Network Navigation: Theory and Interpretation

Yuan Shen, *Student Member, IEEE*, Santiago Mazuelas, *Member, IEEE*, and Moe Z. Win, *Fellow, IEEE*

***Abstract*—Real-time and reliable location information of mobile nodes is a key enabler for many emerging wireless network applications. Such information can be obtained via *network navigation*, a new paradigm in which nodes exploit both spatial and temporal cooperation to infer their positions. In this paper, we establish a theoretical foundation for network navigation and determine the fundamental limits of navigation accuracy using equivalent Fisher information analysis. We then introduce the notion of carry-over information and provide a geometrical interpretation for the evolution of navigation information. Our framework unifies the navigation information obtained from spatial and temporal cooperation, leading to a deep understanding of information evolution and cooperation benefits in navigation networks.**

***Index Terms*—Cooperative network, localization, navigation, Cramér-Rao bound (CRB), equivalent Fisher information (EFI).**

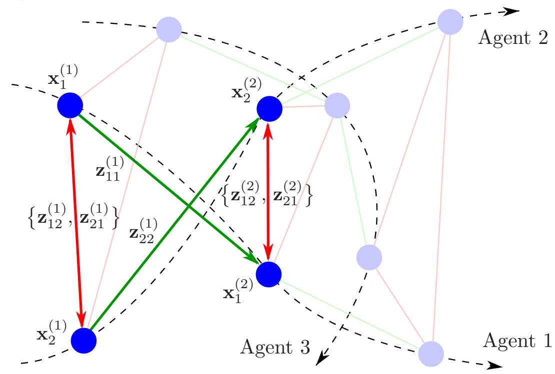

Fig. 1. Network navigation: three mobile agents (blue circle), in three different time steps, aiming to locate themselves by using inter-node measurements $\{\mathbf{z}_{kj}^{(n)}\}$ (red arrows) and intra-node measurements $\{\mathbf{z}_{kk}^{(n)}\}$ (green arrows).

I. Introduction

REAL-TIME reliable localization and tracking capability is a key enabler for numerous emerging applications in commercial, public safety, and military sectors. These include logistics, security tracking, medical services, underwater explorations, search and rescue operations, vehicle networking, and military operations [1]–[11]. This wide range of potential applications has motivated an increasing research interest in localization and tracking technologies for wireless networks [12]–[23].

Navigation encompasses both localization and tracking of mobile nodes (agents) in a network, where the agents infer their positional states[1] based on measurements and prior knowledge. In conventional systems, each agent individually determines its positional state from relative measurements taken with respect to (w.r.t.) fixed infrastructures and/or self measurements using inertial sensors. For instance, in the Global Positioning System (GPS), each agent determines its position from the pseudorange measurements taken w.r.t. multiple satellites with known positions [24]; and in self-tracking systems, each agent determines its positional state from the inertial measurements about its movement [25]. However, these conventional techniques fail to provide satisfactory performance in many scenarios: GPS-based techniques become inaccurate in harsh or indoor environments due to signal blockage, while inertial-based techniques become inaccurate over time due to velocity drift.

Cooperative techniques are attracting increasing interest for localization [6]–[8], [26]–[28], driven by their success in wireless communications [29]–[32]. Such techniques have been shown to improve localization performance due to sharing of information among spatial neighbors [6]–[8]. In addition to spatial cooperation, the agents in the network can also benefit from information obtained in different time instants (see Fig. 1). Traditionally, this temporal information is independently utilized by each agent via filtering techniques in a process commonly known as tracking [25]. Here we advocate a new paradigm that jointly uses both spatial and temporal cooperation for real-time localization and tracking in wireless networks, which we refer to as network navigation.

In the framework of network navigation, agents in a network jointly infer their positional states by sharing information in both the spatial and temporal domains [1]. In the spatial domain, each node obtains information about its positional state relative to that of others by inter-node measurements (e.g., ranges) together with prior spatial knowledge.[2] In the temporal domain, each node obtains information about its positional state evolution by intra-node measurements (e.g., velocities) together with prior temporal knowledge.[3]

The proper use of all relevant information through cooperation in a navigation network can lead to dramatic performance improvements over conventional approaches. However, realizing the benefits of spatial and temporal cooperation

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The authors are with the Laboratory for Information and Decision Systems (LIDS), Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139 USA (e-mail: shenyuan@mit.edu, mazuelas@mit.edu, moewin@mit.edu).

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[1]The positional state can include the position, velocity, acceleration, orientation, and angular velocity.

[2]Examples of inter-node measurement sensors include RF radios, vision sensors, and GPS devices; examples of prior spatial knowledge include positions of certain nodes and map information.

[3]Examples of intra-node measurement sensors include inertial measurement unit, odometer, and compass; examples of prior temporal knowledge include mobility models and types of moving agents.

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incurs associated costs such as additional communication and more complicated information fusion. Thus, understanding the fundamental limits of cooperative navigation networks is important not only for providing performance benchmarks but also for guiding network design and operation under the performance/complexity trade-off.

In this paper, we establish a theoretical foundation for network navigation to determine the fundamental limits of navigation accuracy. Built on our results of static localization using only spatial cooperation [6], we incorporate intra-node measurements and mobility knowledge as another mode of cooperation, i.e., temporal cooperation. The main contributions of the paper are summarized as follows.

- We determine the fundamental limits of network navigation, in terms of navigation information, considering both Bayesian and deterministic models for cooperative networks.
- We derive the navigation information by equivalent Fisher information (EFI) analysis, and show that the EFI matrix (EFIM) can be decomposed into basic building blocks, each associated with certain measurement and prior knowledge.
- We introduce the notion of carry-over information, and develop a geometrical interpretation for the navigation information, providing insights into the information evolution and cooperation benefits.

The rest of the paper is organized as follows. Section II presents the network setting, the problem formulation, and the notion of EFI. In Section III, we derive the EFIMs for network navigation based on both Bayesian and deterministic models. In Section IV, we investigate the EFIM for a 2-D navigation problem and develop a geometrical interpretation. Finally, a summary of the paper is given in the last section.

Notation: $\mathbb{E}_{\mathbf{x}}\{\cdot\}$ denotes the expectation operator w.r.t. the random vector $\mathbf{x}$; $\mathbf{A} \succeq \mathbf{B}$ denotes that the matrix $\mathbf{A}-\mathbf{B}$ is positive semi-definite; $\mathrm{tr}\{\cdot\}$, $[\cdot]^\dagger$, $|\cdot|$, and $\mathrm{adj}(\cdot)$ denote the trace, conjugate transpose, determinant, and adjugate matrix of its argument, respectively; $\|\cdot\|$ denotes the Euclidean norm of its argument; $\mathbb{S}^n_{++}$ and $\mathbb{S}^n_{+}$ denote the set of $n\times n$ positive-definite and positive-semidefinite matrices, respectively; $\mathbf{u}_\phi$ denotes the unit vector $[\cos\phi \;\; \sin\phi]^\dagger$ and $\mathbf{u}_\phi^\perp := \mathbf{u}_{\phi+\pi/2}$; and $\mathbf{J}_\mathrm{r}(\phi) := \mathbf{u}_\phi \mathbf{u}_\phi^\dagger$ and $\mathbf{J}_\mathrm{r}(\phi, \phi+\pi/2) := \big[\mathbf{u}_\phi \, \mathbf{u}_\phi^{\perp\,\dagger} + \mathbf{u}_\phi^\perp \, \mathbf{u}_\phi^\dagger\big]/2$. The notation $\mathbf{x}_{k_1:k_2}$ is used for concatenating the set of vectors $\{\mathbf{x}_{k_1}, \mathbf{x}_{k_1+1}, \dots, \mathbf{x}_{k_2}\}$ and similarly $\mathbf{x}_{k_1:k_2}^{(t_1:t_2)}$ for $\big\{\mathbf{x}_{k_1:k_2}^{(t_1)}, \mathbf{x}_{k_1:k_2}^{(t_1+1)}, \dots, \mathbf{x}_{k_1:k_2}^{(t_2)}\big\}$, for $k_1 \le k_2, t_1 \le t_2$. We denote $\otimes$ as Kronecker product and $\mathbf{E}^N_{i,j}$ be a $N\times N$ matrix with all zeros except a 1 on the ith row and jth column. We also denote by $f(\mathbf{x})$ the probability density function (pdf) $f_\mathbf{X}(\mathbf{x})$ of the random vector $\mathbf{X}$ unless otherwise specified.

II. Problem Formulation

In this section, we describe the network setting, formulate the problem of network navigation, and briefly review the notion of EFI that will be used to characterize the fundamental limits of navigation accuracy.

A. Network Setting

Consider a cooperative wireless network consisting of N_a agents (or targets) and N_b anchors (or beacons), where each agent is equipped with both inter- and intra-node measurement sensors. The sets of agents and anchors are denoted by $\mathcal{N}_\mathrm{a} = \{1, 2, \dots, N_\mathrm{a}\}$ and $\mathcal{N}_\mathrm{b} = \{N_\mathrm{a}+1, N_\mathrm{a}+2, \dots, N_\mathrm{a}+N_\mathrm{b}\}$, respectively. Both the measurements and inference processes are made in discrete time instants t_n's $(n = 1, 2, \dots, N)$. Let $\mathbf{x}_k^{(n)} \in \mathbb{R}^D$ be the positional state of node k at time t_n, where those of the agents are to be determined. The positional state includes the position $\mathbf{p}_k^{(n)}$ and other mobility parameters such as velocity, acceleration, orientation, and angular velocity.[4]

The parameter vector at time t_n is given by

$$\boldsymbol{\theta}^{(n)} = \left[\; \mathbf{x}_{1:N_\mathrm{a}}^{(n)\,\dagger} \quad \boldsymbol{\kappa}_{1:N_\mathrm{a}}^{(n)\,\dagger} \;\right]^\dagger$$

where $\boldsymbol{\kappa}_k^{(n)}$ include the parameters associated with the inter- and intra-node measurements of node k at time t_n.[5] In particular, $\boldsymbol{\kappa}_k^{(n)}$ is the concatenation of the set of vectors $\big\{\boldsymbol{\kappa}_{kj}^{(n)} : j \in \mathcal{N}_\mathrm{a} \cup \mathcal{N}_\mathrm{b}\big\}$, where $\boldsymbol{\kappa}_{kj}^{(n)}$ $(k \neq j)$ is associated with the inter-node measurements of node k w.r.t. node j, and $\boldsymbol{\kappa}_{kk}^{(n)}$ is associated with the intra-node measurements of node k. Correspondingly, the set of measurements made at time t_n is denoted by $\mathbf{z}^{(n)} = \big\{\mathbf{z}_{kj}^{(n)} : k \in \mathcal{N}_\mathrm{a},\, j \in \mathcal{N}_\mathrm{a} \cup \mathcal{N}_\mathrm{b}\big\}$, where $\mathbf{z}_{kj}^{(n)}$ $(k \neq j)$ denotes the inter-node measurements made at node k w.r.t. node j, and $\mathbf{z}_{kk}^{(n)}$ denotes the intra-node measurements made at node k.

B. Bayesian and Deterministic Models

In this section, we describe the Bayesian and deterministic models for network navigation.

1) Bayesian model: The agents' positional states and measurement parameters can be described as random variables. In this case, the evolution in time of such variables and measurements is characterized by a hidden Markov model (HMM),[6] leading to the joint pdf of the measurements and parameters from time t_1 to t_N to be

$$f(\mathbf{z}, \boldsymbol{\theta}) = \prod_{n=1}^{N} f\big(\boldsymbol{\theta}^{(n)} | \boldsymbol{\theta}^{(n-1)}\big) f\big(\mathbf{z}^{(n)} | \boldsymbol{\theta}^{(n)}\big) \tag{1}$$

where $\boldsymbol{\theta} := \boldsymbol{\theta}^{(1:N)}$, $\mathbf{z} := \mathbf{z}^{(1:N)}$, and $\boldsymbol{\theta}_0 = \emptyset$ for notational convenience. We consider that (i) the dynamics of different nodes are mutually independent, (ii) the dynamics of positional states are independent of those of measurement parameters, and (iii) the dynamics of different measurement parameters are mutually independent. Therefore, the dynamic model for the parameters in (1) can be decomposed as

$$\begin{aligned} f\big(\boldsymbol{\theta}^{(n)} | \boldsymbol{\theta}^{(n-1)}\big) = \prod_{k\in\mathcal{N}_\mathrm{a}} \Big[\, & f\big(\boldsymbol{\kappa}_{kk}^{(n)} \big| \mathbf{x}_k^{(n-1)}, \boldsymbol{\kappa}_{kk}^{(n-1)}\big) \\ \times\, & f\big(\mathbf{x}_k^{(n)} \big| \mathbf{x}_k^{(n-1)}\big) \prod_{j\in\mathcal{N}_\mathrm{a}\cup\mathcal{N}_\mathrm{b}\backslash\{k\}} f\big(\boldsymbol{\kappa}_{kj}^{(n)} \big| \mathbf{x}_{kj}^{(n-1)}, \boldsymbol{\kappa}_{kj}^{(n-1)}\big) \Big] \end{aligned} \tag{2}$$

[4]For example, $D = 8$ for 2-D navigation in this case.

[5]Examples of parameters associated with inter- and intra-node measurements include those of wireless medium and clock drift, respectively.

[6]HMMs have been widely applied as a statistical modeling tool since they allow modeling complex real-world problems with reasonable computational complexity [33].

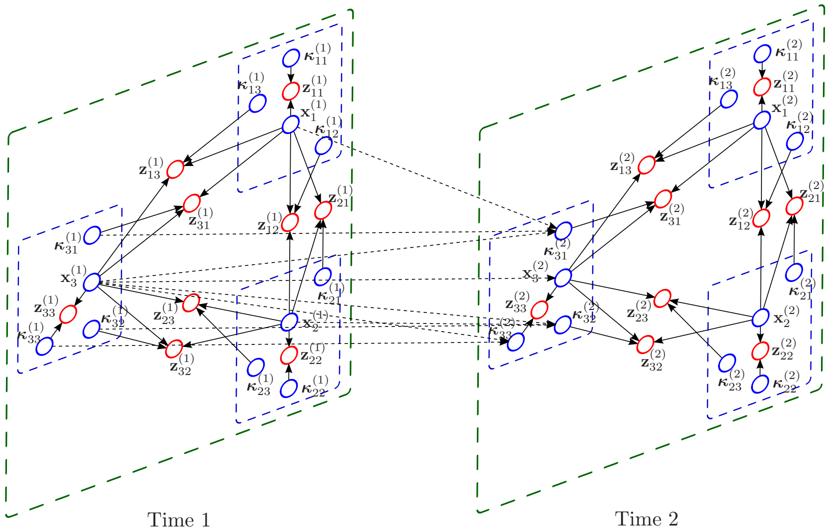


Fig. 2. Graphical model describing the interrelationships among the parameters and measurements present in the Bayesian model for network navigation. The temporal cooperation is illustrated only for agent 3.

where $\mathbf{x}_{kj}^{(n-1)} := \mathbf{x}_k^{(n-1)} - \mathbf{x}_j^{(n-1)}$. Moreover, since the measurements by different sensors are mutually independent conditioned on the positional state and measurement parameters, the measurement model (likelihood) in (1) is given by

$$f\big(\mathbf{z}^{(n)}|\boldsymbol{\theta}^{(n)}\big) = \prod_{k\in\mathcal{N}_\mathrm{a}} \Big[f\big(\mathbf{z}_{kk}^{(n)}\big|\mathbf{x}_k^{(n)}, \boldsymbol{\kappa}_{kk}^{(n)}\big) \times \prod_{j\in\mathcal{N}_\mathrm{a}\cup\mathcal{N}_\mathrm{b}\backslash\{k\}} f\big(\mathbf{z}_{kj}^{(n)}\big|\mathbf{x}_{kj}^{(n)}, \boldsymbol{\kappa}_{kj}^{(n)}\big)\Big]. \quad (3)$$

Figure 2 illustrates the graphical model for the factorizations in (2) and (3).

Since inter-node measurements depend on the relative positional states, we consider in (2) and (3) that these measurements and the dynamics of corresponding parameters depend on the difference of the positional states between two nodes, i.e., $\mathbf{x}_{kj}^{(n)}$ rather than $\{\mathbf{x}_k^{(n)}, \mathbf{x}_j^{(n)}\}$.

2) Deterministic model: The agents' positional states and the measurement parameters can also be described as deterministic unknowns. In this case, there is no prior dynamic knowledge, and the measurements depend on the agents' positions and orientations in a set of consecutive time instants. Hence, the positional state $\mathbf{x}_k^{(n)}$ includes only the position and orientation of agent k, and the likelihood for the deterministic model can be written as[7]

$$f(\mathbf{z}|\boldsymbol{\theta}) = \prod_{n=1}^{N} \prod_{k\in\mathcal{N}_\mathrm{a}} \Big[f\big(\mathbf{z}_{kk}^{(n)}\big|\mathbf{x}_k^{(n-n_0:n+n_0')}, \boldsymbol{\kappa}_{kk}^{(n)}\big) \times \prod_{j\in\mathcal{N}_\mathrm{a}\cup\mathcal{N}_\mathrm{b}\backslash\{k\}} f\big(\mathbf{z}_{kj}^{(n)}\big|\mathbf{x}_{kj}^{(n-n_0:n+n_0')}, \boldsymbol{\kappa}_{kj}^{(n)}\big)\Big] \quad (4)$$

where the measurements at time t_n depend only on the agents' positions and orientations from t_{n-n_0} to $t_{n+n_0'}$ as well as the measurement parameters at t_n. We also consider the independence condition of the measurements as in the Bayesian model (see Fig. 3 for the graphical model).

[7]Throughout the paper, the notation $(n - n_0 : n + n_0')$ is shorthand for $(\max\{n - n_0, 1\} : \min\{n + n_0', N\})$.

Note that the difference between the Bayesian and deterministic models is that the latter (i) considers all parameters to be deterministic unknowns, i.e., assumes no prior knowledge about dynamics, and (ii) includes only the position and orientation in the positional state and the measurements depend on such positional states in consecutive time instants. For example, if the intra-node measurement is node velocity, the deterministic model can describe the measurement as a function of the positions by

$$\mathbf{z}_{kk}^{(n)} = \frac{\mathbf{p}_k^{(n)} - \mathbf{p}_k^{(n-1)}}{\Delta_t} + \mathbf{w}_k^{(n)}$$

where Δ_t is the sampling time interval and $\mathbf{w}_k^{(n)}$ is the residual error. Correspondingly in (4), the likelihood of the velocity measurement can be expressed as $f\big(\mathbf{z}_{kk}^{(n)}\big|\mathbf{p}_k^{(n-1:n)}, \boldsymbol{\kappa}_{kk}^{(n)}\big)$.

C. Equivalent Fisher Information

The mean squared error (MSE) matrix of an estimate $\hat{\mathbf{x}}_k^{(n)}$ for the kth agent's positional state is bounded below as

$$\mathbb{E}_{\mathbf{z},\boldsymbol{\theta}}\Big\{(\hat{\mathbf{x}}_k^{(n)} - \mathbf{x}_k^{(n)})(\hat{\mathbf{x}}_k^{(n)} - \mathbf{x}_k^{(n)})^\dagger\Big\} \succeq \big[\mathbf{J}_{\boldsymbol{\theta}}^{-1}\big]_{\mathbf{x}_k^{(n)}} \quad (5)$$

where $\mathbf{J}_{\boldsymbol{\theta}}$ is the Fisher information matrix (FIM) for $\boldsymbol{\theta}$ [34] and $[\mathbf{J}_{\boldsymbol{\theta}}^{-1}]_{\mathbf{x}_k^{(n)}}$ denotes the square submatrix on the diagonal of $\mathbf{J}_{\boldsymbol{\theta}}^{-1}$ corresponding to $\mathbf{x}_k^{(n)}$ [5]. We next apply the EFI analysis to reduce the dimension of the FIM while retaining all the information for the parameters of interest [5].

Definition 1 (EFIM): Given a parameter vector $\boldsymbol{\theta} = [\boldsymbol{\theta}_1^\dagger \;\; \boldsymbol{\theta}_2^\dagger]^\dagger$ and the FIM $\mathbf{J}_{\boldsymbol{\theta}}$ of the form

$$\mathbf{J}_{\boldsymbol{\theta}} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^\dagger & \mathbf{C} \end{bmatrix}$$

where $\boldsymbol{\theta} \in \mathbb{R}^N$, $\boldsymbol{\theta}_1 \in \mathbb{R}^n$, $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times(N-n)}$, and $\mathbf{C} \in \mathbb{R}^{(N-n)\times(N-n)}$ with $1 \le n < N$, the EFIM for $\boldsymbol{\theta}_1$ is given by

$$\mathbf{J}_\mathrm{e}(\boldsymbol{\theta}_1) := \mathbf{A} - \mathbf{B}\mathbf{C}^{-1}\mathbf{B}^\dagger \,. \quad (6)$$

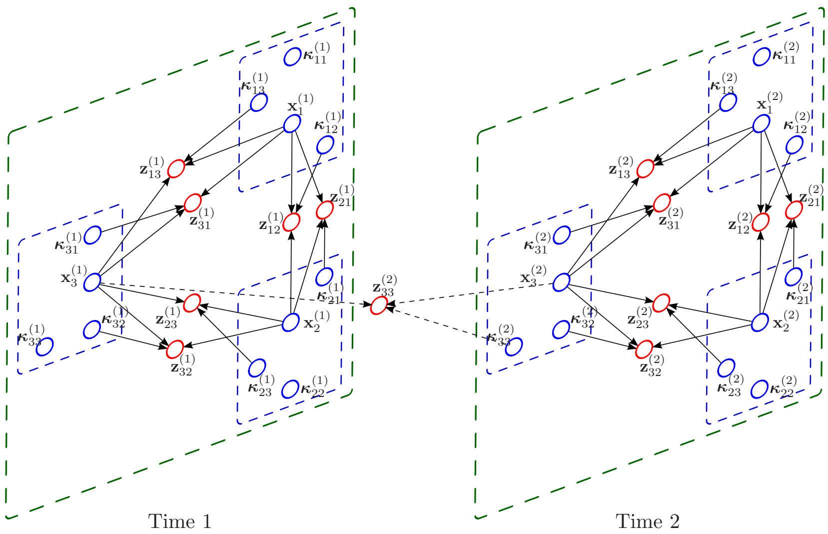


Fig. 3. Graphical model describing the interrelationships among the parameters and measurements present in the deterministic model for network navigation. The temporal cooperation is illustrated only for agent 3 with $n_0 = 1$ and $n_0' = 0$.

We call matrix $\mathbf{B}$ the *cross information* between $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ in $\mathbf{J}_{\boldsymbol{\theta}}$. The right-hand side of (6) is known as the Schur's complement of matrix $\mathbf{A}$ [35]. The EFIM retains all the necessary information to derive the information inequality for parameter $\boldsymbol{\theta}_1$, in the sense that $[\mathbf{J}_{\boldsymbol{\theta}}^{-1}]_{\boldsymbol{\theta}_1} = [\mathbf{J}_{\mathrm{e}}(\boldsymbol{\theta}_1)]^{-1}$.

III. EFIM for Network Navigation

In this section, we first derive the EFIMs for the agents' positional states for both Bayesian and deterministic models, and then discuss the implications of the EFIMs as well as the challenges in realizing network navigation. For notational convenience, we denote $\mathbf{x} := \mathbf{x}_{1:N_{\mathrm{a}}}^{(1:N)}$, $\boldsymbol{\kappa} := \boldsymbol{\kappa}_{1:N_{\mathrm{a}}}^{(1:N)}$, and functionals

$$\boldsymbol{\Phi}_{\boldsymbol{\alpha},\boldsymbol{\beta}}(f) := \mathbb{E}_{\mathbf{z},\boldsymbol{\theta}}\Big\{-\frac{\partial^2 \ln f}{\partial \boldsymbol{\alpha}\, \partial \boldsymbol{\beta}^{\dagger}}\Big\}$$

$$\boldsymbol{\Psi}_{\boldsymbol{\alpha},\boldsymbol{\beta}}^{\boldsymbol{\gamma}}(f) := \boldsymbol{\Phi}_{\boldsymbol{\alpha},\boldsymbol{\beta}}(f) - \boldsymbol{\Phi}_{\boldsymbol{\alpha},\boldsymbol{\gamma}}(f)\big[\boldsymbol{\Phi}_{\boldsymbol{\gamma},\boldsymbol{\gamma}}(f)\big]^{-1}\boldsymbol{\Phi}_{\boldsymbol{\gamma},\boldsymbol{\beta}}(f)\,.$$

A. EFIM for Bayesian Model

We first consider the Bayesian model of network navigation as introduced in Section II-B1 and derive the EFIM for the agents' positional states based on (1). In the following theorem, we show that the EFIM can be decomposed into the sum of information corresponding to the mobility model, spatial cooperation, and temporal cooperation. This result implies that the prior knowledge and each mode of cooperation contribute to navigation information in an additive manner.

Theorem 1: For the Bayesian model of network navigation, the EFIM for the agents' positional states $\mathbf{x}$ from time t_1 to t_N is given by

$$\mathbf{J}_{\mathrm{e}}(\mathbf{x}) = \mathbf{J}_{\mathrm{e}}^{\mathrm{m}}(\mathbf{x}) + \mathbf{J}_{\mathrm{e}}^{\mathrm{s}}(\mathbf{x}) + \mathbf{J}_{\mathrm{e}}^{\mathrm{t}}(\mathbf{x}) \tag{7}$$

where $\mathbf{J}_{\mathrm{e}}^{\mathrm{m}}(\mathbf{x})$ is the EFIM corresponding to the mobility model, given by a diagonally-striped matrix

$$\mathbf{J}_{\mathrm{e}}^{\mathrm{m}}(\mathbf{x}) = \sum_{n=1}^{N} \mathbf{E}_{n,n}^{N} \otimes \mathbf{P}^{(n,n)} + \sum_{n=1}^{N-1}\Big(\mathbf{E}_{n,n+1}^{N} \otimes \mathbf{P}^{(n,n+1)} + \mathbf{E}_{n+1,n}^{N} \otimes \mathbf{P}^{(n+1,n)}\Big) \tag{8}$$

in which $\mathbf{P}^{(n,m)} = \sum_{k\in\mathcal{N}_{\mathrm{a}}} \mathbf{E}_{k,k}^{N_{\mathrm{a}}} \otimes \mathbf{P}_k^{(n,m)}$ with $\mathbf{P}_k^{(n,m)}$ given by (27); $\mathbf{J}_{\mathrm{e}}^{\mathrm{s}}(\mathbf{x})$ is the EFIM corresponding to spatial cooperation, given by

$$\mathbf{J}_{\mathrm{e}}^{\mathrm{s}}(\mathbf{x}) = \sum_{n=1}^{N}\sum_{m=1}^{N} \mathbf{E}_{n,m}^{N} \otimes \mathbf{S}^{(n,m)} \tag{9}$$

in which

$$\mathbf{S}^{(n,m)} = \sum_{k\in\mathcal{N}_{\mathrm{a}}}\ \sum_{j\in\mathcal{N}_{\mathrm{a}}\cup\mathcal{N}_{\mathrm{b}}\backslash\{k\}} \mathbf{E}_{k,k}^{N_{\mathrm{a}}} \otimes \mathbf{S}_{kj}^{(n,m)} - \sum_{k\in\mathcal{N}_{\mathrm{a}}}\ \sum_{j\in\mathcal{N}_{\mathrm{a}}\backslash\{k\}} \mathbf{E}_{k,j}^{N_{\mathrm{a}}} \otimes \mathbf{S}_{kj}^{(n,m)} \tag{10}$$

with $\mathbf{S}_{kj}^{(n,m)}$ given by (28); and $\mathbf{J}_{\mathrm{e}}^{\mathrm{t}}(\mathbf{x})$ is the EFIM corresponding to temporal cooperation, given by

$$\mathbf{J}_{\mathrm{e}}^{\mathrm{t}}(\mathbf{x}) = \sum_{n=1}^{N}\sum_{m=1}^{N} \mathbf{E}_{n,m}^{N} \otimes \mathbf{T}^{(n,m)} \tag{11}$$

in which $\mathbf{T}^{(n,m)} = \sum_{k\in\mathcal{N}_{\mathrm{a}}} \mathbf{E}_{k,k}^{N_{\mathrm{a}}} \otimes \mathbf{T}_k^{(n,m)}$ with $\mathbf{T}_k^{(n,m)}$ given by (29).

Proof: See Appendix A for the outline of the proof. The detailed proof is omitted due to the space constraint. □

Remark 1: Theorem 1 shows that the EFIM for the positional states can be decomposed into three terms, i.e., the information corresponding to the mobility model, spatial cooperation, and temporal cooperation. Each term has a specific structure explained in the following.

Mobility model: $\mathbf{J}_{\mathrm{e}}^{\mathrm{m}}(\mathbf{x})$ characterizes the navigation information, i.e., information about the positional states, corresponding to the mobility model. Every matrix $\mathbf{P}^{(n,n)}$ on the diagonal of $\mathbf{J}_{\mathrm{e}}^{\mathrm{m}}(\mathbf{x})$ is block-diagonal with each block of size $D\times D$ corresponding to $\mathbf{x}_k^{(n)}$'s, since the mobilities of different agents are independent. Moreover, since the mobility model characterizes the positional states in two consecutive time instants, only the matrices $\mathbf{P}^{(n,n+1)}$ in off-diagonals of $\mathbf{J}_{\mathrm{e}}^{\mathrm{m}}(\mathbf{x})$ are non-zero. Furthermore, the matrices $\mathbf{P}^{(n,n+1)}$ are block-diagonal with each block of size $D \times D$ corresponding to the statistical distribution of mobility $f\big(\mathbf{x}_k^{(n+1)}|\mathbf{x}_k^{(n)}\big)$, again since the mobilities of different agents at different time instants are independent.

Spatial cooperation: $\mathbf{J}_\mathrm{e}^\mathrm{s}(\mathbf{x})$ characterizes the navigation information corresponding to the cooperation via inter-node measurements. Every matrix $\mathbf{S}^{(n,n)}$ on the diagonal of $\mathbf{J}_\mathrm{e}^\mathrm{s}(\mathbf{x})$ has the specific structure shown in (10): (i) the submatrix $\mathbf{S}_{kj}^{(n,n)}$ characterizes the navigation information obtained from the inter-node measurement between nodes k and j, (ii) the kth block on the diagonal is the summation of such information between agent k and all other nodes, and (iii) the off-diagonal blocks are the negative of such information between each pair of agents due to the uncertainty of the agents's positional states in cooperation [6]. The off-diagonal matrices $\mathbf{S}^{(n,m)}$ in $\mathbf{J}_\mathrm{e}^\mathrm{s}(\mathbf{x})$ have the same structure as $\mathbf{S}^{(n,n)}$, and these matrices arise due to the elimination of the parameters $\{\boldsymbol{\kappa}_{kj}\}$.

Temporal cooperation: $\mathbf{J}_\mathrm{e}^\mathrm{t}(\mathbf{x})$ characterizes the navigation information corresponding to the cooperation via intra-node measurements. Every matrix $\mathbf{T}^{(n,n)}$ on the diagonal of $\mathbf{J}_\mathrm{e}^\mathrm{t}(\mathbf{x})$ is block-diagonal with blocks of size $D \times D$ corresponding to $\mathbf{x}_k^{(n)}$'s, since the intra-node measurements of different agents are independent. Furthermore, the off-diagonal matrices $\mathbf{T}^{(n,m)}$ are block-diagonal with each block $\mathbf{T}_k^{(n,m)}$ of size $D \times D$ corresponding to the cross information of the positional states $\mathbf{x}_k^{(n)}$ and $\mathbf{x}_k^{(m)}$ in $\mathbf{J}_\mathrm{e}^\mathrm{t}(\mathbf{x})$. This cross information arises due to the elimination of the parameters $\{\boldsymbol{\kappa}_{kk}\}$. The absence of cross information between the positional states of different agents, i.e., the zero cross information for $\mathbf{x}_k^{(n)}$ and $\mathbf{x}_j^{(m)}$ $(k \neq j)$, is due to the independence of all the intra-node measurements among the agents.

Moreover, one can show from Theorem 1 that when an agent has infinite Fisher information about its positional states, its role in the navigation network is the same as an anchor. Therefore, there is no fundamental difference between agents and anchors, and our framework unifies these nodes in the network from the Bayesian point of view.

Theorem 1 presents the EFIM for network navigation based on the general Bayesian model. When the measurement parameters have unknown or simple dynamics, they can be modeled as independent of the positional states and for different time instants, in which case $f\big(\boldsymbol{\kappa}_{kk}^{(n)}\big|\mathbf{x}_k^{(n-1)}, \boldsymbol{\kappa}_{kk}^{(n-1)}\big) = f\big(\boldsymbol{\kappa}_{kk}^{(n)}\big)$ and $f\big(\boldsymbol{\kappa}_{kj}^{(n)}\big|\mathbf{x}_{kj}^{(n-1)}, \boldsymbol{\kappa}_{kj}^{(n-1)}\big) = f\big(\boldsymbol{\kappa}_{kj}^{(n)}\big)$ in (2). We derive the EFIM for this special case in the following corollary.

Corollary 1: When the measurement parameters are independent of the positional states and for different time instants, the EFIM for the agents' positional states $\mathbf{x}$ from time t_1 to t_N is given by

$$\mathbf{J}_\mathrm{e}(\mathbf{x}) = \mathbf{J}_\mathrm{e}^\mathrm{m}(\mathbf{x}) + \mathbf{J}_\mathrm{e}^\mathrm{s}(\mathbf{x}) + \mathbf{J}_\mathrm{e}^\mathrm{t}(\mathbf{x}) \tag{12}$$

where $\mathbf{J}_\mathrm{e}^\mathrm{m}(\mathbf{x})$ is given by (8); $\mathbf{J}_\mathrm{e}^\mathrm{s}(\mathbf{x}) = \sum_{n=1}^{N} \mathbf{E}_{n,n}^{N} \otimes \mathbf{S}^{(n,n)}$ is block-diagonal, in which $\mathbf{S}^{(n,n)}$ has the same structure of (10); and $\mathbf{J}_\mathrm{e}^\mathrm{t}(\mathbf{x}) = \sum_{n=1}^{N} \mathbf{E}_{n,n}^{N} \otimes \mathbf{T}^{(n,n)}$ is also block-diagonal, with $\mathbf{T}^{(n,n)} = \sum_{k\in\mathcal{N}_\mathrm{a}} \mathbf{E}_{k,k}^{N_\mathrm{a}} \otimes \mathbf{T}_k^{(n,n)}$. In these equations, the matrix $\mathbf{P}_k^{(n,n)}$ is given by (27), and

$$\mathbf{S}_{kj}^{(n,n)} = \boldsymbol{\Psi}_{\mathbf{x}_k^{(n)},\,\mathbf{x}_k^{(n)}}^{\boldsymbol{\kappa}_{kj}^{(n)}} \Big(f\big(\boldsymbol{\kappa}_{kj}^{(n)}\big) \cdot f\big(\mathbf{z}_{kj}^{(n)} | \mathbf{x}_{kj}^{(n)}, \boldsymbol{\kappa}_{kj}^{(n)}\big) \Big)$$
$$+ \boldsymbol{\Psi}_{\mathbf{x}_k^{(n)},\,\mathbf{x}_k^{(n)}}^{\boldsymbol{\kappa}_{jk}^{(n)}} \Big(f\big(\boldsymbol{\kappa}_{jk}^{(n)}\big) \cdot f\big(\mathbf{z}_{jk}^{(n)} | \mathbf{x}_{jk}^{(n)}, \boldsymbol{\kappa}_{jk}^{(n)}\big) \Big) \tag{13}$$

$$\mathbf{T}_k^{(n,n)} = \boldsymbol{\Psi}_{\mathbf{x}_k^{(n)},\,\mathbf{x}_k^{(n)}}^{\boldsymbol{\kappa}_{kk}^{(n)}} \Big(f\big(\boldsymbol{\kappa}_{kk}^{(n)}\big) \cdot f\big(\mathbf{z}_{kk}^{(n)} | \mathbf{x}_k^{(n)}, \boldsymbol{\kappa}_{kk}^{(n)}\big) \Big). \tag{14}$$

Proof: (Outline) Due to the independence condition, the off-diagonal submatrices (or cross information) in $\mathbf{J}_{\boldsymbol{\theta}}$ are zeros, e.g.,

$$\boldsymbol{\Phi}_{\boldsymbol{\kappa}_{kk}^{(n)},\,\boldsymbol{\kappa}_{kk}^{(n+1)}} \big(f(\boldsymbol{\kappa}_{kk}^{(n+1)} | \mathbf{x}_k^{(n)}, \boldsymbol{\kappa}_{kk}^{(n)}) \big) = \mathbf{0}\,.$$

Based on this equality, (13) and (14) can be obtained after some algebra. □

Remark 2: Under the independence condition of the measurement parameters, Corollary 1 shows that both the EFIMs corresponding to the spatial and temporal cooperation are block-diagonal. In other words, since the correlation between the measurement parameters in time no longer exists, the inter- and intra-node measurements do not induce any cross information in the navigation information for different time instants.

B. EFIM for Deterministic Model

We now consider all the parameters to be deterministic unknowns as introduced in Section II-B2. In the following theorem, we derive the corresponding EFIM for the agents' positional states, i.e., positions and orientations in this case, based on the model in (4). The decomposition of the EFIM is analogous to that in the previous section, but with the absence of the term corresponding to the mobility model. The proof of the theorem follows from a similar derivation of Theorem 1, and we omit it due to the space constraint.

Theorem 2: For the deterministic model of network navigation, the EFIM for the agents' positional states $\mathbf{x}$ from time t_1 to t_N is given by

$$\mathbf{J}_\mathrm{e}(\mathbf{x}) = \mathbf{J}_\mathrm{e}^\mathrm{s}(\mathbf{x}) + \mathbf{J}_\mathrm{e}^\mathrm{t}(\mathbf{x}) \tag{15}$$

where $\mathbf{J}_\mathrm{e}^\mathrm{s}(\mathbf{x})$ is the EFIM from spatial cooperation, structured as (9), and $\mathbf{J}_\mathrm{e}^\mathrm{t}(\mathbf{x})$ is the EFIM from temporal cooperation, structured as (11), in which the block matrices $\mathbf{S}_{kj}^{(n,m)} = \mathbf{S}_{kj}^{(m,n)\,\dagger}$ and $\mathbf{T}_k^{(n,m)} = \mathbf{T}_k^{(m,n)\,\dagger}$ are given by

$$\mathbf{S}_{kj}^{(n,m)} = \begin{cases} \displaystyle\sum_{l=m-n_0'}^{n+n_0} \Bigg[\boldsymbol{\Psi}_{\mathbf{x}_k^{(n)},\,\mathbf{x}_k^{(m)}}^{\boldsymbol{\kappa}_{kj}^{(l)}} \Big(f\big(\mathbf{z}_{kj}^{(l)} | \mathbf{x}_{kj}^{(l-n_0:l+n_0')}, \boldsymbol{\kappa}_{kj}^{(l)}\big) \Big) \\ \qquad + \boldsymbol{\Psi}_{\mathbf{x}_k^{(n)},\,\mathbf{x}_k^{(m)}}^{\boldsymbol{\kappa}_{jk}^{(l)}} \Big(f\big(\mathbf{z}_{jk}^{(l)} | \mathbf{x}_{jk}^{(l-n_0:l+n_0')}, \boldsymbol{\kappa}_{jk}^{(l)}\big) \Big) \Bigg], & n \le m \le n + n_0 + n_0' \\ \mathbf{0}\,, & m > n + n_0 + n_0' \end{cases} \tag{16}$$

$$\mathbf{T}_k^{(n,m)} = \begin{cases} \displaystyle\sum_{l=m-n_0'}^{n+n_0} \boldsymbol{\Psi}_{\mathbf{x}_k^{(n)},\,\mathbf{x}_k^{(m)}}^{\boldsymbol{\kappa}_{kk}^{(l)}} \Big(f\big(\mathbf{z}_{kk}^{(l)} | \mathbf{x}_k^{(l-n_0:l+n_0')}, \boldsymbol{\kappa}_{kk}^{(l)}\big) \Big), & n \le m \le n + n_0 + n_0' \\ \mathbf{0}\,, & m > n + n_0 + n_0'\,. \end{cases} \tag{17}$$

Remark 3: Compared to the EFIM for the Bayesian case in (7) of Theorem 1, the EFIM for the deterministic case in (15) does not contain the components corresponding to the dynamic models for the mobility and measurement parameters. Theorem 2 shows that the EFIM in (15) can be decomposed into two terms, i.e., the information from spatial and temporal cooperation, where $\mathbf{J}_{\mathrm{e}}^{\mathrm{s}}(\mathbf{x})$ and $\mathbf{J}_{\mathrm{e}}^{\mathrm{t}}(\mathbf{x})$ as well as their submatrices have the same decomposition as their counterparts in Theorem 1. Hence, the remarks drawn for the Bayesian case on spatial and temporal cooperation can be extended to this case.

Specifically for this case, since the measurements made at time t_n are only related to the positional states from t_{n-n_0} to $t_{n+n_0'}$, the cross information $\mathbf{T}_k^{(n,m)}$ and $\mathbf{S}_{kj}^{(n,m)}$ between $\mathbf{x}_k^{(n)}$ and $\mathbf{x}_k^{(m)}$ depend only on the measurements from $t_{m-n_0'}$ to t_{n+n_0}. In other words, this cross information is zero for pairs of positional states with time separation larger than $n_0 + n_0'$, i.e., $\mathbf{T}_k^{(n,m)} = \mathbf{0}$ for $m > n + n_0 + n_0'$.

C. Discussion

The above results show that joint spatial and temporal cooperation can significantly improve the navigation performance, since each mode of cooperation corresponds to a positive semi-definite matrix in the sum for the total EFIM. However, realizing this benefit in practical implementation incurs associated costs such as additional communication and more complicated information fusion: (i) the communication among nodes required for cooperation can jeopardize the benefits of cooperation if such communication is not properly designed [1], [13], and (ii) the non-diagonal structure of the above EFIMs implies strong correlation in agents' position estimates and hence hinders the development of efficient distributed information fusion algorithms for medium- and large-scale networks [1], [36], [37]. Hence, for realistic network design and operation, it is crucial to develop efficient communication strategies. These include message representation and scheduling techniques, tailored specifically to network navigation, as well as efficient information fusion techniques that can accurately combine information from various modes of cooperation in a distributed manner.

To handle the trade-off between performance and complexity, it is essential to grasp the essence of the network navigation process. Hence, we will further explore the implications of the navigation information and develop a geometrical interpretation of information evolution to gain insights for network design and operation.

IV. Navigation Information Evolution

In this section, we investigate the EFIM for the agents' positions in navigation networks, and develop a geometrical interpretation of navigation information to illustrate the information evolution.

A. Navigation Information for Positions

We focus on the positions of the agents in a 2-D navigation network and assume that their orientations are known for simplicity, i.e., the positional state only includes the position and its derivatives. Each agent obtains inter-node measurements for the distances to its neighboring nodes and intra-node measurements for its velocity.[8] Without loss of generality, we will focus on the deterministic case, since the EFIM for the deterministic case has the same structure as that for the Bayesian case with a simple mobility model.

The models for the inter- and intra-node measurements are given by $f\big(\mathbf{z}_{kj}^{(n)}|\|\mathbf{p}_k^{(n)} - \mathbf{p}_j^{(n)}\|, \boldsymbol{\kappa}_{kj}^{(n)}\big)$ and $f\big(\mathbf{z}_{kk}^{(n)}|\mathbf{p}_k^{(n)} - \mathbf{p}_k^{(n-1)}, \boldsymbol{\kappa}_{kk}^{(n)}\big)$, respectively. Let the angle $\phi_{kj}^{(n)}$ describe the direction from $\mathbf{p}_k^{(n)}$ to $\mathbf{p}_j^{(n)}$, $\phi_{kk}^{(n)}$ describe the direction from $\mathbf{p}_k^{(n-1)}$ to $\mathbf{p}_k^{(n)}$, $d_{kj}^{(n)} := \|\mathbf{p}_k^{(n)} - \mathbf{p}_j^{(n)}\|$, and $d_{kk}^{(n)} := \|\mathbf{p}_k^{(n)} - \mathbf{p}_k^{(n-1)}\|$. The next theorem gives the simple structure of the EFIM, showing that there is only cross information between consecutive time instants and the information from each measurement is characterized by a 2×2 matrix. The proof follows from a similar derivation of Theorem 1, and we omit it due to the space constraint.

Theorem 3: For the deterministic model of 2-D network navigation with measurements of the agent velocity and inter-node distance, the EFIM for the agents' positions $\mathbf{p}$ from time t_1 to t_N is given by

$$\mathbf{J}_{\mathrm{e}}(\mathbf{p}) = \sum_{n=1}^{N} \mathbf{E}_{n,n}^{N} \otimes \big(\mathbf{S}^{(n)} + \mathbf{T}^{(n)} + \mathbf{T}^{(n+1)}\big) - \sum_{n=1}^{N-1} \big(\mathbf{E}_{n,n+1}^{N} + \mathbf{E}_{n+1,n}^{N}\big) \otimes \mathbf{T}^{(n+1)} \tag{18}$$

where the EFIM from spatial cooperation is $\mathbf{S}^{(n)}$ structured as (10) and the EFIM from temporal cooperation is $\mathbf{T}^{(n)} = \sum_{k\in\mathcal{N}_{\mathrm{a}}} \mathbf{E}_{k,k}^{N_{\mathrm{a}}} \otimes \mathbf{T}_k^{(n)}$ with $\mathbf{T}_k^{(1)} = \mathbf{T}_k^{(N_{\mathrm{a}}+1)} = \mathbf{0}$.[9] In the above expressions, $\mathbf{S}_{kj}^{(n)}$, $\mathbf{T}_k^{(n)} \in \mathbb{S}_+^2$ are given by

$$\mathbf{S}_{kj}^{(n)} = \lambda_{kj}^{(n)} \cdot \mathbf{J}_{\mathrm{r}}(\phi_{kj}^{(n)})$$

$$\mathbf{T}_k^{(n)} = \lambda_{kk}^{(n)} \cdot \mathbf{J}_{\mathrm{r}}(\phi_{kk}^{(n)}) + \nu_{kk}^{(n)} \cdot \mathbf{J}_{\mathrm{r}}(\phi_{kk}^{(n)} + \pi/2) + \xi_{kk}^{(n)} \cdot \mathbf{J}_{\mathrm{r}}(\phi_{kk}^{(n)}, \phi_{kk}^{(n)} + \pi/2)$$

where the coefficients are given by

$$\lambda_{kj}^{(n)} = \boldsymbol{\Psi}^{\boldsymbol{\kappa}_{kj}^{(n)}}_{d_{kj}^{(n)}, d_{kj}^{(n)}}\Big(f\big(\mathbf{z}_{kj}^{(n)}|d_{kj}^{(n)}, \boldsymbol{\kappa}_{kj}^{(n)}\big)\Big)$$

and $\lambda_{kk}^{(n)} = [\breve{\mathbf{T}}_k^{(n)}]_{1,1}$, $\xi_{kk}^{(n)} = [\breve{\mathbf{T}}_k^{(n)}]_{1,2}/d_{kk}^{(n)}$, $\nu_{kk}^{(n)} = [\breve{\mathbf{T}}_k^{(n)}]_{2,2}/d_{kk}^{(n)\,2}$ with

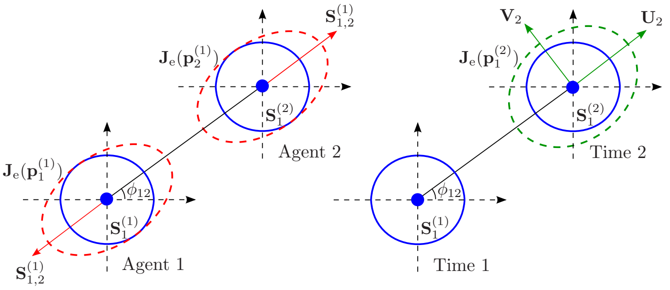
$$\breve{\mathbf{T}}_k^{(n)} = \boldsymbol{\Psi}^{\boldsymbol{\kappa}_{kk}^{(n)}}_{[d_{kk}^{(n)}\ \phi_{kk}^{(n)}],[d_{kk}^{(n)}\ \phi_{kk}^{(n)}]}\Big(f\big(\mathbf{z}_{kk}^{(n)}|d_{kk}^{(n)}, \phi_{kk}^{(n)}, \boldsymbol{\kappa}_{kk}^{(n)}\big)\Big).$$

In particular, $\xi_{kk}^{(n)} = 0$ when the amplitude and direction measurements of the velocity are independent.

Remark 4: The EFIM for the agents' positions in (18) has the diagonally-striped structure, with each building block of size $2N_{\mathrm{a}} \times 2N_{\mathrm{a}}$. The EFIM $\mathbf{S}^{(n)}$ characterizes the spatial cooperation at time t_n, and $\mathbf{T}^{(n)}$ characterizes the temporal cooperation from time t_{n-1} to t_n. Both of them consist of 2×2 building blocks, i.e., $\mathbf{S}_{kj}^{(n,n)}$ and $\mathbf{T}_k^{(n)}$ corresponding to

[8] For example, the ranges can be measured from RF signals transmitted between nodes, and the velocity can be measured by inertial sensors or Doppler radars.

[9] The notations $\mathbf{S}^{(n)}$, $\mathbf{S}_{kj}^{(n)}$, $\mathbf{T}^{(n)}$, and $\mathbf{T}_k^{(n)}$ are shorthands for $\mathbf{S}^{(n,n)}$, $\mathbf{S}_{kj}^{(n,n)}$, $\mathbf{T}^{(n,n)}$, and $\mathbf{T}_k^{(n,n)}$, respectively.



(a) Spatial Cooperation (b) Temporal Cooperation

Fig. 4. Spatial and temporal cooperation: (a) Spatial cooperation increases the information in the direction formed by the two nodes' positions; (b) Temporal cooperation increases the information both in the direction formed by the node positions in different times and in its orthogonal direction.

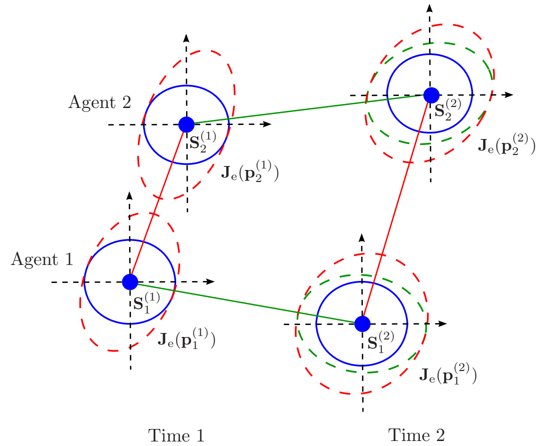


Fig. 5. Information evolution in time for network navigation with two agents in two consecutive time instants. The red and green ellipses denote the navigation information after spatial and temporal cooperation, respectively.

inter- and intra-node measurements, respectively. The EFIM from each inter-node measurement is of rank 1, where the eigenvector corresponding to the nonzero eigenvalue is in the direction connecting the two nodes. On the other hand, when the amplitude and direction measurement of the velocity are independent, the corresponding $\mathbf{T}_k^{(n)}$ can be decomposed into the direction of movement (from the amplitude measurement) and its orthogonal direction (from the direction measurement).

Figures 4(a), 4(b), and 5 illustrate the contribution from spatial cooperation, temporal cooperation, and joint cooperation in terms of information ellipse, respectively.[10] Figures 6 and 7 show the squared position error bound (SPEB) [5] obtained from the EFIM for the agents' positions with different modes of cooperation, where the contribution of the spatial and temporal cooperation increases with the network size and the time steps, respectively. These figures also show the significant performance improvement that can be achieved by joint spatial and temporal cooperation.

B. Carry-Over Information

The diagonally-striped structure of (18) allows derivation of the EFIM $\mathbf{J}_{\mathrm{e}}(\mathbf{p}^{(N)})$ for the agents' positions $\mathbf{p}^{(N)}$ at time instant t_N recursively. We next define the notion of *carry-over* information, which characterizes the useful information transferred from one time instant to the next through temporal cooperation.

Definition 2 (Carry-over information): The carry-over information from t_{n-1} to t_n is defined to be the matrix $\widetilde{\mathbf{T}}^{(n)} \in \mathbb{S}_+^{2N_{\mathrm{a}}}$ such that the EFIM $\mathbf{J}_{\mathrm{e}}(\mathbf{p}^{(n:N)})$ can be written as

$$\begin{aligned}\mathbf{J}_{\mathrm{e}}(\mathbf{p}^{(n:N)}) &= \mathbf{E}_{1,1}^{N-n+1} \otimes \left(\mathbf{S}^{(n)} + \widetilde{\mathbf{T}}^{(n)} + \mathbf{T}^{(n+1)}\right) \\ &\quad + \sum_{m=n+1}^{N} \mathbf{E}_{m-n+1,m-n+1}^{N-n+1} \otimes \left(\mathbf{S}^{(m)} + \mathbf{T}^{(m)} + \mathbf{T}^{(m+1)}\right) \\ &\quad - \sum_{m=n+1}^{N} \left(\mathbf{E}_{m-n,m-n+1}^{N-n+1} + \mathbf{E}_{m-n+1,m-n}^{N-n+1}\right) \otimes \mathbf{T}^{(m)}.\end{aligned} \tag{19}$$

[10] The information ellipse is characterized by $\{\mathbf{p} \in \mathbb{R}^2 : \mathbf{p}^{\dagger}\mathbf{J}_{\mathrm{e}}^{-1}\mathbf{p} = 1\}$ for an EFIM $\mathbf{J}_{\mathrm{e}} \in \mathbb{S}_{++}^2$ [6].

Note that the carry-over information $\widetilde{\mathbf{T}}^{(n)}$ retains all the information from t_1 to t_{n-1} for the EFIM $\mathbf{J}_{\mathrm{e}}(\mathbf{p}^{(n:N)})$. In the following proposition, we show that such carry-over information always exists and derive its expression.

Proposition 1: The carry-over information $\widetilde{\mathbf{T}}^{(n)} \in \mathbb{S}_+^{2N_{\mathrm{a}}}$ always exists and is unique. It is given by

$$\widetilde{\mathbf{T}}^{(n)} = \mathbf{T}^{(n)} - \mathbf{T}^{(n)}\left(\mathbf{S}^{(n-1)} + \widetilde{\mathbf{T}}^{(n-1)} + \mathbf{T}^{(n)}\right)^{-1}\mathbf{T}^{(n)} \tag{20}$$

with $\widetilde{\mathbf{T}}^{(1)} := \mathbf{0}$.

Remark 5: The proposition shows that the carry-over information for navigation can be obtained recursively at each time instant and used as prior knowledge of the agents' positions for the next time instant. In the following, we characterize the properties of this important matrix to gain insights into temporal cooperation.

The spatial cooperation results in coupled inference, manifested by the non block-diagonal structure of $\mathbf{S}^{(n-1)}$ in (20). In other words, the efficient estimates of the agents' positions are correlated due to spatial cooperation [37]. However, in distributed networks, the agents usually do not capture such correlation and only obtain individual (marginal) position distributions. Hence, after spatial cooperation at each time instant, the navigation accuracy limits of individual agents can be characterized by their own EFIMs.

For distributed networks, we consider a new EFIM consisting of the EFIMs for individual agents, given by

$$\widetilde{\mathbf{S}}^{(n-1)} = \sum_{k \in \mathcal{N}_{\mathrm{a}}} \mathbf{E}_{k,k}^{N_{\mathrm{a}}} \otimes \widetilde{\mathbf{S}}_k^{(n-1)} \tag{21}$$

where $\widetilde{\mathbf{S}}_k^{(n-1)} = \left\{\left[(\mathbf{S}^{(n-1)} + \widetilde{\mathbf{T}}^{(n-1)})^{-1}\right]_{\mathbf{p}_k^{(n-1)}}\right\}^{-1}$ is the individual EFIM for agent k at time t_{n-1} after spatial cooperation. By ignoring the correlation carry-over information, we approximate the carry-over information for individual agents in the following proposition.

Proposition 2: The carry-over information in distributed networks can be approximated as

$$\widetilde{\mathbf{T}}^{(n)} = \sum_{k \in \mathcal{N}_{\mathrm{a}}} \mathbf{E}_{k,k}^{N_{\mathrm{a}}} \otimes \widetilde{\mathbf{T}}_k^{(n)} \tag{22}$$

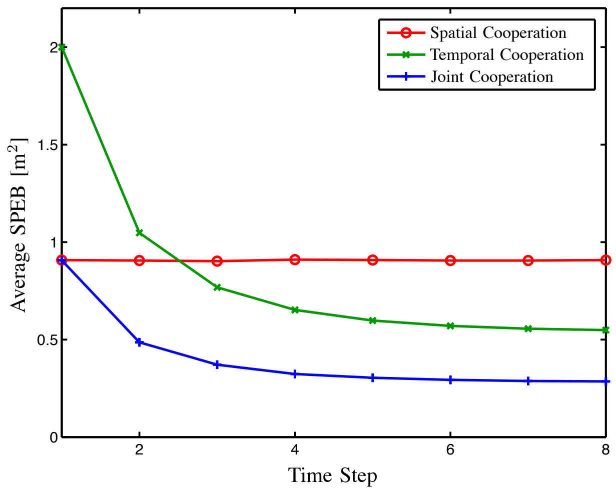


Fig. 6. Average SPEB vs. the number of consecutive time steps. Four agents randomly locate in an area of 20 m by 20 m and follow a Gaussian random walk, with $\lambda_{kk}^{(n)} = \nu_{kk}^{(n)} = \lambda_{kj}^{(n)} = 5\,\mathrm{m}^{-2}$ and $\xi_{kk}^{(n)} = 0$. The EFIM from anchors at each time instant is assumed to be the identity matrix.

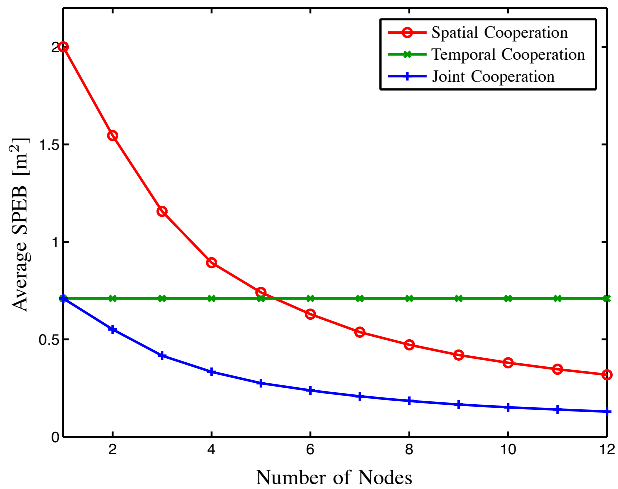


Fig. 7. Average SPEB vs. the number of agents at time step 3. Agents randomly locate in an area of 20 m by 20 m and follow a Gaussian random walk, with $\lambda_{kk}^{(n)} = \nu_{kk}^{(n)} = \lambda_{kj}^{(n)} = 5\,\mathrm{m}^{-2}$ and $\xi_{kk}^{(n)} = 0$. The EFIM from anchors at each time instant is assumed to be the identity matrix.

where $\widetilde{\mathbf{T}}_k^{(n)} = \mathbf{T}_k^{(n)} - \mathbf{T}_k^{(n)}\big(\widetilde{\mathbf{S}}_k^{(n-1)} + \mathbf{T}_k^{(n)}\big)^{-1}\mathbf{T}_k^{(n)}$ is the carry-over information of agent k from time t_{n-1} to t_n. Moreover, (20) reduces to (22) for noncooperative navigation since the distributed condition is slack.

Remark 6: This proposition shows how the navigation information evolves in distributed navigation networks: at each time instant, each agent uses its own carry-over information as prior knowledge, updates its position distribution through spatial cooperation with its neighbors, and finally obtains the carry-over information for the next time instant based on its position distribution and temporal cooperation. This insight reveals that the complex joint cooperation can be decomposed into the spatial and temporal steps. Such decomposition can significantly reduce the complexity and facilitate the design of distributed network navigation algorithms.

C. Geometrical Interpretation

To visualize the information evolution, we now provide a geometrical interpretation of navigation information, particularly the carry-over information, for distributed network navigation.[11] Since the EFIM (22) is a block-diagonal matrix with block size of 2×2, we can focus on one agent and simplify the notation of the carry-over information as

$$\widetilde{\mathbf{T}} = \mathbf{T} - \mathbf{T}\big(\mathbf{S} + \mathbf{T}\big)^{-1}\mathbf{T} \tag{23}$$

where $\mathbf{T} = \lambda\,\mathbf{J}_{\mathrm{r}}(\psi) + \nu\,\mathbf{J}_{\mathrm{r}}(\psi + \pi/2) =: \mathbf{U} + \mathbf{V}$ and $\mathbf{S} = \mu\,\mathbf{J}_{\mathrm{r}}(\beta) + \eta\,\mathbf{J}_{\mathrm{r}}(\beta + \pi/2)$, by eigenvalue decomposition for matrices in $\mathbb{S}_{++}^2$ [35], in which $\lambda \geq \nu \geq 0$, $\mu \geq \eta \geq 0$, and $\psi, \beta \in [0, \pi)$. We next simplify the expression of the carry-over information in two ways, providing different insights into such information for navigation.

Proposition 3: The carry-over navigation information (23) can be written as

$$\widetilde{\mathbf{T}} = \frac{|\mathbf{T}|}{|\mathbf{S} + \mathbf{T}|}\,\mathbf{S} + \frac{|\mathbf{S}|}{|\mathbf{S} + \mathbf{T}|}\,\mathbf{T}\,. \tag{24}$$

Proof: See Appendix B. □

Remark 7: The carry-over information can be written as a weighted sum of the EFIM from spatial cooperation $\mathbf{S}$ and that from temporal cooperation $\mathbf{T}$. The intuition is that the carry-over contribution depends on both the original position information (after spatial cooperation) and the information obtained from the intra-node measurements. The carry-over information satisfies $\widetilde{\mathbf{T}} \preceq \mathbf{T}$, where the equality is achieved when $\mathbf{S}$ goes to $\mathrm{diag}\{\infty, \infty\}$. This implies that the information from the intra-node measurement cannot be fully utilized due to the uncertainty of the previous position characterized by $\mathbf{S}$. We next examine some special cases:

- When the intra-node measurement is only the amplitude of the velocity, e.g., $\mathbf{T} = \mathbf{U}$, we have $|\mathbf{T}| = 0$ and hence $\widetilde{\mathbf{T}} = |\mathbf{S}|/|\mathbf{U} + \mathbf{S}| \cdot \mathbf{U}$. That is, the carry-over information is proportional to the rank-1 matrix $\mathbf{U}$, where the proportional constant is between 0 and 1 depending on the directional position uncertainty of the agent at the previous time instant.[12] Similar observations hold when the intra-node measurement is only the direction of the velocity.
- When $\beta = \psi$ or $\beta = \psi + \pi/2$, the carry-over information in the two orthogonal directions is decoupled, and can be calculated as

$$\widetilde{\mathbf{T}} = \begin{cases} \frac{\lambda\mu}{\lambda+\mu}\,\mathbf{J}_{\mathrm{r}}(\psi) + \frac{\eta\nu}{\eta+\nu}\,\mathbf{J}_{\mathrm{r}}(\psi + \pi/2)\,, & \beta = \psi\,, \\ \frac{\lambda\eta}{\lambda+\eta}\,\mathbf{J}_{\mathrm{r}}(\psi + \pi/2) + \frac{\eta\nu}{\eta+\nu}\,\mathbf{J}_{\mathrm{r}}(\psi)\,, & \beta = \psi + \pi/2\,. \end{cases}$$

The result (24) relies on a property of $\mathbb{S}_{++}^2$, which cannot be generalized to 3-D cases. We next show another geometrical

[11] The geometrical interpretation for spatial cooperation can be found in [6]. Here, the focus is on the discussion of the temporal cooperation.

[12] Detailed discussion about directional position uncertainty can be found in [6].

interpretation of the carry-over information based on general matrix calculation.

Proposition 4: The carry-over navigation information (23) can be written as

$$\widetilde{\mathbf{T}} = \zeta_1\,\mathbf{U} + \zeta_2\,\mathbf{V} + \kappa\,\mathbf{J}_{\mathrm{r}}(\psi, \psi+\pi/2) \tag{25}$$

where the coefficients are given by

$$\zeta_1 = \left(1+\lambda\,\mathbf{u}_\psi^\dagger(\mathbf{S}+\mathbf{V})^{-1}\mathbf{u}_\psi\right)^{-1}$$
$$\zeta_2 = \left(1+\nu\,\mathbf{u}_\psi^{\perp\,\dagger}(\mathbf{S}+\mathbf{U})^{-1}\mathbf{u}_\psi^\perp\right)^{-1}$$
$$\kappa = -2\,\lambda\,\nu\cdot\mathbf{u}_\psi^\dagger(\mathbf{S}+\mathbf{U}+\mathbf{V})^{-1}\mathbf{u}_\psi^\perp\,.$$

In particular, for 2-D cases, the coefficient of the third term can be simplified as

$$\kappa = \frac{\lambda\,\nu\,(\mu-\eta)\sin(2(\beta-\psi))}{|\mathbf{S}+\mathbf{U}+\mathbf{V}|}\,. \tag{26}$$

Proof: See Appendix C. □

Remark 8: Proposition 4 shows that the carry-over information can be expressed as a sum of three terms: the first and second terms are respectively proportional to $\mathbf{U}$ and $\mathbf{V}$, where the proportional constants are between 0 and 1 depending on the directional position uncertainty of the agent at the previous time instant; and the third term accounts for the coupling of the information from the spatial and temporal cooperation.

For the coupling term, note that the matrix $\mathbf{J}_{\mathrm{r}}(\psi,\psi+\pi/2)$ can be decomposed as

$$\mathbf{J}_{\mathrm{r}}(\psi,\psi+\pi/2) = \frac{1}{2}\,\mathbf{J}_{\mathrm{r}}(\psi+\pi/4) - \frac{1}{2}\,\mathbf{J}_{\mathrm{r}}(\psi-\pi/4)$$

i.e., the eigenvalues and eigenvectors are $(1/2, \mathbf{u}_{\psi+\pi/4})$ and $(-1/2, \mathbf{u}_{\psi-\pi/4})$. Hence, the coupling terms increases the EFIM with $\kappa/2$ in direction of $\psi+\pi/4$ and decreases with the same amount in the direction of $\psi-\pi/4$ if $\beta \geq \psi$, and vice versa if $\beta < \psi$. The quantity κ given in (26) is proportional to the product of $\sin(2(\beta-\psi))$, λ, ν, as well as the difference $\mu-\eta$. Thus, this quantity vanishes when (i) $\eta=\mu$, (ii) $|\beta-\psi| = 0$ or $\pi/2$, and (iii) either λ or ν equals 0. In the first two cases, the eigenvectors of $\mathbf{T}$ aligns with those of $\mathbf{S}$, and in the last case the navigation information from temporal cooperation degenerates to a rank-1 matrix.

V. Summary

In this paper, we established a theoretical foundation for network navigation and determined the fundamental limits of navigation accuracy. We considered both Bayesian and deterministic models for cooperative networks, and applied the EFI analysis to derive the navigation information. We showed that such information can be expressed as the sum of three terms that correspond to the mobility model, spatial cooperation, and temporal cooperation; these terms can be further decomposed into basic building blocks, each associated with certain measurement and prior knowledge. We also introduced the notion of the carry-over information and developed a geometrical interpretation for the evolution of navigation information, yielding important insights into the essence of network navigation. Our results provide performance benchmarks for cooperative navigation networks, as well as guidelines for network design and operation under performance/complexity trade-off.

VI. Acknowledgments

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Appendix A
Proof of Theorem 1

Proof: (Outline) The derivation of the EFIM for $\mathbf{x}$ can be done in two steps: (i) identify the structure of the original FIM $\mathbf{J}(\boldsymbol{\theta})$, and (ii) apply the EFI analysis to obtain the EFIM.

Spatial cooperation: we eliminate $\boldsymbol{\kappa}_{kj}^{(n)}$ in time sequence. The matrix $\widetilde{\mathbf{C}}_{kj}^{(n)}$ denotes the EFIM for $\boldsymbol{\kappa}_{kj}^{(n)}$ when $\boldsymbol{\kappa}_{kj}^{(1:n-1)}$ are eliminated by the EFI process, given by

$$\widetilde{\mathbf{C}}_{kj}^{(n)} = \begin{cases} \mathbf{C}_k^{(1,1)}, & n=1 \\ \mathbf{C}_{kj}^{(n,n)} - \mathbf{C}_{kj}^{(n,n-1)}\big[\widetilde{\mathbf{C}}_{kj}^{(n-1,n-1)}\big]^{-1}\mathbf{C}_{kj}^{(n-1,n)}, & n>1 \end{cases}$$

in which $\mathbf{C}_{kj}^{(n,m)} = \mathbf{C}_{kj}^{(m,n)\,\dagger}$ is given by

$$\mathbf{C}_{kj}^{(n,m)} = \begin{cases} \boldsymbol{\Phi}_{\boldsymbol{\kappa}_{kj}^{(n)},\boldsymbol{\kappa}_{kj}^{(n)}}\Big(f\big(\boldsymbol{\kappa}_{kj}^{(n)}|\mathbf{x}_{kj}^{(n-1)},\boldsymbol{\kappa}_{kj}^{(n-1)}\big) \\ \quad\cdot f\big(\boldsymbol{\kappa}_{kj}^{(n+1)}|\mathbf{x}_{kj}^{(n)},\boldsymbol{\kappa}_{kj}^{(n)}\big)\,f\big(\mathbf{z}_{kj}^{(n)}|\mathbf{x}_{kj}^{(n)},\boldsymbol{\kappa}_{kj}^{(n)}\big)\Big), & m=n \\ \boldsymbol{\Phi}_{\boldsymbol{\kappa}_{kj}^{(n)},\boldsymbol{\kappa}_{kj}^{(n+1)}}\Big(f\big(\boldsymbol{\kappa}_{kj}^{(n+1)}|\mathbf{x}_{kj}^{(n)},\boldsymbol{\kappa}_{kj}^{(n)}\big)\Big), & m=n+1\,; \end{cases}$$

and $\widetilde{\mathbf{E}}_{kj}^{(n-l,n+1)}$ denotes the cross information for $\mathbf{x}_k^{(n-l)}$ and $\boldsymbol{\kappa}_{kj}^{(n+1)}$ when $\boldsymbol{\kappa}_{kj}^{(1:n-1)}$ are eliminated by the EFI process, given by

$$\widetilde{\mathbf{E}}_{kj}^{(n)} = \mathbf{E}_{kj}^{(n,n)}$$
$$\widetilde{\mathbf{E}}_{kj}^{(n-l,n+1)} = \begin{cases} \mathbf{E}_{kj}^{(n,n+1)} - \mathbf{E}_{kj}^{(n,n)}\big[\widetilde{\mathbf{C}}_{kj}^{(n)}\big]^{-1}\mathbf{C}_{kj}^{(n,n+1)}, & l=0 \\ -\widetilde{\mathbf{E}}_{kj}^{(n-l,n)}\big[\widetilde{\mathbf{C}}_{kj}^{(n)}\big]^{-1}\mathbf{C}_{kj}^{(n,n+1)}, & 1\leq l\leq n-1 \end{cases}$$

in which $\mathbf{E}_{kj}^{(n,m)} = \mathbf{E}_{kj}^{(m,n)\,\dagger}$ is given by

$$\mathbf{E}_{kj}^{(n,m)} = \begin{cases} \boldsymbol{\Phi}_{\mathbf{x}_k^{(n)},\boldsymbol{\kappa}_{kj}^{(n)}}\Big(f\big(\boldsymbol{\kappa}_{kj}^{(n+1)}|\mathbf{x}_{kj}^{(n)},\boldsymbol{\kappa}_{kj}^{(n)}\big) \\ \quad\cdot f\big(\mathbf{z}_{kj}^{(n)}|\mathbf{x}_{kj}^{(n)},\boldsymbol{\kappa}_{kj}^{(n)}\big)\Big), & m=n \\ \boldsymbol{\Phi}_{\mathbf{x}_k^{(n)},\boldsymbol{\kappa}_{kj}^{(n+1)}}\Big(f\big(\boldsymbol{\kappa}_{kj}^{(n+1)}|\mathbf{x}_{kj}^{(n)},\boldsymbol{\kappa}_{kj}^{(n)}\big)\Big), & m=n+1\,. \end{cases}$$

In particular, for notational convenience, we set $\widetilde{\mathbf{E}}_{jk}^{(n,m)} = \mathbf{0}$ for $j \in \mathcal{N}_{\mathrm{b}}$.

Temporal cooperation: we eliminate $\boldsymbol{\kappa}_{kk}^{(n)}$ in time sequence. The matrix $\widetilde{\mathbf{B}}_k^{(n)}$ denotes the EFIM for $\boldsymbol{\kappa}_{kk}^{(n)}$ when $\boldsymbol{\kappa}_{kk}^{(1:n-1)}$ are eliminated by the EFI process, given by

$$\widetilde{\mathbf{B}}_k^{(n)} = \begin{cases} \mathbf{B}_k^{(1,1)}, & n=1 \\ \mathbf{B}_k^{(n,n)} - \mathbf{B}_k^{(n,n-1)}\big[\widetilde{\mathbf{B}}_k^{(n-1,n-1)}\big]^{-1}\mathbf{B}_k^{(n-1,n)}, & n>1 \end{cases}$$

in which $\mathbf{B}_k^{(n,m)} = \mathbf{B}_k^{(m,n)\dagger}$ is given by

$$\mathbf{B}_k^{(n,m)} = \begin{cases} \mathbf{\Phi}_{\boldsymbol{\kappa}_{kk}^{(n)},\boldsymbol{\kappa}_{kk}^{(n)}}\Big(f\big(\boldsymbol{\kappa}_{kk}^{(n)}|\mathbf{x}_k^{(n-1)},\boldsymbol{\kappa}_{kk}^{(n-1)}\big) \\ \qquad \cdot f\big(\boldsymbol{\kappa}_{kk}^{(n+1)}|\mathbf{x}_k^{(n)},\boldsymbol{\kappa}_{kk}^{(n)}\big)\, f\big(\mathbf{z}_{kk}^{(n)}|\mathbf{x}_k^{(n)},\boldsymbol{\kappa}_{kk}^{(n)}\big)\Big), \\ \qquad\qquad m=n \\ \mathbf{\Phi}_{\boldsymbol{\kappa}_{kk}^{(n)},\boldsymbol{\kappa}_{kk}^{(n+1)}}\Big(f\big(\boldsymbol{\kappa}_{kk}^{(n+1)}|\mathbf{x}_k^{(n)},\boldsymbol{\kappa}_{kk}^{(n)}\big)\Big), \\ \qquad\qquad m=n+1\,; \end{cases}$$

and $\widetilde{\mathbf{D}}_k^{(n-l,n+1)}$ denotes the cross information for $\mathbf{x}_k^{(n-l)}$ and $\boldsymbol{\kappa}_{kk}^{(n+1)}$ when $\boldsymbol{\kappa}_{kk}^{(1:n-1)}$ are eliminated by the EFI process, given by

$$\widetilde{\mathbf{D}}_k^{(n)} = \mathbf{D}_k^{(n,n)}$$

$$\widetilde{\mathbf{D}}_k^{(n-l,n+1)} = \begin{cases} \mathbf{D}_k^{(n,n+1)} - \mathbf{D}_k^{(n,n)}\big[\widetilde{\mathbf{B}}_k^{(n,n)}\big]^{-1}\mathbf{B}_k^{(n,n+1)}, \\ \qquad\qquad l=0 \\ -\widetilde{\mathbf{D}}_k^{(n-l,n)}\big[\widetilde{\mathbf{B}}_k^{(n,n)}\big]^{-1}\mathbf{B}_k^{(n,n+1)}, \\ \qquad\qquad 1\le l\le n-1 \end{cases}$$

in which $\mathbf{D}_k^{(n,m)} = \mathbf{D}_k^{(m,n)\dagger}$ is given by

$$\mathbf{D}_k^{(n,m)} = \begin{cases} \mathbf{\Phi}_{\mathbf{x}_k^{(n)},\boldsymbol{\kappa}_{kk}^{(n)}}\Big(f\big(\boldsymbol{\kappa}_{kk}^{(n+1)}|\mathbf{x}_k^{(n)},\boldsymbol{\kappa}_{kk}^{(n)}\big) \\ \qquad \cdot f\big(\mathbf{z}_{kk}^{(n)}|\mathbf{x}_k^{(n)},\boldsymbol{\kappa}_{kk}^{(n)}\big)\Big), \quad m=n \\ \mathbf{\Phi}_{\mathbf{x}_k^{(n)},\boldsymbol{\kappa}_{kk}^{(n+1)}}\Big(f\big(\boldsymbol{\kappa}_{kk}^{(n+1)}|\mathbf{x}_k^{(n)},\boldsymbol{\kappa}_{kk}^{(n)}\big)\Big), \\ \qquad\qquad m=n+1\,; \end{cases}$$

Combining all contributions from the EFI process, we have the EFIM for the positional state $\mathbf{x}$ as in (7), where $\mathbf{P}_k^{(n,m)} = \mathbf{P}_k^{(m,n)\dagger}$ is given by

$$\mathbf{P}_k^{(n,m)} = \begin{cases} \mathbf{\Phi}_{\mathbf{x}_k^{(n)},\mathbf{x}_k^{(n)}}\Big(f\big(\mathbf{x}_k^{(n)}|\mathbf{x}_k^{(n-1)}\big)\cdot f\big(\mathbf{x}_k^{(n+1)}|\mathbf{x}_k^{(n)}\big)\Big), \\ \qquad\qquad m=n \\ \mathbf{\Phi}_{\mathbf{x}_k^{(n)},\mathbf{x}_k^{(n+1)}}\Big(f\big(\mathbf{x}_k^{(n+1)}|\mathbf{x}_k^{(n)}\big)\Big), \quad m=n+1 \\ \mathbf{0}, \qquad \text{otherwise}\,, \end{cases} \tag{27}$$

and $\mathbf{S}_{kj}^{(n,m)}$ and $\mathbf{T}_k^{(n,m)}$ are given by (28) and (29), respectively, shown at the bottom of the page. □

Appendix B
Proof of Proposition 3

Proof: Note that since $\mathbf{T}$, $\mathbf{S} \in \mathbb{S}_+^2$ and $\mathbf{T}+\mathbf{S} \in \mathbb{S}_{++}^2$, we have

$$(\mathbf{T}+\mathbf{S})^{-1} = \frac{\text{adj}(\mathbf{T}+\mathbf{S})}{|\mathbf{T}+\mathbf{S}|}\,.$$

Then, the carry-over information can be derived as

$$\begin{aligned}\widetilde{\mathbf{T}} &= \frac{|\mathbf{T}+\mathbf{S}|\,\mathbf{T} - \mathbf{T}\cdot\text{adj}(\mathbf{T}+\mathbf{S})\cdot\mathbf{T}}{|\mathbf{T}+\mathbf{S}|} \\ &= \frac{\big[(\mathbf{T}+\mathbf{S})\cdot\text{adj}(\mathbf{T}+\mathbf{S}) - \mathbf{T}\cdot\text{adj}(\mathbf{T}+\mathbf{S})\big]\cdot\mathbf{T}}{|\mathbf{T}+\mathbf{S}|} \\ &= \frac{\mathbf{S}\cdot\text{adj}(\mathbf{T}+\mathbf{S})\cdot\mathbf{T}}{|\mathbf{T}+\mathbf{S}|} \\ &= \frac{|\mathbf{T}|}{|\mathbf{T}+\mathbf{S}|}\,\mathbf{S} + \frac{|\mathbf{S}|}{|\mathbf{T}+\mathbf{S}|}\,\mathbf{T}\end{aligned}$$

where we have used $|\mathbf{A}|\cdot\mathbf{I} = \mathbf{A}\cdot\text{adj}(\mathbf{A})$ for any $\mathbf{A}\in\mathbb{S}_+^2$. □

Appendix C
Proof of Proposition 4

Proof: Since $\mathbf{T} = \mathbf{U}+\mathbf{V}$, the carry-over information can be written as a sum of three components

$$\begin{aligned}\widetilde{\mathbf{T}} = \big[\,\mathbf{U} - \mathbf{U}(\mathbf{S}+\mathbf{T})^{-1}\mathbf{U}\,\big] + \big[\,\mathbf{V} - \mathbf{V}(\mathbf{S}+\mathbf{T})^{-1}\mathbf{V}\,\big] \\ - \big[\,\mathbf{U}(\mathbf{S}+\mathbf{T})^{-1}\mathbf{V} + \mathbf{V}(\mathbf{S}+\mathbf{T})^{-1}\mathbf{U}\,\big]\,.\end{aligned} \tag{30}$$

Note that $\mathbf{U} = \lambda\,\mathbf{u}_\psi\mathbf{u}_\psi^\dagger$, we have that for any $\mathbf{A}\in\mathbb{S}_{++}$

$$\begin{aligned}\mathbf{A}(\mathbf{A}+\mathbf{U})^{-1}\mathbf{U} &= \mathbf{A}\Big(\mathbf{A}^{-1} - \frac{\lambda\,\mathbf{A}^{-1}\,\mathbf{u}_\psi\,\mathbf{u}_\psi^\dagger\,\mathbf{A}^{-1}}{1+\lambda\,\mathbf{u}_\psi^\dagger\,\mathbf{A}^{-1}\,\mathbf{u}_\psi}\Big)\,\lambda\,\mathbf{u}_\psi\mathbf{u}_\psi^\dagger \\ &= \lambda\,\mathbf{u}_\psi\mathbf{u}_\psi^\dagger - \lambda\,\mathbf{u}_\psi\mathbf{u}_\psi^\dagger\cdot\frac{\lambda\,\mathbf{u}_\psi^\dagger\,\mathbf{A}^{-1}\,\mathbf{u}_\psi}{1+\lambda\,\mathbf{u}_\psi^\dagger\,\mathbf{A}^{-1}\,\mathbf{u}_\psi} \\ &= \frac{1}{1+\lambda\,\mathbf{u}_\psi^\dagger\,\mathbf{A}^{-1}\,\mathbf{u}_\psi}\cdot\mathbf{U}\,.\end{aligned}$$

Hence, the first component in (30) can be simplified as

$$\mathbf{U} - \mathbf{U}(\mathbf{S}+\mathbf{T})^{-1}\mathbf{U} = (\mathbf{S}+\mathbf{V})(\mathbf{S}+\mathbf{V}+\mathbf{U})^{-1}\mathbf{U} = \zeta_1\,\mathbf{U}$$

$$\mathbf{S}_{kj}^{(n,m)} = \begin{cases} \mathbf{\Phi}_{\mathbf{x}_k^{(n)},\mathbf{x}_k^{(n)}}\Big(f\big(\boldsymbol{\kappa}_{kj}^{(n+1)}|\mathbf{x}_{kj}^{(n)},\boldsymbol{\kappa}_{kj}^{(n)}\big)\cdot f\big(\mathbf{z}_{kj}^{(n)}|\mathbf{x}_{kj}^{(n)},\boldsymbol{\kappa}_{kj}^{(n)}\big)\Big) - \sum\limits_{l=n}^{N}\widetilde{\mathbf{E}}_{kj}^{(n,l)}\big[\widetilde{\mathbf{C}}_{kj}^{(l)}\big]^{-1}\widetilde{\mathbf{E}}_{kj}^{(l,n)} \\ \quad + \mathbf{\Phi}_{\mathbf{x}_k^{(n)},\mathbf{x}_k^{(n)}}\Big(f\big(\boldsymbol{\kappa}_{jk}^{(n+1)}|\mathbf{x}_{jk}^{(n)},\boldsymbol{\kappa}_{jk}^{(n)}\big)\cdot f\big(\mathbf{z}_{jk}^{(n)}|\mathbf{x}_{jk}^{(n)},\boldsymbol{\kappa}_{jk}^{(n)}\big)\Big) - \sum\limits_{l=n}^{N}\widetilde{\mathbf{E}}_{jk}^{(n,l)}\big[\widetilde{\mathbf{C}}_{jk}^{(l)}\big]^{-1}\widetilde{\mathbf{E}}_{jk}^{(l,n)}, & m=n \\ -\sum\limits_{l=m}^{N}\Big[\widetilde{\mathbf{E}}_{kj}^{(n,l)}\big[\widetilde{\mathbf{C}}_{kj}^{(l)}\big]^{-1}\widetilde{\mathbf{E}}_{kj}^{(l,m)} + \widetilde{\mathbf{E}}_{jk}^{(n,l)}\big[\widetilde{\mathbf{C}}_{jk}^{(l)}\big]^{-1}\widetilde{\mathbf{E}}_{jk}^{(l,m)}\Big], & n<m\le N \end{cases} \tag{28}$$

$$\mathbf{T}_k^{(n,m)} = \begin{cases} \mathbf{\Phi}_{\mathbf{x}_k^{(n)},\mathbf{x}_k^{(n)}}\Big(f\big(\boldsymbol{\kappa}_{kk}^{(n+1)}|\mathbf{x}_k^{(n)},\boldsymbol{\kappa}_{kk}^{(n)}\big)\cdot f\big(\mathbf{z}_{kk}^{(n)}|\mathbf{x}_k^{(n)},\boldsymbol{\kappa}_{kk}^{(n)}\big)\Big) - \sum\limits_{l=n}^{N}\widetilde{\mathbf{D}}_k^{(n,l)}\big[\widetilde{\mathbf{B}}_k^{(l)}\big]^{-1}\widetilde{\mathbf{D}}_k^{(l,n)}, & m=n \\ -\sum\limits_{l=m}^{N}\widetilde{\mathbf{D}}_k^{(n,l)}\big[\widetilde{\mathbf{B}}_k^{(l)}\big]^{-1}\widetilde{\mathbf{D}}_k^{(l,m)}, & n<m\le N \end{cases} \tag{29}$$

and the second one can be simplified in a similar way. Moreover, we can simplify the third component to the corresponding term in (25) by

$$\begin{aligned}-\mathbf{U}(\mathbf{S}+\mathbf{T})^{-1}\mathbf{V} &= -\lambda\,\mathbf{u}_\psi\mathbf{u}_\psi^\dagger\,(\mathbf{S}+\mathbf{T})^{-1}\nu\,\mathbf{u}_\psi^\perp\mathbf{u}_\psi^{\perp\,\dagger}\\ &= \frac{1}{2}\,\kappa\,\mathbf{u}_\psi\mathbf{u}_\psi^{\perp\,\dagger}\end{aligned}$$

and

$$-\mathbf{V}(\mathbf{S}+\mathbf{T})^{-1}\mathbf{U} = \frac{1}{2}\,\kappa\,\mathbf{u}_\psi^\perp\mathbf{u}_\psi^\dagger\,.$$

Finally, for 2-D cases, the expression (26) can be obtained by using the equation that

$$\mathbf{u}_\psi^\dagger(\mathbf{S}+\mathbf{U}+\mathbf{V})^{-1}\mathbf{u}_\psi^\perp = \frac{\mathbf{u}_\psi^\dagger\,\mathrm{adj}\{\mathbf{S}\}\,\mathbf{u}_\psi^\perp}{|\mathbf{S}+\mathbf{U}+\mathbf{V}|}$$

where we have used $\mathbf{u}_\psi^\dagger\,\mathrm{adj}\{\mathbf{U}+\mathbf{V}\}\,\mathbf{u}_\psi^\perp = 0$. □

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Yuan Shen (S'05) received his B.S. degree (with highest honor) from Tsinghua University, China, in 2005, and S.M. degree from the Massachusetts Institute of Technology (MIT), Cambridge, MA, in 2008, both in electrical engineering.

Since 2005, he has been with Wireless Communications and Network Science Laboratory at MIT, where he is now a Ph.D. candidate. He was with the Wireless Communications Laboratory at The Chinese University of Hong Kong in summer 2010, the Hewlett-Packard Labs in winter 2009, the Corporate R&D of Qualcomm Inc. in summer 2008, and the Intelligent Sensing Laboratory at Tsinghua University from 2003 to 2005. His research interests include communication theory, information theory, and statistical inference. His current research focuses on network localization and navigation, filtering techniques, resource allocation, cooperative networks, and wideband communication systems.

Mr. Shen served as a member of the Technical Program Committee (TPC) for the IEEE GLOBECOM in 2010–2012, the IEEE ICC in 2010–2012, the IEEE WCNC in 2009–2012, and the IEEE ICUWB in 2011–2012. He is a recipient of the Marconi Society Paul Baran Young Scholar Award in 2010, Ernst A. Guillemin Thesis Award (first place) for the best S.M. thesis from the Department of Electrical Engineering and Computer Science at MIT in 2008, the Roberto Padovani Scholarship from Qualcomm Inc. in 2008, and the MIT Walter A. Rosenblith Presidential Fellowship in 2005. He also received the Best Paper Awards from the IEEE GLOBECOM in 2011, the IEEE ICUWB in 2011, and the IEEE WCNC in 2007.

Santiago Mazuelas (M'10) received his Ph.D. in mathematics and in telecommunications engineering from the University of Valladolid, Spain, in 2009 and 2011, respectively.

Since 2009, he has been a postdoctoral fellow in the Wireless Communication and Network Sciences Laboratory at MIT. He previously worked from 2006 to 2009 as a Researcher and Project Manager in a Spanish technological center, as well as a junior lecturer in the University of Valladolid. His research interests are the application of mathematical and statistical theories to communications, localization, and navigation networks.

Dr. Mazuelas has served as a member of the Technical Program Committee (TPC) for the IEEE Global Communications Conference (GLOBECOM) in 2010 and 2011, the IEEE International Conference on Ultra-Wideband (ICUWB) in 2011, the IEEE 3rd Latin-American Conference on Communications (LATINCOM) in 2011, and the IEEE Wireless Telecommunications Symposium (WTS) in 2010 and 2011. He has received the Special prize end of degree for the best student record in Telecommunications Technical Engineering, the young scientists prize for the best communication at the Union Radio-Scientifique Internationale (URSI) XXII Symposium in 2007, and Best Paper Award from the IEEE ICUWB in 2011.

Moe Z. Win (S'85-M'87-SM'97-F'04) (S'85-M'87-SM'97-F'04) received both the Ph.D. in Electrical Engineering and M.S. in Applied Mathematics as a Presidential Fellow at the University of Southern California (USC) in 1998. He received an M.S. in Electrical Engineering from USC in 1989, and a B.S. (*magna cum laude*) in Electrical Engineering from Texas A&M University in 1987.

He is a Professor at the Massachusetts Institute of Technology (MIT). Prior to joining MIT, he was at AT&T Research Laboratories for five years and at the Jet Propulsion Laboratory for seven years. His research encompasses fundamental theories, algorithm design, and experimentation for a broad range of real-world problems. His current research topics include network localization and navigation, network interference exploitation, intrinsic wireless secrecy, adaptive diversity techniques, and ultra-wide bandwidth systems.

Professor Win is an elected Fellow of the AAAS, the IEEE, and the IET, and was an IEEE Distinguished Lecturer. He was honored with two IEEE Technical Field Awards: the IEEE Kiyo Tomiyasu Award (2011) and the IEEE Eric E. Sumner Award (2006, jointly with R. A. Scholtz). Together with students and colleagues, his papers have received numerous awards including the IEEE Communications Society's Leonard G. Abraham Prize Paper Award (2011), the IEEE Aerospace and Electronic Systems Society's M. Barry Carlton Award (2011), the IEEE Communications Society's Guglielmo Marconi Prize Paper Award (2008), and the IEEE Antennas and Propagation Society's Sergei A. Schelkunoff Transactions Prize Paper Award (2003). Highlights of his international scholarly initiatives are the Copernicus Fellowship (2011), the Royal Academy of Engineering Distinguished Visiting Fellowship (2009), and the Fulbright Fellowship (2004). Other recognitions include the Outstanding Service Award of the IEEE ComSoc Radio Communications Committee (2010), the Laurea Honoris Causa from the University of Ferrara (2008), the Technical Recognition Award of the IEEE ComSoc Radio Communications Committee (2008), the Wireless Educator of the Year Award (2007), and the U.S. Presidential Early Career Award for Scientists and Engineers (2004).

Dr. Win is an elected Member-at-Large on the IEEE Communications Society Board of Governors (2011–2013). He was the chair (2004–2006) and secretary (2002–2004) for the Radio Communications Committee of the IEEE Communications Society. Over the last decade, he has organized and chaired numerous international conferences. He is currently an Editor for IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, and served as Area Editor (2003–2006) and Editor (1998–2006) for the IEEE TRANSACTIONS ON COMMUNICATIONS. He was Guest-Editor for the PROCEEDINGS OF THE IEEE (2009) and IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS (2002).