T}}(t) \triangleq J(t) + n(t)$, is given by $R_{n_{\text{T}}}(\tau) = R_{\text{J}}(\tau) + \frac{N_0}{2}\delta(\tau)$, where $R_{\text{J}}(\tau) =$

$\mathbb{E}\{J(t)J(t+\tau)\}$. Since the bandwidth of typical NBI is smaller than that of the transmitted pulse, the autocorrelation function of $\tilde{n}_{\text{T}}(t)$, the bandpass filtered version of $n_{\text{T}}(t)$, is given by

$$R_{\tilde{n}_{\text{T}}}(\tau) = R_{\text{J}}(\tau) + WN_0 \text{sinc}(W\tau) \cos(2\pi f_c \tau). \quad (34)$$

As in [23], we model the NBI as a single-tone continuous-wave signal given by

$$J(t) = \alpha_{\text{J}} \sqrt{2J_0} \cos(2\pi f_{\text{J}} t + \theta), \quad (35)$$

where J_0 is the average NBI power, α_{J} is a slowly-varying Rayleigh distributed r.v. with $\mathbb{E}\{\alpha_{\text{J}}^2\} = 1$, f_{J} is the NBI carrier frequency, and θ is the random phase, uniformly distributed over $[0, 2\pi)$.¹³ Thus, $R_{\text{J}}(\tau) = J_0 \cos(2\pi f_{\text{J}} \tau)$, which means that $\tilde{n}_{\text{T}}(t)$ is colored, and the samples of $\tilde{n}_{\text{T}}(t)$ taken at an interval of $1/W$ are correlated. In the following, we derive the BEP of TR and DTR signaling with an AcR in the presence of NBI, where we define $\text{SIR} \triangleq E_s/(N_s T_{\text{f}} J_0)$. The extension to modified AcR is straightforward and omitted for brevity.

A. Transmitted-Reference

By incorporating the NBI given in (35) and using the sampling expansion approach, we can rewrite U_j in (14) as

$$U_j = \frac{1}{2W} \sum_{m=1}^{2WT} \left[d_0 w_{j,m}^2 + w_{j,m} (\xi_{2,j,m} + \eta_{2,j,m}) + d_0 w_{j,m} (\xi_{1,j,m} + \eta_{1,j,m}) + (\xi_{1,j,m} + \eta_{1,j,m}) (\xi_{2,j,m} + \eta_{2,j,m}) \right], \quad (36)$$

where $\xi_{1,j,m}$ and $\xi_{2,j,m}$, for odd m (even m) respectively are the real (imaginary) parts of the samples of the equivalent low-pass version of $\xi_{1,j}(t)$ and $\xi_{2,j}(t)$, given by

$$\begin{aligned} \xi_{1,j}(t) &= \alpha_{\text{J}} \sqrt{2J_0} \cos[2\pi f_{\text{J}}(t + j2T_{\text{f}} + c_j T_{\text{p}}) + \theta], \\ \xi_{2,j}(t) &= \alpha_{\text{J}} \sqrt{2J_0} \cos[2\pi f_{\text{J}}(t + j2T_{\text{f}} + c_j T_{\text{p}} + T_{\text{r}}) + \theta]. \end{aligned} \quad (37)$$

The rest of the terms in (36) are defined similarly to those in (14). By conditioning on d_0 , we can rewrite (36) in the form of (15) and (16), where $\beta_{1,j,m} = \frac{1}{2\sqrt{2W}}(\eta_{2,j,m} + \xi_{2,j,m} + \eta_{1,j,m} + \xi_{1,j,m})$ and $\beta_{2,j,m} = \frac{1}{2\sqrt{2W}}(\eta_{2,j,m} + \xi_{2,j,m} - \eta_{1,j,m} - \xi_{1,j,m})$. Further conditioning on θ , α_{J} , and $\{c_j\}$, the quantities $\xi_{1,j,m}$ and $\xi_{2,j,m}$ in (37) are deterministic, and the conditional variance σ_{TR}^2 of $\beta_{1,j,m}$ and $\beta_{2,j,m}$ is simply $\frac{N_0}{4}$. Thus, the statistical characterization of U_j when conditioned on θ , α_{J} , $\{c_j\}$, $\{a_j\}$, and $\{\alpha_l\}$ is no longer symmetric with respect to d_0 due to the presence of the interference term. Note that $U_j|_{d_0=+1}$ is simply the difference of two noncentral chi-squared r.v.'s with the same degrees of freedom, but different non-centrality parameters. As a result, we need to separately calculate the conditional BEP with respect to d_0 to obtain the overall BEP.

¹³Unlike UWB signals, NBI experiences frequency flat fading and the amplitude α_{J} is assumed to be constant over at least two symbols of TR signaling.

¹²The notation $\mathbb{V}\{x\}$ denotes the variance of r.v. x .

$$\begin{aligned}
\mu_{\text{TR},Y_1}^{(\text{NBI})} &\triangleq \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T \left[w_j(t) + \frac{\xi_{1,j}(t) + \xi_{2,j}(t)}{2} \right]^2 dt \\
&\approx \frac{E_s}{N_0} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2 + \frac{\alpha_J^2 N_s J_0 T}{2N_0} + \frac{\alpha_J^2 N_s J_0 T}{2N_0} \cos(2\pi f_J T_r) \\
&\quad + \frac{4\alpha_J |\widehat{P}(f_J)| \sqrt{2E_p J_0} \cos(\pi f_J T_r)}{N_0} \sum_{j=0}^{\frac{N_s}{2}-1} a_j \sum_{l=1}^{L_{\text{CAP}}} \alpha_l \cos(2\pi f_J (\tau_l + j2T_t + c_j T_p + T_r/2) + \varphi), \quad (38)
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}\{Z_{\text{TR}} \leq 0 | d_0 = +1\} &= \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \left(\frac{1}{1+v^2} \right)^{q_{\text{TR}}} \Re \left\{ \frac{\mathbb{E}_\Psi \left\{ \exp\left(\frac{-jv\mu_{\text{TR},Y_1}^{(\text{NBI})}}{1+jv}\right) \exp\left(\frac{jv\mu_{\text{TR},Y_2}^{(\text{NBI})}}{1-jv}\right) \right\}}{jv} \right\} dv, \quad (42) \\
\mathbb{P}\{Z_{\text{TR}} \leq 0 | d_0 = +1\} &\simeq \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \left(\frac{1}{1+v^2} \right)^{q_{\text{TR}}} \Re \left\{ \frac{\mathbb{E}_{\{\alpha_l, \alpha_J\}} \left\{ \exp\left(\frac{-jv\mu_{\text{TR},Y_1}^{(\text{NBI})}}{1+jv}\right) \exp\left(\frac{jv\mu_{\text{TR},Y_2}^{(\text{NBI})}}{1-jv}\right) \right\}}{jv} \right\} dv \\
&= \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \left(\frac{1}{1+v^2} \right)^{q_{\text{TR}}} \Re \left\{ \frac{\psi_{\mu_{\text{TR}}}\left(\frac{-jv}{1+jv}\right) \psi_J(g_{\text{TR},d_0=+1}(jv))}{jv} \right\} dv \\
&\triangleq P_e^{(\text{NBI})}\left(\psi_{\mu_{\text{TR}}}(jv), \psi_J(g_{\text{TR},d_0=+1}(jv)), q_{\text{TR}}\right), \quad (43)
\end{aligned}$$

First, we derive the non-centrality parameters of Y_1 and Y_2 when conditioned on Ψ in (38) (at the top of this page) and (39), respectively:

$$\begin{aligned}
\mu_{\text{TR},Y_2}^{(\text{NBI})} &\triangleq \frac{1}{8\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T (\xi_{2,j}(t) - \xi_{1,j}(t))^2 dt \\
&\approx \frac{\alpha_J^2 N_s J_0 T}{2N_0} - \frac{\alpha_J^2 N_s J_0 T}{2N_0} \cos(2\pi f_J T_r), \quad (39)
\end{aligned}$$

where Ψ is defined for notational convenience as $\Psi \triangleq \{\alpha_J, \{\alpha_l\}_{l=1}^L, \varphi, \{c_j\}_{j=1}^{N_s/2}, \{a_j\}_{j=1}^{N_s/2}\}$. The detailed derivation and the justification of the approximations leading to (38) and (39), as well as the definition of φ and $\widehat{P}(f_j)$ can be found in Appendix I. Note that Y_2 in (17) is now a noncentral chi-squared r.v. due to the presence of NBI. From (38) and (39), it is interesting to see that the NBI affects the performance by changing the conditional means and variances of Y_1 and Y_2 .¹⁴ Using (38) and (39), the conditional pdfs of Y_1 and Y_2 are given by

$$f_{Y_1|\Psi}(y_1) = f_{\text{NC}}\left(y_1, \mu_{\text{TR},Y_1}^{(\text{NBI})}, q_{\text{TR}}\right), \quad (40)$$

$$f_{Y_2|\alpha_J}(y_2) = f_{\text{NC}}\left(y_2, \mu_{\text{TR},Y_2}^{(\text{NBI})}, q_{\text{TR}}\right), \quad (41)$$

where we have suppressed the conditioning r.v.'s $\{\alpha_l\}$, φ , $\{c_j\}$, and $\{a_j\}$ in (41) since (39) does not depend on these r.v.'s. Now, to evaluate the BEP for $Z_{\text{TR}} \leq 0$ when

¹⁴The mean and variance of a non-central chi-squared r.v. are given by $(k + \mu)$ and $2(k + 2\mu)$ respectively, where k is the degrees of freedom and μ is the non-centrality parameter.

$d_0 = +1$, we use the inversion theorem [59] to obtain (42), shown at the top of this page, where $\Re\{\cdot\}$ denotes the real part. By resorting to a quasi-analytical method, the statistical expectation in (42) can be calculated by numerically averaging each argument within the expectation with respect to its corresponding r.v.'s. Alternatively, we can resort to an approximate analytical method, where we consider the last term in (38) negligible compared to the first two terms. As a result, we can further suppress the conditioning r.v.'s φ , $\{c_j\}$, and $\{a_j\}$ in (40), since the dependence of $\mu_{\text{TR},Y_1}^{(\text{NBI})}$ on these r.v.'s is now negligible. The approximate BEP conditioned on $d_0 = +1$ can be rewritten as (43), shown at the top of this page, where $\psi_{\mu_{\text{TR}}}(jv)$ is the CF of μ_{TR} given in (18). Given that $\psi_J(jv)$ is the CF of α_J^2 , $g_{\text{TR},d_0=+1}(jv)$ in (43) is defined as follows:

$$\begin{aligned}
g_{\text{TR},d_0=+1}(jv) &\triangleq \frac{-jv}{1+jv} \cdot \frac{N_s J_0 T}{2N_0} \left[1 + \cos(2\pi f_J T_r) \right] \\
&\quad + \frac{jv}{1-jv} \cdot \frac{N_s J_0 T}{2N_0} \left[1 - \cos(2\pi f_J T_r) \right]. \quad (44)
\end{aligned}$$

In the absence of NBI or when $J_0 = 0$, (43) gives us an alternative, but equivalent, expression to (21) for the BEP of TR signaling with an Acr.¹⁵

From (16), we can observe that $U_{j|d_0=-1}$, when conditioned on Ψ , is also the difference of two noncentral chi-squared

¹⁵Note that the difference in the expressions lies in the fact that we have used the inversion theorem [58], [59] to derive $\mathbb{P}\{Z_{\text{TR}} \leq 0 | d_0 = +1\}$ in (43).

$$\begin{aligned} \mu_{\text{TR},Y_3}^{(\text{NBI})} &\approx \frac{E_s}{N_0} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2 + \frac{\alpha_J^2 N_s J_0 T}{2N_0} - \frac{\alpha_J^2 N_s J_0 T}{2N_0} \cos(2\pi f_J T_r) \\ &+ \frac{4\alpha_J |\widehat{P}(f_J)| \sqrt{2E_p J_0} \sin(-\pi f_J T_r)}{N_0} \sum_{j=0}^{\frac{N_s}{2}-1} a_j \sum_{l=1}^{L_{\text{CAP}}} \alpha_l \sin(2\pi f_J (\tau_l + j2T_f + c_j T_p + T_r/2) + \varphi), \end{aligned} \quad (45)$$

r.v.'s with same degrees of freedom, but with different non-centrality parameters. Following the derivation from (38) to (42), we can again resort to the quasi-analytical method to evaluate $\mathbb{P}\{Z_{\text{TR}} > 0 | d_0 = -1\}$. We first derive the non-centrality parameters of Y_3 and Y_4 in (17) conditioned on Ψ , as shown in (45) (at the top of this page) and (46), respectively:

$$\mu_{\text{TR},Y_4}^{(\text{NBI})} \approx \frac{\alpha_J^2 N_s J_0 T}{2N_0} + \frac{\alpha_J^2 N_s J_0 T}{2N_0} \cos(2\pi f_J T_r), \quad (46)$$

where the detailed derivation and the justification of the approximations leading to (45) and (46) can be found in Appendix II. We then replace $\mu_{\text{TR},Y_1}^{(\text{NBI})}$ and $\mu_{\text{TR},Y_2}^{(\text{NBI})}$ in (42) with $\mu_{\text{TR},Y_3}^{(\text{NBI})}$ and $\mu_{\text{TR},Y_4}^{(\text{NBI})}$ to obtain $\mathbb{P}\{Z_{\text{TR}} > 0 | d_0 = -1\}$. Alternatively, under the approximate analytical method leading to (43) and (44) when the dependence of $\mu_{\text{TR},Y_3}^{(\text{NBI})}$ on φ , $\{c_j\}$ and $\{a_j\}$ is negligible, we can ignore the last term in (45). The approximate BEP conditioned on $d_0 = -1$ is then given by

$$\begin{aligned} &\mathbb{P}\{Z_{\text{TR}} > 0 | d_0 = -1\} \\ &\simeq P_e^{(\text{NBI})}(\psi_{\mu_{\text{TR}}}(jv), \psi_J(g_{\text{TR},d_0=-1}(jv)), q_{\text{TR}}), \end{aligned} \quad (47)$$

where $g_{\text{TR},d_0=-1}(jv)$ in (47) is defined as follows:

$$\begin{aligned} g_{\text{TR},d_0=-1}(jv) &\triangleq \frac{-jv}{1+jv} \cdot \frac{N_s J_0 T}{2N_0} \left[1 - \cos(2\pi f_J T_r) \right] \\ &+ \frac{jv}{1-jv} \cdot \frac{N_s J_0 T}{2N_0} \left[1 + \cos(2\pi f_J T_r) \right]. \end{aligned} \quad (48)$$

Using (43) and (47), it follows that the approximate BEP of TR signaling with an AcR in the presence of NBI is given by

$$\begin{aligned} &P_{e,\text{TR}}^{(\text{NBI})} \\ &\simeq \frac{1}{2} \left[P_e^{(\text{NBI})}(\psi_{\mu_{\text{TR}}}(jv), \psi_J(g_{\text{TR},d_0=+1}(jv)), q_{\text{TR}}) \right. \\ &\quad \left. + P_e^{(\text{NBI})}(\psi_{\mu_{\text{TR}}}(jv), \psi_J(g_{\text{TR},d_0=-1}(jv)), q_{\text{TR}}) \right]. \end{aligned} \quad (49)$$

Note that the fidelity of the above approximation depends on the insignificance of the last terms in both $\mu_{\text{TR},Y_1}^{(\text{NBI})}$ and $\mu_{\text{TR},Y_3}^{(\text{NBI})}$. As shown in Appendix I and in Section VI, the approximation is in good agreement with the quasi-analytical results for cases of practical interest.¹⁶

¹⁶However, in cases when this approximation fails, we can always resort to the quasi-analytical method.

B. Differential Transmitted-Reference

Following the sampling expansion approach and incorporating the NBI in (35), we can rewrite U_j in (24) as

$$\begin{aligned} U_j &= \frac{1}{2W} \sum_{m=1}^{2WT} \left[d_0 w_{j,m}^2 + e_{-1} w_{j,m} (\xi_{2,j,m} + \eta_{2,j,m}) \right. \\ &\quad \left. + e_0 w_{j,m} (\xi_{1,j,m} + \eta_{1,j,m}) \right. \\ &\quad \left. + (\xi_{1,j,m} + \eta_{1,j,m})(\xi_{2,j,m} + \eta_{2,j,m}) \right], \end{aligned} \quad (50)$$

where $\xi_{1,j,m}$ and $\xi_{2,j,m}$, for odd m (even m) are the real (imaginary) parts of the samples of the equivalent low-pass version of $\xi_{1,j}(t) \triangleq J(t + jT_f + c_j T_p - N_s T_f)$ and $\xi_{2,j}(t) \triangleq J(t + jT_f + c_j T_p)$, respectively, in the interval $[0, T]$, and the rest of the terms in (50) are defined similarly as in (24). Conditioned on d_0 , we can rewrite (50) in the form of (15) and (16), where $\beta_{1,j,m} = \frac{1}{2\sqrt{2W}}(e_{-1}\eta_{2,j,m} + e_{-1}\xi_{2,j,m} + e_0\eta_{1,j,m} + e_0\xi_{1,j,m})$ and $\beta_{2,j,m} = \frac{1}{2\sqrt{2W}}(e_{-1}\eta_{2,j,m} + e_{-1}\xi_{2,j,m} - e_0\eta_{1,j,m} - e_0\xi_{1,j,m})$. Further conditioning on θ , α_J , $\{c_j\}$, and $d_0 = +1$, the conditional variance σ_{DTR}^2 of $\beta_{1,j,m}$ and $\beta_{2,j,m}$ is $\frac{N_0}{4}$.¹⁷ Following the discussion for TR signaling, we will develop the approximate analytical method below. Under the approximation presented in Appendix I, the conditional non-centrality parameters of Y_1 and Y_2 in (17) are given by

$$\begin{aligned} \mu_{\text{DTR},Y_1}^{(\text{NBI})} &\approx \frac{2E_s}{N_0} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2 + \frac{\alpha_J^2 N_s J_0 T}{N_0} \\ &\quad + \frac{\alpha_J^2 N_s J_0 T}{N_0} \cos(2\pi f_J N_s T_f), \end{aligned} \quad (51)$$

$$\mu_{\text{DTR},Y_2}^{(\text{NBI})} \approx \frac{\alpha_J^2 N_s J_0 T}{N_0} - \frac{\alpha_J^2 N_s J_0 T}{N_0} \cos(2\pi f_J N_s T_f). \quad (52)$$

The conditional non-centrality parameters of Y_3 and Y_4 in (17) are given by

$$\begin{aligned} \mu_{\text{DTR},Y_3}^{(\text{NBI})} &\approx \frac{2E_s}{N_0} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2 + \frac{\alpha_J^2 N_s J_0 T}{N_0} \\ &\quad - \frac{\alpha_J^2 N_s J_0 T}{N_0} \cos(2\pi f_J N_s T_f), \end{aligned} \quad (53)$$

$$\mu_{\text{DTR},Y_4}^{(\text{NBI})} \approx \frac{\alpha_J^2 N_s J_0 T}{N_0} + \frac{\alpha_J^2 N_s J_0 T}{N_0} \cos(2\pi f_J N_s T_f), \quad (54)$$

where the above derivations follow straightforwardly from Appendix I and II. Using (51) to (54), the approximate BEP

¹⁷Note that this conditional variance σ_{DTR}^2 remains the same even when $d_0 = -1$.

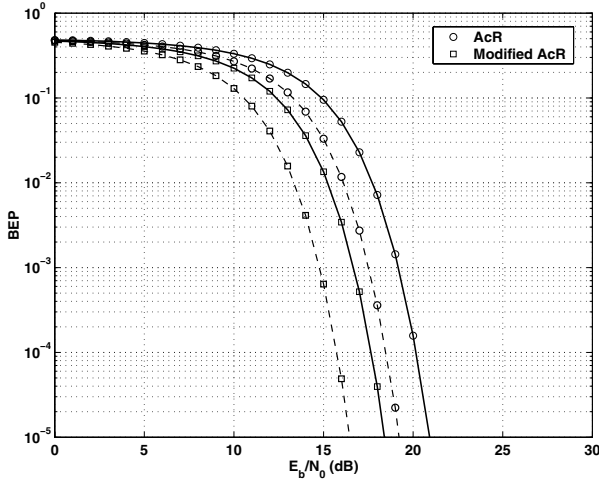


Fig. 3. BEP performance of TR and DTR signaling with an AcR and a modified AcR for $(L, \epsilon, m) = (32, 0, 3)$ and $WT = L$. The solid and dashed lines indicate the TR and DTR signaling, respectively.

of DTR signaling with an AcR in the presence of NBI is then given by

$$P_{e,DTR}^{(NBI)} \simeq \frac{1}{2} \left[P_e^{(NBI)} \left(\psi_{\mu_{DTR}}(jv), \psi_J(g_{DTR,d_0=+1}(jv)), q_{DTR} \right) + P_e^{(NBI)} \left(\psi_{\mu_{DTR}}(jv), \psi_J(g_{DTR,d_0=-1}(jv)), q_{DTR} \right) \right], \quad (55)$$

where $\psi_{\mu_{DTR}}(jv)$ is the CF of μ_{DTR} defined after (28), $g_{DTR,d_0=+1}(jv)$ and $g_{DTR,d_0=-1}(jv)$ are defined as follows:

$$g_{DTR,d_0=+1}(jv) \triangleq \frac{-jv}{1+jv} \cdot \frac{N_s J_0 T}{N_0} \left[1 + \cos(2\pi f_J N_s T_f) \right] + \frac{jv}{1-jv} \cdot \frac{N_s J_0 T}{N_0} \left[1 - \cos(2\pi f_J N_s T_f) \right], \quad (56)$$

$$g_{DTR,d_0=-1}(jv) \triangleq \frac{-jv}{1+jv} \cdot \frac{N_s J_0 T}{N_0} \left[1 - \cos(2\pi f_J N_s T_f) \right] + \frac{jv}{1-jv} \cdot \frac{N_s J_0 T}{N_0} \left[1 + \cos(2\pi f_J N_s T_f) \right]. \quad (57)$$

We remark that (49) and (55) can be evaluated for a broad class of fading channels, including Nakagami, Rice, and Rayleigh, whose CFs are known in closed-form [58].

VI. NUMERICAL EXAMPLES

In this section, we evaluate the performance of both TR and DTR signaling, with and without NBI, based on the unified analysis developed in previous sections. We consider a band-pass UWB system with pulse duration $T_p = 0.5$ ns, average repetition period $T_f = 100$ ns, and $N_s = 16$. For simplicity, T_r is set such that there is no ISI or isi in the system, i.e., $T_r = 2T_f - T_g - N_h T_p$. We consider a TH sequence of all ones ($c_j = 1$ for all j) and $N_h = 2$. The NBI carrier frequency is $f_J = 2.45$ GHz.¹⁸ Since the NBI experiences

¹⁸For our numerical results, we assume that the NBI is within the band of interest.

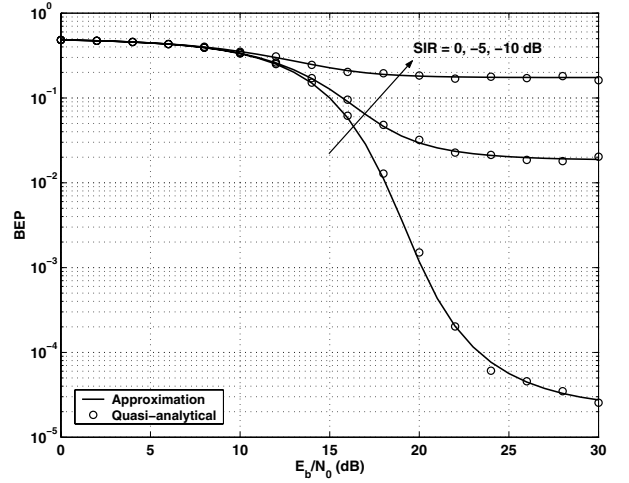


Fig. 4. BEP performance of TR signaling with an AcR in the presence of NBI for $(L, \epsilon, m) = (32, 0, 3)$ and $WT = L$.

flat Rayleigh fading, the CF of α_j^2 is $\psi_J(jv) = 1/(1-jv)$. For UWB channels, it has been verified experimentally that the multipath gains can be modeled as Nakagami- m r.v.'s [8]. As a result, we consider a dense resolvable multipath channel, where each multipath gain is Nakagami distributed with fading severity index m and average power $\mathbb{E}\{\alpha_l^2\}$, where $\mathbb{E}\{\alpha_l^2\} = \mathbb{E}\{\alpha_1^2\} \exp[-\epsilon(l-1)]$, for $l = 1, \dots, L$, are normalized such that $\sum_{l=1}^L \mathbb{E}\{\alpha_l^2\} = 1$. For simplicity, the fading severity index m is assumed to be identical for all paths. The average power of the first arriving multipath component is given by $\mathbb{E}\{\alpha_1^2\}$, and ϵ is the power decay factor. With this model, we consider two sets of parameters, $(L, \epsilon, m) = (32, 0, 3)$ for uniform PDP and $(32, 0.4, 3)$ for exponential PDP.

Figure 3 compares the BEP performance of TR and DTR signaling with an AcR and a modified AcR for $(L, \epsilon, m) = (32, 0, 3)$ and $WT = L$ in the absence of NBI. The difference between TR and DTR signaling is about 2 dB, slightly less than the 3 dB expected from the doubling of the non-centrality parameter shown in Table I. This loss of 1 dB can be attributed to more noise accumulation as $q_{DTR} = 2q_{TR}$. Note that the optimum WT is L for uniform PDP [39]. However, when the PDP is exponential, the optimum WT is much smaller than L , and the degradation due to excessive noise accumulation is less significant, as apparent in Fig. 7. By comparing the performance between an AcR and a modified AcR, it can be observed in Fig. 3 that the modified AcR performs better than the AcR by about 3 dB for both signaling schemes. This accounts for the increase of about a factor of two in the instantaneous received SNR through the non-centrality parameter when a modified AcR is used, as indicated in Table I.

To better understand the validity of the approximation developed in Section V, we compare the BEP performance of TR signaling with an AcR in the presence of NBI when $a_j = 1$ for all j and $|\hat{P}(f_J)| \approx \sqrt{T_p}$.¹⁹ Fig. 4 shows the BEP performance of TR signaling for different SIR values, $(L, \epsilon, m) = (32, 0, 3)$ and $WT = L$. It can be observed that

¹⁹For simplicity, we have considered the case where the frequency response of $p(t)$ is flat over the bandwidth W .

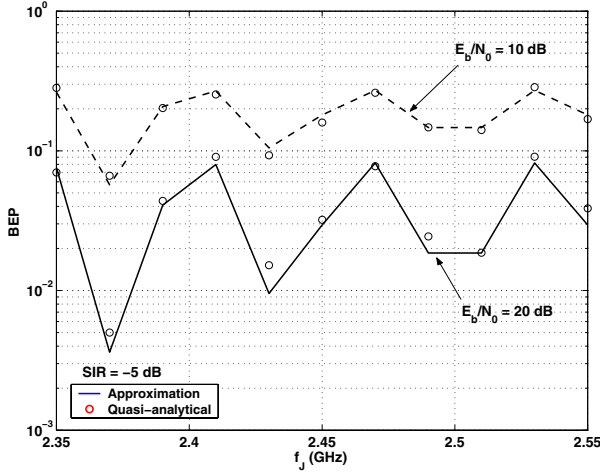


Fig. 5. Effect of NBI carrier frequency on BEP performance of TR signaling with an AcR for $(L, \epsilon, m) = (32, 0, 3)$ and $WT = L$.

the approximate analytical results are in good agreement with the quasi-analytical results. We further investigate the effect of the NBI carrier frequency on the fidelity of the approximation in Fig. 5 for $(L, \epsilon, m) = (32, 0, 3)$, $WT = L$, and $SIR = -5$ dB. Similar to the results in Fig. 4, the approximate analytical method is in good agreement with the quasi-analytical results, showing the usefulness of the approximation for investigating the performance of TR signaling schemes in the presence of NBI.

To understand the effect of NBI and PDP on the choice of integration interval T of an AcR, we first plot the BEP of TR signaling in Fig. 6 as a function of time-bandwidth product, WT , using an analytical approximation for $(L, \epsilon, m) = (32, 0, 3)$. Fig. 6 shows that with this PDP, the optimum T is always equal to T_g (i.e., $WT = L$), regardless of the presence of NBI. It can also be observed that the performance gain for using the optimum T is significant in the absence of NBI, especially at high E_b/N_0 . This is because, in the absence of NBI, more useful energy is captured with increasing WT until optimum WT is reached for high E_b/N_0 . However, in the presence of NBI, interference energy is also accumulated for every increase in WT , causing performance degradation. This trade-off is more subtle for channels with non-uniform PDP, as illustrated in Fig. 7 for $(L, \epsilon, m) = (32, 0.4, 3)$. It can be seen that the optimum T is no longer at T_g , since the gain from collecting more residual multipath energies inherent in the channel with exponential PDP is not sufficient to compensate for the noise accumulation beyond the optimum point. Moreover, we observe that the optimum T increases with E_b/N_0 and SIR , due to decreasing noise and interference accumulation. In general, the optimum T depends on the channel PDP, the operating E_b/N_0 , and the SIR . Consequently, it is important that the AcR is designed with an appropriate choice of T . Some of our work in this direction is reported in [60].

The effect of NBI on TR and DTR signaling with an AcR is plotted in Fig. 8 for different SIR values, $(L, \epsilon, m) = (32, 0.4, 3)$, and optimum T chosen for each E_b/N_0 and SIR . First, we can observe that the error floor for large values of E_b/N_0 becomes more significant as SIR decreases for both signaling schemes. In the absence of NBI, DTR signaling has a

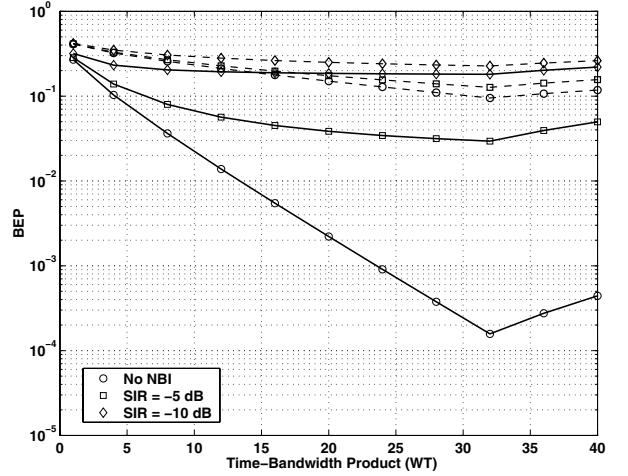


Fig. 6. Effect of integration interval T on BEP performance of TR signaling with an AcR for uniform PDP. The solid and dashed lines indicate $E_b/N_0 = 20$ dB and $E_b/N_0 = 15$ dB, respectively.

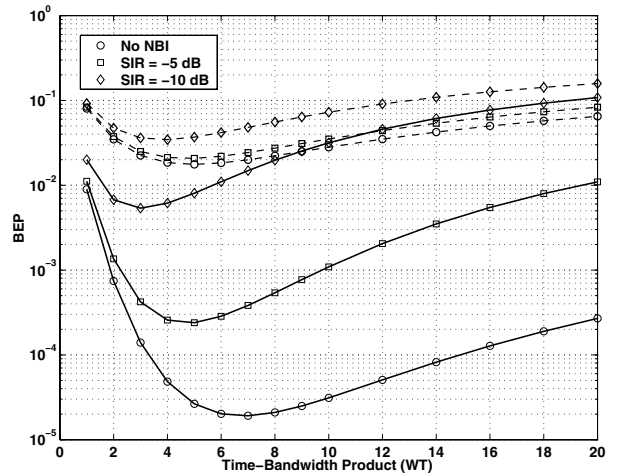


Fig. 7. Effect of integration interval T on BEP performance of TR signaling with an AcR for exponential PDP. The solid and dashed lines indicate $E_b/N_0 = 20$ dB and $E_b/N_0 = 15$ dB, respectively.

gain of about 3 dB compared to TR signaling. However, this gain diminishes as SIR decreases. After certain points (e.g., $E_b/N_0 = 20, 16, 10$ dB for $SIR = -5, -10, -20$ dB, respectively), DTR signaling performs worse than TR signaling, as indicated by the error floor. In the interference-limited regime, it is particularly interesting to observe that TR signaling is more robust against NBI compared to DTR signaling. This is because interference is more severe in DTR signaling due to the presence of more noise and interference terms as $q_{DTR} = 2q_{TR}$. Despite a doubling of the received multipath energies in DTR signaling compared to TR signaling, the presence of more interference terms essentially outweighs this gain, as indicated by the crossing of the curves in Fig. 8.

VII. CONCLUSION

In this paper, we developed an analytical framework and provided a unified performance analysis of TR and DTR signaling for both an AcR and a modified AcR in dense resolvable multipath channels. The framework is based on the sampling expansion approach without the conventional

$$\mu_{\text{TR},Y_1}^{(\text{NBI})} = \underbrace{\frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T w_j^2(t) dt}_{\triangleq \mu_A} + \underbrace{\frac{1}{8\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T (\xi_{1,j}(t) + \xi_{2,j}(t))^2 dt}_{\triangleq \mu_B} + \underbrace{\frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T w_j(t) (\xi_{1,j}(t) + \xi_{2,j}(t)) dt}_{\triangleq \mu_C}, \quad (58)$$

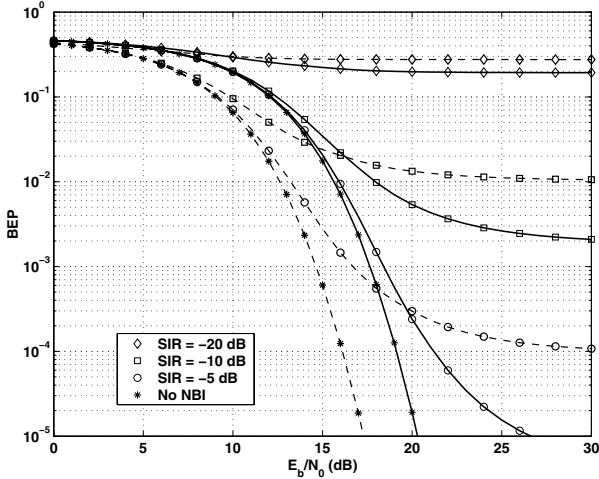


Fig. 8. Effect of NBI on BEP performance of TR and DTR signaling with an AcR for $(L, \epsilon, m) = (32, 0.4, 3)$ and optimum T chosen for each SNR and SIR. The solid and dashed lines indicate the TR and DTR signaling, respectively.

Gaussian approximation, and is valid for a broad class of fading channels. We first analyzed the case without NBI, then extended our analysis to take into account the effect of NBI. We developed a quasi-analytical method as well as an approximate analytical method to evaluate the BEP of TR and DTR signaling in the presence of NBI. We showed that the approximate analytical method is particularly useful in obtaining BEP expressions that provide insight into the effect of NBI on the performance of TR signaling schemes. Numerical results revealed that the approximation is in good agreement with the quasi-analytical results. We quantified the effects of NBI and channel PDP on the optimum integration interval of an AcR, showing that NBI imposes a practical limit on the amount of multipath energy that can be captured by an AcR. In particular, we showed that the optimum integration interval strongly depends on the channel PDP and the SNR, as well as the SIR. We compared TR and DTR signaling in terms of their sensitivity to NBI and revealed that the BEP improvement provided by DTR signaling is entirely different in noise-limited and interference-limited regimes.

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APPENDIX I DERIVATION OF $\mu_{\text{TR},Y_1}^{(\text{NBI})}$ AND $\mu_{\text{TR},Y_2}^{(\text{NBI})}$

In this appendix, the derivation of the non-centrality parameters $\mu_{\text{TR},Y_1}^{(\text{NBI})}$ in (38) and $\mu_{\text{TR},Y_2}^{(\text{NBI})}$ in (39) are shown. We begin

first with the derivation of $\mu_{\text{TR},Y_1}^{(\text{NBI})}$ in (58) as shown at the top of this page, where we can simplify μ_A and μ_B as

$$\mu_A = \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T w_j^2(t) dt = \frac{E_s}{N_0} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2, \quad (59)$$

$$\mu_B \approx \frac{\alpha_J^2 N_s J_0 T}{2N_0} + \frac{\alpha_J^2 N_s J_0 T}{2N_0} \cos(2\pi f_J T_r).$$

To obtain μ_B in (59), we simply need to expand all the terms of μ_B in (58) as follows:

$$\frac{1}{8\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T \xi_{1,j}^2(t) dt = \frac{\alpha_J^2 J_0}{2N_0} \sum_{j=0}^{\frac{N_s}{2}-1} \left[T + \frac{\sin(4\pi f_J (T + j2T_f + c_j T_p) + \theta)}{4\pi f_J} - \frac{\sin(4\pi f_J (j2T_f + c_j T_p) + \theta)}{4\pi f_J} \right] \approx \frac{\alpha_J^2 N_s J_0 T}{4N_0}$$

$$\frac{1}{8\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T \xi_{2,j}^2(t) dt \approx \frac{\alpha_J^2 N_s J_0 T}{4N_0}$$

where we have made the above approximations by observing that $T \gg \frac{1}{4\pi f_J}$ and $|\sin(\phi)| \leq 1$. In addition,

$$\frac{1}{4\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T \xi_{1,j}(t) \xi_{2,j}(t) dt \approx \mu_D. \quad (60)$$

where $\mu_D = \frac{\alpha_J^2 N_s J_0 T}{2N_0} \cos(2\pi f_J T_r)$ when $T \cos(2\pi f_J T_r) \gg \frac{1}{4\pi f_J}$. Otherwise, μ_D is on the same order as $\frac{1}{4\pi f_J}$, which is negligible compared to the first term of μ_B in (59). As a result, we can ignore the latter case and consider only the scenario when $T \cos(2\pi f_J T_r) \gg \frac{1}{4\pi f_J}$ as shown in (59). Next, we can rewrite μ_C in (58) as (61) shown at the top of next page, where $|\hat{P}(f_j)|$ is the magnitude of the frequency response of $p(t)$ at frequency f_j . The composite random phase is given by $\varphi \triangleq \arg\{\hat{P}(f_j)\} + \theta$, where $\arg\{\hat{P}(f_j)\}$ is the angle of the frequency response of $p(t)$ at frequency f_j , and φ is uniformly distributed over $[0, 2\pi)$. In summary, we obtain $\mu_{\text{TR},Y_1}^{(\text{NBI})}$ in (38) using (59) and (61).

Using (58), (59) and (61), we can determine the validity of the approximate analytical method using (62) and (63) shown at the top of the next page. For a given set of system parameters, we simply need to check if $\mu_A + \mu_B \gg \mu_C$ using (62) and (63). For example, this can arise when Walsh-Hadamard sequences are used for $\{a_j\}$. To elaborate on this example, we first numerically average (over $\{\alpha_l\}$ and α_j) the quantities within $|\cdot|$ of (62) and (63) for a typical set of system parameters ($T_f = 100\text{ns}$, $T_p = 0.5\text{ns}$, $T_r = 40\text{ns}$, possible

$$\begin{aligned}\mu_C &= \frac{2\alpha_J\sqrt{2E_pJ_0}}{N_0} \sum_{j=0}^{\frac{N_s}{2}-1} a_j \sum_{l=1}^{L_{CAP}} \alpha_l \int_0^{T_p} p(t) \\ &\quad \times [\cos(2\pi f_J(t + \tau_l + j2T_f + c_jT_p) + \theta) + \cos(2\pi f_J(t + \tau_l + j2T_f + c_jT_p + T_r) + \theta)] dt \\ &= \frac{4\alpha_J|\hat{P}(f_J)|\sqrt{2E_pJ_0} \cos(\pi f_J T_r)}{N_0} \sum_{j=0}^{\frac{N_s}{2}-1} a_j \sum_{l=1}^{L_{CAP}} \alpha_l \cos(2\pi f_J(\tau_l + j2T_f + c_jT_p + T_r/2) + \varphi),\end{aligned}\quad (61)$$

$$\frac{\mu_A}{\mu_C} = \left| \frac{\sum_{l=1}^{L_{CAP}} \alpha_l^2}{\alpha_J \cos(\pi f_J T_r) \sum_{j=0}^{\frac{N_s}{2}-1} a_j \sum_{l=1}^{L_{CAP}} \alpha_l \cos(2\pi f_J(\tau_l + j2T_f + c_jT_p + T_r/2) + \varphi)} \right| \frac{N_s}{4|\hat{P}(f_J)|} \sqrt{\frac{T_f \text{SIR}}{2}}, \quad (62)$$

$$\frac{\mu_B}{\mu_C} = \left| \frac{\alpha_J \cos(\pi f_J T_r)}{\sum_{j=0}^{\frac{N_s}{2}-1} a_j \sum_{l=1}^{L_{CAP}} \alpha_l \cos(2\pi f_J(\tau_l + j2T_f + c_jT_p + T_r/2) + \varphi)} \right| \frac{N_s L_{CAP} T_p}{4|\hat{P}(f_J)|} \sqrt{\frac{1}{2T_f \text{SIR}}}, \quad (63)$$

$$\begin{aligned}\mu_{\text{TR}, Y_3}^{(\text{NBI})} &= \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T w_j^2(t) dt + \frac{1}{8\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T (\xi_{2,j}(t) - \xi_{1,j}(t))^2 dt + \frac{1}{2\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T w_j(t) (\xi_{2,j}(t) - \xi_{1,j}(t)) dt \\ &\approx \frac{E_s}{N_0} \sum_{l=1}^{L_{CAP}} \alpha_l^2 + \frac{\alpha_J^2 N_s J_0 T}{2N_0} - \frac{\alpha_J^2 N_s J_0 T}{2N_0} \cos(2\pi f_J T_r) \\ &\quad + \frac{4\alpha_J|\hat{P}(f_J)|\sqrt{2E_pJ_0} \sin(-\pi f_J T_r)}{N_0} \sum_{j=0}^{\frac{N_s}{2}-1} a_j \sum_{l=1}^{L_{CAP}} \alpha_l \sin(2\pi f_J(\tau_l + j2T_f + c_jT_p + T_r/2) + \varphi),\end{aligned}\quad (65)$$

$f_J = 1.575$ GHz, 3.5 GHz, and 5.745 GHz) and considering $c_j = 1$ for all j and Walsh-Hadamard sequences for $\{a_j\}$. Results have shown that the value of $|\cdot|$ in (62) and (63) is always greater than 1. Approximating these factors by 1 and $|\hat{P}(f_J)| \approx \sqrt{T_p}$, we focus on the rest of the terms in (62) and (63). Since T_f is usually on the order of 100 times T_p , and N_s and L_{CAP} are usually larger than $4\sqrt{2}$, we can verify that $\mu_A + \mu_B \gg \mu_C$ and neglect μ_C when $\text{SIR} \geq -20$ dB.

Using similar approach leading to (59), $\mu_{\text{TR}, Y_2}^{(\text{NBI})}$ in (39) can be approximated as follows:

$$\begin{aligned}\mu_{\text{TR}, Y_2}^{(\text{NBI})} &= \frac{1}{8\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T (\xi_{2,j}(t) - \xi_{1,j}(t))^2 dt \\ &\approx \frac{\alpha_J^2 N_s J_0 T}{2N_0} - \frac{\alpha_J^2 N_s J_0 T}{2N_0} \cos(2\pi f_J T_r).\end{aligned}\quad (64)$$

APPENDIX II

DERIVATION OF $\mu_{\text{TR}, Y_3}^{(\text{NBI})}$ AND $\mu_{\text{TR}, Y_4}^{(\text{NBI})}$

In this appendix, the non-centrality parameters $\mu_{\text{TR}, Y_3}^{(\text{NBI})}$ in (45) and $\mu_{\text{TR}, Y_4}^{(\text{NBI})}$ in (46) are derived. We derive $\mu_{\text{TR}, Y_3}^{(\text{NBI})}$ in (65), shown at the top of this page, using similar steps as in Appendix I. Similarly, $\mu_{\text{TR}, Y_4}^{(\text{NBI})}$ is given by

$$\begin{aligned}\mu_{\text{TR}, Y_4}^{(\text{NBI})} &= \frac{1}{8\sigma_{\text{TR}}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T (\xi_{2,j}(t) + \xi_{1,j}(t))^2 dt \\ &\approx \frac{\alpha_J^2 N_s J_0 T}{2N_0} + \frac{\alpha_J^2 N_s J_0 T}{2N_0} \cos(2\pi f_J T_r).\end{aligned}\quad (66)$$

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