Virtual Branch Analysis of Symbol Error Probability for Hybrid Selection/Maximal-Ratio Combining in Rayleigh Fading

Moe Z. Win, *Senior Member, IEEE*, and Jack H. Winters, *Fellow, IEEE*

***Abstract*—In this paper, we derive analytical expressions for the symbol error probability (SEP) for a hybrid selection/maximal-ratio combining (H-S/MRC) diversity system in multipath-fading wireless environments. With H-S/MRC, L out of N diversity branches are selected and combined using maximal-ratio combining (MRC). We consider coherent detection of M-ary phase-shift keying (MPSK) and quadrature amplitude modulation (MQAM) using H-S/MRC for the case of independent Rayleigh fading with equal signal-to-noise ratio averaged over the fading. The proposed problem is made analytically tractable by transforming the ordered physical diversity branches, which are correlated, into independent and identically distributed (i.i.d.) "virtual branches," which results in a simple derivation of the SEP for *arbitrary* L and N. We further obtain a *canonical structure* for the SEP of H-S/MRC as a weighted sum of the elementary SEP's, which are the SEP's using MRC with i.i.d. diversity branches in Rayleigh fading, or equivalently the SEP's of the nondiversity (single-branch) system in Nakagami fading, whose closed-form expressions are well-known. We present numerical examples illustrating that H-S/MRC, even with $L \ll N$, can achieve performance close to that of N-branch MRC.**

***Index Terms*—Diversity combining, error probability, fading channel, maximal ratio combining, selection diversity, virtual branch technique.**

I. INTRODUCTION

THE CAPACITY of wireless systems in a multipath environment can be increased by diversity techniques [1], such as selection diversity (SD) [2]–[4] or maximal-ratio combining (MRC) [4]. SD is the simplest form of diversity system whereby the received signal is selected from *one* out of N available diversity branches. In MRC, the received signals from *all* the diversity branches are weighted and combined to maximize the *instantaneous* signal-to-noise ratio (SNR) at the combiner output.

Though a high diversity order is possible in many situations, it may not be feasible to utilize all of the available branches. For example, a large order of antenna diversity may be obtained easily, especially at higher frequencies such as the PCS bands, using spatial separation and/or orthogonal polarizations. Even for a handset, the main diversity-order limitation is typically not the handset size (which determines the maximum number of antenna elements) but the power consumption and cost of the RF electronics for each diversity branch [5].

This has motivated studies [6]–[11] of diversity combining techniques that process only a *subset* of the available diversity branches with limited resources (i.e., power, RF electronics), but achieve better performance than SD. These reduced-complexity combining systems select L branches (from N available diversity branches) and combine them based on a chosen criterion. Here, we consider a hybrid selection/maximal-ratio combining (H-S/MRC) diversity system which selects the L branches with largest receive SNR at each instant, and then combines these branches to maximize the instantaneous output SNR. This potentially reduces the number of required RF chains from N to L. We assume that instantaneous channel estimation using a scanning receiver across all possible diversity branches is feasible, such as with slow fading. However, H-S/MRC also offers improvement in fast fading conditions, and our results serve as a lower bound on the symbol error probability (SEP) performance when perfect channel estimates are not available.

In [7], the bit error rate performance of H-S/MRC with $L = 2$ and $L = 3$ out of N branches was analyzed, and it was pointed out that "the expressions become extremely unwieldy" for $L > 3$. The *average* SNR of H-S/MRC was derived in [8]. In [10], a "virtual branch" technique was introduced (see also [11]) to succinctly derive the mean as well as the variance of the combiner output SNR of the H-S/MRC diversity system. Concurrent and independent work on the performance analysis of H-S/MRC can also be found in [12], where H-S/MRC is referred to as "generalized selection combining."

In this paper, we derive *exact* expressions for the SEP of a H-S/MRC diversity system with *arbitrary* L and N. We consider coherent detection of M-ary phase-shift keying (MPSK) and quadrature amplitude modulation (MQAM) for the case of independent Rayleigh fading with equal SNR averaged over the fading. The proposed problem is made analytically tractable by transforming the ordered physical diversity branches, which are correlated, into independent and identically distributed (i.i.d.) *virtual branches*.[1] We further obtain a *canonical structure* for the SEP of H-S/MRC as a weighted sum of the elementary SEP's, which are the SEP's using MRC with i.i.d. branches in Rayleigh fading, or equivalently the SEP's of a nondiversity (single-branch) system in Nakagami fading, whose closed-form

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The authors are with the Wireless Systems Research Department, AT&T Labs–Research, Middletown, NJ 07748-4801 USA (e-mail: win@research.att.com; jhw@research.att.com).

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[1]When the average branch SNR's are not necessarily equal, it can be shown that the virtual branch technique still applies, but the virtual branches are *conditionally* independent [9].

expressions are well known. We also present numerical examples illustrating that H-S/MRC, even with $L \ll N$, can achieve performance close to that of N-branch MRC.

II. Diversity Combining Analysis

A. Virtual Branch Technique: The Key Idea

The analysis of H-S/MRC based on a chosen ordering of the branches at first appears to be complicated, since the SNR statistics of the ordered branches are *not* independent. Here, we alleviate this problem by transforming the ordered-branch variables into a new set of i.i.d. *virtual branches*, and expressing the ordered-branch SNR variables as a linear function of i.i.d. virtual branch SNR variables. The key advantage of this formulation is that it allows greater flexibility in the selection process of the ordered instantaneous SNR values, and permits the combiner output SNR to be expressed in terms of the i.i.d. virtual branch SNR variables. In this framework, the derivation of the SEP for H-S/MRC, involving the evaluation of nested N-fold integrals, essentially reduces to the evaluation of a single integral with *finite* limits. The well-known results for SD and MRC are shown to be special cases of our results.

B. General Theory

Let γ_i denote the instantaneous SNR of the ith diversity branch defined by

$$\gamma_i \triangleq \alpha_i^2 \frac{E_s}{N_{0i}} \tag{1}$$

where $2E_s$ is the average symbol energy, and α_i is the instantaneous fading amplitude and $2N_{0i}$ is the two-sided noise power spectral density of the ith branch. We model the γ_i's as continuous random variables with (probability density function (pdf) $f_{\gamma_i}(x)$ and mean $\Gamma_i = \mathbb{E}\{\gamma_i\}$.

Let us first consider a general diversity combining (GDC) system with the instantaneous output SNR of the form[2]

$$\gamma_{\text{GDC}} = \langle \boldsymbol{a}, \boldsymbol{\gamma}_{[N]} \rangle \tag{2}$$

where $\boldsymbol{a}$ and $\boldsymbol{\gamma}_{[N]}$ are $N \times 1$ vectors with N denoting the number of available diversity branches. The selection vector $\boldsymbol{a}$ is binary-valued with ith element $a_i \in \{0, 1\}$. The ordered vector $\boldsymbol{\gamma}_{[N]} \triangleq [\gamma_{[1]}, \gamma_{[2]}, \ldots, \gamma_{[N]}]^t$, where $\{\gamma_{[i]}\}$ is the ordered set of $\{\gamma_i\}$, i.e., $\gamma_{[1]} > \gamma_{[2]} > \cdots > \gamma_{[N]}$ and $(\cdot)^t$ denotes transpose. Note that GDC selectively combines the branches with instantaneous SNR $\gamma_{[i]}$ corresponding to nonzero elements ($a_i \neq 0$) of the selection vector $\boldsymbol{a}$. It will be apparent later that several diversity combining schemes, including H-S/MRC, turn out to be special cases of (2). Note that the possibility of at least two equal $\gamma_{[i]}$'s is excluded, since $\gamma_{[i]} \neq \gamma_{[j]}$ *almost surely* for continuous random variables γ_i's.[3]

[2]The notation $\langle \boldsymbol{x}, \boldsymbol{y} \rangle$ for $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^N$ is used to denote the usual inner product on $\mathbb{R}^N$ defined by $\langle \boldsymbol{x}, \boldsymbol{y} \rangle \triangleq \sum_{i=1}^N x_i y_i$. For a linear transformation $\boldsymbol{T}$: $\mathbb{R}^N \to \mathbb{R}^N$, we will use the fact that $\langle \boldsymbol{x}, \boldsymbol{T}\boldsymbol{y} \rangle = \langle \boldsymbol{T}^t \boldsymbol{x}, \boldsymbol{y} \rangle$ [13], [14].

[3]In our context, the notion of "almost sure" or "almost everywhere" can be stated mathematically as: $\gamma_{[i]} \neq \gamma_{[j]}$ *almost surely* if and only if $\Pr\{\gamma_{[i]} = \gamma_{[j]}\} = 0$ [15], [16].

For a Rayleigh fading channel, the pdf of α_i's is given by

$$f_{\alpha_i}(r) = \frac{2r}{\Omega_i} e^{-r^2/\Omega_i}, \qquad r \geq 0 \tag{3}$$

where $\Omega_i = \mathbb{E}\{\alpha_i^2\}$. Then, the pdf of the instantaneous branch SNR is given by

$$f_{\gamma_i}(x) = \begin{cases} \dfrac{1}{\Gamma_i} e^{-(x/\Gamma_i)}, & 0 \leq x < \infty \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

where the mean $\Gamma_i = \mathbb{E}\{\gamma_i\} = \mathbb{E}\{\alpha_i^2\}(E_s/N_{0i}) = \Omega_i(E_s/N_{0i})$. If γ_is are independent with equal average SNR, i.e., $\Gamma_i = \Gamma$ for $i = 1, \ldots, N$, then $f_{\gamma_i}(x) = f(x)$ for $i = 1, \ldots, N$.

The joint pdf of $\gamma_{[1]}, \gamma_{[2]}, \ldots, \gamma_{[N]}$ can be derived using the theory of "order statistics" [17] as

$$f_{\boldsymbol{\gamma}_{[N]}}\left(\{\gamma_{[i]}\}_{i=1}^N\right) = \begin{cases} N! f(\gamma_{[1]}) f(\gamma_{[2]}) \cdots f(\gamma_{[N]}), \\ \qquad \gamma_{[1]} > \gamma_{[2]} > \cdots > \gamma_{[N]} \\ 0, \qquad \text{otherwise.} \end{cases} \tag{5}$$

Therefore

$$f_{\boldsymbol{\gamma}_{[N]}}\left(\{\gamma_{[i]}\}_{i=1}^N\right) = \begin{cases} N! \left(\dfrac{1}{\Gamma}\right)^N e^{-(1/\Gamma)\langle \mathbf{1}_N, \boldsymbol{\gamma}_{[N]} \rangle}, \\ \qquad \gamma_{[1]} > \gamma_{[2]} > \cdots > \gamma_{[N]} > 0 \\ 0, \qquad \text{otherwise} \end{cases} \tag{6}$$

where $\mathbf{1}_N$ denotes the vector of length N whose elements are all ones. It is important to note that the $\gamma_{[i]}$'s are *no* longer independent, even though the underlying γ_i's are independent.

C. Symbol Error Probability for GDC Over the Channel Ensemble

The SEP for GDC in multipath-fading environment is obtained by averaging the conditional SEP over the channel ensemble. This can be accomplished by averaging the $\Pr\{e|\gamma_{\text{GDC}}\}$ over the pdf of γ_{GDC} as

$$\begin{aligned} P_{e,\text{GDC}} &= \mathbb{E}_{\gamma_{\text{GDC}}}\{\Pr\{e|\gamma_{\text{GDC}}\}\} \\ &= \int_0^\infty \Pr\{e|\gamma\} f_{\gamma_{\text{GDC}}}(\gamma) d\gamma \end{aligned} \tag{7}$$

where $\Pr\{e|\gamma_{\text{GDC}}\}$ is the *conditional* SEP, conditioned on the random variable γ_{GDC}, and $f_{\gamma_{\text{GDC}}}(\cdot)$ is the pdf of the combiner output SNR [18]–[20]. Alternatively, averaging over the channel ensemble can be accomplished, using the technique of [21], [22], by substituting the expression for γ_{GDC} directly in terms of the physical branch variables given in (2), as

$$\begin{aligned} P_{e,\text{GDC}} &= \mathbb{E}_{\{\gamma_{[i]}\}}\left\{\Pr\left\{e|\gamma_{\text{GDC}} = \langle \boldsymbol{a}, \boldsymbol{\gamma}_{[N]} \rangle\right\}\right\} \\ &= \int_0^\infty \int_0^{\gamma_{[1]}} \cdots \int_0^{\gamma_{[N-1]}} \Pr\left\{e|\langle \boldsymbol{a}, \boldsymbol{\gamma}_{[N]} \rangle\right\} \\ &\quad \times f_{\boldsymbol{\gamma}_{[N]}}\left(\{\gamma_{[i]}\}_{i=1}^N\right) d\gamma_{[N]} \cdots d\gamma_{[2]} d\gamma_{[1]}. \end{aligned} \tag{8}$$

Since the statistics of the ordered-branches are *no* longer independent, the evaluation of (8) involves nested N-fold integrals, which are in general cumbersome and complicated to compute. This can be alleviated by transforming the instantaneous SNR of the ordered diversity branches, $\gamma_{[i]}$'s, into a new set of *virtual branch* instantaneous SNR's, V_n's, using the following relation:

$$\boldsymbol{\gamma}_{[N]} = \boldsymbol{T}_{\rm VB}\boldsymbol{V}_N \tag{9}$$

where $\boldsymbol{T}_{\rm VB}$: $\mathbb{R}^N \to \mathbb{R}^N$ is the upper triangular virtual branch transformation matrix given by

$$\boldsymbol{T}_{\rm VB} = \begin{bmatrix} \frac{\Gamma}{1} & \frac{\Gamma}{2} & \cdots & \frac{\Gamma}{N} \\ & \frac{\Gamma}{2} & \cdots & \frac{\Gamma}{N} \\ & & \ddots & \vdots \\ & & & \frac{\Gamma}{N} \end{bmatrix} \tag{10}$$

and $\boldsymbol{V}_N \triangleq [V_1, V_2, \ldots, V_N]^t$. Using the distribution theory for transformations of random vectors [17], the joint pdf of $V_1, V_2, \ldots, V_N$ can written as

$$f_{\boldsymbol{V}_N}\left(\{v_n\}_{n=1}^N\right) = J f_{\boldsymbol{\gamma}_{[N]}}\left(\left\{\gamma_{[i]}\right\}_{i=1}^N\right)\Big|_{\boldsymbol{\gamma}_{[N]}=\boldsymbol{T}_{\rm VB}\boldsymbol{v}_N} \tag{11}$$

where J is the Jacobian of the virtual branch transformation and $\boldsymbol{v}_N \triangleq [v_1, v_2, \ldots, v_N]^t$.

Denoting $I_N^{(i)}$ be the ith column of the $N \times N$ identity matrix I_N, we derive the recursion

$$\begin{aligned}\gamma_{[i]} &= \left\langle I_N^{(i)}, \boldsymbol{\gamma}_{[N]}\right\rangle = \left\langle \boldsymbol{T}_{\rm VB}^t I_N^{(i)}, \boldsymbol{V}_N\right\rangle \\ &= \underbrace{\left\langle \boldsymbol{T}_{\rm VB}^t I_N^{(i+1)}, \boldsymbol{V}_N\right\rangle}_{=\gamma_{i+1}} + \frac{\Gamma}{i}V_i\end{aligned} \tag{12}$$

where $\gamma_{N+1} \triangleq 0$ (or equivalently $I_N^{(N+1)} \triangleq 0$) and $(\Gamma/i)V_i$ can be interpreted as the "difference between the adjacent ordered instantaneous SNR's." This implies that $0 < V_n < \infty$. Since the virtual branch transformation is linear and $\boldsymbol{T}_{\rm VB}$ is an upper triangular matrix,

$$J = |\boldsymbol{T}_{\rm VB}| = \prod_{n=1}^N \frac{\Gamma}{n} = \frac{\Gamma^N}{N!} \tag{13}$$

where $|\cdot|$ denotes the determinant [23]. Note also that

$$\frac{1}{\Gamma}\left\langle \mathbf{1}_N, \boldsymbol{\gamma}_{[N]}\right\rangle = \frac{1}{\Gamma}\left\langle \boldsymbol{T}_{\rm VB}^t \mathbf{1}_N, \boldsymbol{V}_N\right\rangle = \left\langle \mathbf{1}_N, \boldsymbol{V}_N\right\rangle. \tag{14}$$

Substituting (13) and (14), in (11) together with (6), the joint pdf of $V_1, V_2, \ldots, V_N$ becomes

$$f_{\boldsymbol{V}_N}\left(\{v_n\}_{n=1}^N\right) = \begin{cases} e^{-\langle \mathbf{1}_N, \boldsymbol{v}_N\rangle}, & 0 \le v_n < \infty \\ 0, & \text{otherwise.}\end{cases} \tag{15}$$

Therefore

$$f_{\boldsymbol{V}_N}\left(\{v_n\}_{n=1}^N\right) = \prod_{n=1}^N f_{V_n}(v_n) \tag{16}$$

where $f_{V_n}(\cdot)$ is the pdf V_n given by

$$f_{V_n}(v) = \begin{cases} e^{-v}, & 0 \le v < \infty \\ 0, & \text{otherwise.}\end{cases} \tag{17}$$

This shows that the instantaneous SNR's of the virtual branches are i.i.d. with characteristic function (c.f.) given by

$$\psi_{V_n}(j\nu) \triangleq \mathbb{E}\{e^{+j\nu V_n}\} = \frac{1}{1 - j\nu}. \tag{18}$$

The instantaneous SNR of the combiner output can now be expressed in terms of the instantaneous SNR of the virtual branches as

$$\gamma_{\rm GDC} = \langle \boldsymbol{a}, \boldsymbol{T}_{\rm VB}\boldsymbol{V}_N\rangle. \tag{19}$$

Using the *independent* virtual branches, the N-fold nested integrals of (8) reduce to

$$\begin{aligned}P_{e,{\rm GDC}} &= \mathbb{E}_{\{V_n\}}\left\{\Pr\left\{e\,|\gamma_{\rm GDC} = \langle \boldsymbol{a}, \boldsymbol{T}_{\rm VB}\boldsymbol{V}_N\rangle\right\}\right\} \\ &= \int_0^\infty\int_0^\infty\cdots\int_0^\infty \Pr\left\{e\,\middle|\langle \boldsymbol{T}_{\rm VB}^t\boldsymbol{a}, \boldsymbol{v}_N\rangle\right\}\prod_{n=1}^N f_{V_n}(v_n)dv_n.\end{aligned} \tag{20}$$

For many important modulation techniques, $\Pr\{e|\langle \boldsymbol{a}, \boldsymbol{\gamma}_{[N]}\rangle\}$ factors into a product of N terms, where each term depends *only* on one of the $\gamma_{[i]}$'s. Similarly, $\Pr\{e|\langle \boldsymbol{T}_{\rm VB}^t\boldsymbol{a}, \boldsymbol{V}_N\rangle\}$ factors into a product of N terms, each dependent only on one V_n. We will illustrate this by the following two important examples.

1) SEP for MPSK With GDC: For coherent detection of M-ary phase-shift keying (PSK), an alternative representation $\Pr\{e|\gamma_{\rm GDC}\}$, involving a definite integral with *finite* limits, is given by [18], [19], [24]–[27]

$$\Pr\{e_{\rm MPSK}|\gamma_{\rm GDC}\} = \frac{1}{\pi}\int_0^\Theta \exp\left(-\frac{c_{\rm MPSK}}{\sin^2\theta}\gamma_{\rm GDC}\right)d\theta \tag{21}$$

where $c_{\rm MPSK} = \sin^2(\pi/M)$ and $\Theta = \pi(M-1)/M$. Substituting (21) into (7), the SEP for MPSK becomes

$$\begin{aligned}P_{e,{\rm GDC}}^{\rm MPSK} &= \frac{1}{\pi}\int_0^\Theta \mathbb{E}_{\gamma_{\rm GDC}}\left\{\exp\left(-\frac{c_{\rm MPSK}}{\sin^2\theta}\gamma_{\rm GDC}\right)\right\}d\theta \\ &= \frac{1}{\pi}\int_0^\Theta\int_0^\infty \exp\left(-\frac{c_{\rm MPSK}}{\sin^2\theta}\gamma\right)f_{\gamma_{\rm GDC}}(\gamma)d\gamma d\theta.\end{aligned} \tag{22}$$

Note that the inner integral of (22) is the c.f. of $\gamma_{\rm GDC}$ evaluated at $+j(c_{\rm MPSK}/\sin^2\theta)$, and the SEP analysis that uses the c.f. of the combiner output SNR can be found in [19].

Although, the evaluation of (22) involves a single integration for averaging over the channel ensemble, it requires the knowledge of the pdf (or equivalently the c.f.) of $\gamma_{\rm GDC}$. Alternatively, we substitute (21) into (8), and the SEP for MPSK becomes

$$P_{e,{\rm GDC}}^{\rm MPSK} = \frac{1}{\pi}\int_0^\Theta \mathbb{E}_{\{\gamma_{[i]}\}}\left\{\exp\left(-\frac{c_{\rm MPSK}}{\sin^2\theta}\langle \boldsymbol{a}, \boldsymbol{\gamma}_{[N]}\rangle\right)\right\}d\theta \tag{23}$$

$$\begin{aligned}&= \frac{1}{\pi}\int_0^\Theta\int_0^\infty\int_0^{\gamma_{[1]}}\cdots\int_0^{\gamma_{[N-1]}} \exp\left(-\frac{c_{\rm MPSK}}{\sin^2\theta}\langle \boldsymbol{a}, \boldsymbol{\gamma}_{[N]}\rangle\right) \\ &\quad \times f_{\boldsymbol{\gamma}_{[N]}}\left(\left\{\gamma_{[i]}\right\}_{i=1}^N\right)d\gamma_{[N]}\cdots d\gamma_{[2]}d\gamma_{[1]}d\theta.\end{aligned} \tag{24}$$

Note in (24) that, since the ordered physical branches are *no* longer independent, direct use of the methods given in [21] and [22] requires an N-fold nested integration for the expectation operation in (23). This is alleviated using the virtual branch technique by substituting (21) into (20) as

$$P_{e,\mathrm{GDC}}^{\mathrm{MPSK}} = \frac{1}{\pi}\int_0^{\Theta} \mathbb{E}_{\{V_n\}}\left\{\exp\left(-\frac{c_{\mathrm{MPSK}}}{\sin^2\theta}\langle \boldsymbol{a}, \boldsymbol{T}_{\mathrm{VB}}\boldsymbol{V}_N\rangle\right)\right\} d\theta$$
$$= \frac{1}{\pi}\int_0^{\Theta}\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} \exp\left(-\frac{c_{\mathrm{MPSK}}}{\sin^2\theta}\langle \boldsymbol{T}_{\mathrm{VB}}^t\boldsymbol{a}, \boldsymbol{v}_N\rangle\right) \times \prod_{n=1}^{N} f_{V_n}(v_n)dv_n d\theta. \quad (25)$$

Exploiting the fact that V_n's are independent, (25) becomes

$$P_{e,\mathrm{GDC}}^{\mathrm{MPSK}} = \frac{1}{\pi}\int_0^{\Theta}\prod_{n=1}^{N} \mathbb{E}_{V_n}\left\{\exp\left(-\frac{c_{\mathrm{MPSK}}b_n}{\sin^2\theta}V_n\right)\right\} d\theta$$
$$= \frac{1}{\pi}\int_0^{\Theta}\prod_{n=1}^{N} \psi_{V_n}\left(-\frac{c_{\mathrm{MPSK}}b_n}{\sin^2\theta}\right) d\theta \quad (26)$$

where b_n is the nth element of $\boldsymbol{b} = \boldsymbol{T}_{\mathrm{VB}}^t\boldsymbol{a}$. The powerfulness of the virtual branch technique is apparent by observing that the expectation operation in (23) no longer requires an N-fold nested integration.

Substituting (18) into (26) gives

$$P_{e,\mathrm{GDC}}^{\mathrm{MPSK}} = \frac{1}{\pi}\int_0^{\Theta}\prod_{n=1}^{N}\left[\frac{\sin^2\theta}{c_{\mathrm{MPSK}}b_n + \sin^2\theta}\right] d\theta. \quad (27)$$

Thus the derivation of the SEP for coherent detection of MPSK using N-branch GDC, involving the N-fold nested integrals in (24), essentially reduces to a single integral over θ with finite limits. The integrand is an N-fold product of a simple expression involving trigonometric functions. Note that the independence of the virtual branch variables plays a key role in simplifying the derivation.

2) SEP for MQAM With GDC: For coherent detection of squared M-ary quadrature amplitude modulation (QAM) with $M = 2^k$ for even k, $\Pr\{e|\gamma_{\mathrm{MRC}}\}$ is given by [22]

$$\Pr\{e_{\mathrm{MQAM}}|\gamma_{\mathrm{GDC}}\} = q\frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{c_{\mathrm{MQAM}}}{\sin^2\theta}\gamma_{\mathrm{GDC}}\right)d\theta - \frac{q^2}{4}\frac{1}{\pi}\int_0^{\pi/4}\exp\left(-\frac{c_{\mathrm{MQAM}}}{\sin^2\theta}\gamma_{\mathrm{GDC}}\right)d\theta \quad (28)$$

where $q = 4(1-(1/\sqrt{M}))$, and $c_{\mathrm{MQAM}} = 3/(2(M-1))$. Using the virtual branch technique, similar to the steps used for MPSK, the SEP for MQAM becomes

$$P_{e,\mathrm{GDC}}^{\mathrm{MQAM}} = q\frac{1}{\pi}\int_0^{\pi/2}\prod_{n=1}^{N}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}b_n + \sin^2\theta}\right]d\theta - \frac{q^2}{4}\frac{1}{\pi}\int_0^{\pi/4}\prod_{n=1}^{N}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}b_n + \sin^2\theta}\right]d\theta. \quad (29)$$

Again, the derivation of the SEP for coherent detection of MQAM using GDC in Rayleigh fading reduces to two terms, each consisting of a single integral over θ involving trigonometric functions with finite limits.

Note in passing that the two signal constellations for MQAM and MPSK coincide when $M = 4$, and hence their respective performance obtained from our virtual branch analysis are expected to be the same. Using the fact that the integrand in (29) is even symmetric about $\pi/2$ in θ, it is easy to verify that (29) with 4-QAM is identical to (27) 4-PSK.

III. Application of General Theory

The results given in (27) and (29) of Section II-C are for the SEP for coherent detection of MPSK and MQAM, respectively, using N-branch GDC in Rayleigh-fading channels. The b_n's in (27) and (29) depend on the selection vector $\boldsymbol{a}$. The results obtained in (27) and (29) are *general* in the sense that they apply to a variety of diversity combining systems that fit the form of (2), including H-S/MRC, SD, and MRC. In the following, the general theory derived in Section II-C is used to evaluate the performance of H-S/MRC, SD, and MRC.

A. SEP's With H-S/MRC

The instantaneous output SNR of H-S/MRC is

$$\gamma_{\text{H-S/MRC}} = \sum_{i=1}^{L}\gamma_{[i]} \quad (30)$$

where $1 \le L \le N$. Note that $\gamma_{\text{H-S/MRC}} = \gamma_{\mathrm{GDC}}$ with

$$\boldsymbol{a} = [\underbrace{1, 1, \ldots 1}_{L \text{ terms}}, 0, \ldots, 0]^t. \quad (31)$$

In this case,

$$b_n = \begin{cases}\Gamma, & n \le L\\ \Gamma\dfrac{L}{n}, & \text{otherwise.}\end{cases} \quad (32)$$

Substituting (32) into (27) of Section II-C, the SEP for MPSK with H-S/MRC can be easily obtained as

$$P_{e,\text{H-S/MRC}}^{\mathrm{MPSK}} = \frac{1}{\pi}\int_0^{\Theta}\left[\frac{\sin^2\theta}{c_{\mathrm{MPSK}}\Gamma + \sin^2\theta}\right]^L \times \prod_{n=L+1}^{N}\left[\frac{\sin^2\theta}{c_{\mathrm{MPSK}}\Gamma\frac{L}{n} + \sin^2\theta}\right]d\theta. \quad (33)$$

Similarly, the SEP for MQAM with H-S/MRC can be obtained by substituting (32) into (29) of Section II-C as

$$P_{e,\text{H-S/MRC}}^{\mathrm{MQAM}} = q\frac{1}{\pi}\int_0^{\pi/2}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}\Gamma + \sin^2\theta}\right]^L \times \prod_{n=L+1}^{N}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}\Gamma\frac{L}{n} + \sin^2\theta}\right]d\theta - \frac{q^2}{4}\frac{1}{\pi}\int_0^{\pi/4}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}\Gamma + \sin^2\theta}\right]^L \times \prod_{n=L+1}^{N}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}\Gamma\frac{L}{n} + \sin^2\theta}\right]d\theta. \quad (34)$$

B. SEP's With SD

SD is the simplest form of diversity system whereby the received signal from *one* of N diversity branches is selected [4]. The output SNR of SD is

$$\gamma_{\mathrm{SD}} \triangleq \max_i\{\gamma_i\} = \gamma_{[1]}. \tag{35}$$

Note that $\gamma_{\mathrm{SD}} = \gamma_{\mathrm{GDC}}$ with $\boldsymbol{a} = I_N^{(1)}$. In this case, $b_n = \Gamma/n$ for $n = 1, \ldots, N$, and substituting this into (27) and (29) of Section II-C, the SEP for coherent detection of MPSK and MQAM using SD becomes

$$P_{e,\mathrm{SD}}^{\mathrm{MPSK}} = \frac{1}{\pi}\int_0^{\Theta}\prod_{n=1}^{N}\left[\frac{\sin^2\theta}{c_{\mathrm{MPSK}}\Gamma\frac{1}{n}+\sin^2\theta}\right]d\theta \tag{36}$$

and

$$\begin{aligned} P_{e,\mathrm{SD}}^{\mathrm{MQAM}} =& q\frac{1}{\pi}\int_0^{\pi/2}\prod_{n=1}^{N}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}\Gamma\frac{1}{n}+\sin^2\theta}\right]d\theta \\ &- \frac{q^2}{4}\frac{1}{\pi}\int_0^{\pi/4}\prod_{n=1}^{N}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}\Gamma\frac{1}{n}+\sin^2\theta}\right]d\theta \end{aligned} \tag{37}$$

respectively. Since SD is a special case of H-S/MRC with $L = 1$, (36) and (37) can also be obtained from the H-S/MRC results of Section III-A by setting $L = 1$ in (33) and (34).

C. SEP's With MRC

In MRC, the received signals from *all* diversity branches are weighted and combined to maximize the SNR at the combiner output [4]. The output SNR of MRC is

$$\gamma_{\mathrm{MRC}} \triangleq \sum_{i=1}^{N}\gamma_i = \sum_{i=1}^{N}\gamma_{[i]}. \tag{38}$$

Note that $\gamma_{\mathrm{MRC}} = \gamma_{\mathrm{GDC}}$ with $\boldsymbol{a} = \mathbf{1}_N$. In this case, $b_n = \Gamma$ for $n = 1, \ldots, N$, and substituting this into (27) of Section II-C, the SEP for MPSK becomes

$$\begin{aligned} P_{e,\mathrm{MRC}}^{\mathrm{MPSK}} =& \frac{1}{\pi}\int_0^{\Theta}\left[\frac{\sin^2\theta}{c_{\mathrm{MPSK}}\Gamma+\sin^2\theta}\right]^N d\theta \\ \triangleq& P_{e,\mathrm{MRC}}^{\mathrm{MPSK}}(N,\Gamma). \end{aligned} \tag{39}$$

Specifically (39) is the SEP for coherent detection of MPSK using MRC with N *independent* branches having *equal* average SNR's of Γ in Rayleigh fading, and therefore a closed-form expression for (39) can be found in [28]. Comparing to the SEP expression given by [29, eq. (24)], we note that (39) is equivalent to the SEP for *single*-branch reception of MPSK in Nakagami-m fading with fading parameter N (i.e., $m = N$) having an average SNR of $N\Gamma$ [29]–[31].[4]

[4]This is due to the fact that pdf of $(2/\Gamma)\gamma_{\mathrm{MRC}}$ is chi-squared distributed with $2N$ degrees of freedom, and, therefore, γ_{MRC} is equal (in distribution or law) to the square of the Nakagami random variable with $m = N$ and mean $N\Gamma$. The fading parameter of Nakagami fading, usually denoted by the symbol m, is also known as fade parameter, fading severity factor, fading figure, or (inverse) fading-depth parameter.

Similarly, the SEP for MQAM becomes

$$\begin{aligned} P_{e,\mathrm{MRC}}^{\mathrm{MQAM}} =& q\frac{1}{\pi}\int_0^{\pi/2}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}\Gamma+\sin^2\theta}\right]^N d\theta \\ &- \frac{q^2}{4}\frac{1}{\pi}\int_0^{\pi/4}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}\Gamma+\sin^2\theta}\right]^N d\theta \\ \triangleq& P_{e,\mathrm{MRC}}^{\mathrm{MQAM}}(N,\Gamma). \end{aligned} \tag{40}$$

The result of (40) is the SEP for coherent detection of MQAM using MRC with N *independent* branches having *equal* average SNR's of Γ in Rayleigh fading, and therefore a closed-form expression for (40) can be found in [32]. Note again that (40) is equivalent to the SEP for *single*-branch reception of MQAM in Nakagami fading with fading parameter N having an average SNR of $N\Gamma$ [29]–[31].

Since MRC is a special case of H-S/MRC with $L = N$, (39) and (40) can also be obtained from the H-S/MRC results of Section III-A by setting $L = N$ in (33) and (34).

IV. Canonical Form for SEPs

A. Canonical Form for SEP's With GDC

The quest for obtaining insights from (27) and (29) is at its peak, which leads to an expansion for the integrand in (27). Let $\{\tilde{b}_n\}$ be the set of $\tilde{N}$ *distinct* values of $\{b_n\}$ where each $\tilde{b}_n$ has algebraic multiplicity μ_n such that $\sum_{n=1}^{\tilde{N}}\mu_n = N$. Then (27) and (29) can be rewritten as

$$P_{e,\mathrm{GDC}}^{\mathrm{MPSK}} = \frac{1}{\pi}\int_0^{\Theta}\prod_{n=1}^{\tilde{N}}\left[\frac{\sin^2\theta}{c_{\mathrm{MPSK}}\tilde{b}_n+\sin^2\theta}\right]^{\mu_n}d\theta \tag{41}$$

and

$$\begin{aligned} P_{e,\mathrm{GDC}}^{\mathrm{MQAM}} =& q\frac{1}{\pi}\int_0^{\pi/2}\prod_{n=1}^{\tilde{N}}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}\tilde{b}_n+\sin^2\theta}\right]^{\mu_n}d\theta \\ &- \frac{q^2}{4}\frac{1}{\pi}\int_0^{\pi/4}\prod_{n=1}^{\tilde{N}}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}\tilde{b}_n+\sin^2\theta}\right]^{\mu_n}d\theta. \end{aligned} \tag{42}$$

Letting $x = 1/\sin^2\theta$ and $c_n = 1/c_{\mathrm{MPSK}}\tilde{b}_n$, the integrand in (41) fits into the expression of (53) of the Appendix.[5] Using the canonical expansion formula given in (54) of the Appendix, (41) can be rewritten as

$$P_{e,\mathrm{GDC}}^{\mathrm{MPSK}} = \sum_{n=1}^{\tilde{N}}\sum_{k=1}^{\mu_n}A_{n,k}\frac{1}{\pi}\int_0^{\Theta}\left[\frac{\sin^2\theta}{c_{\mathrm{MPSK}}\tilde{b}_n+\sin^2\theta}\right]^k d\theta \tag{43}$$

where the weighting coefficients $A_{n,k}$ are given by (55). Comparing (43) with (39)

$$P_{e,\mathrm{GDC}}^{\mathrm{MPSK}} = \sum_{n=1}^{\tilde{N}}\sum_{k=1}^{\mu_n}A_{n,k}P_{e,\mathrm{MRC}}^{\mathrm{MPSK}}(k,\tilde{b}_n). \tag{44}$$

Interesting insights can now be obtained from (44). The SEP for MPSK using N-branch GDC in Rayleigh fading is simply

[5]Although, there are numerous ways to expand the product of polynomials, we have chosen a specific one that leads to the canonical structure given by (44).

the weighted sum of the elementary SEP's. The weighting coefficients $A_{n,k}$ are given by (55), and the elementary SEP's for the (n,k)-entries are simply the SEP's for the coherent detection of MPSK using MRC with k *independent* branches having *equal* SNR's of $\tilde{b}_n$ in Rayleigh-fading, or equivalently the SEP for *single*-branch reception of MPSK in Nakagami fading with fading parameter equal to k having an average SNR of $k\tilde{b}_n$.

Similarly, (42) can be rewritten as

$$P_{e,\mathrm{GDC}}^{\mathrm{MQAM}} = \sum_{n=1}^{\tilde{N}}\sum_{k=1}^{\mu_n} A_{n,k} q \frac{1}{\pi}\int_0^{\pi/2}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}\tilde{b}_n+\sin^2\theta}\right]^k d\theta - \sum_{n=1}^{\tilde{N}}\sum_{k=1}^{\mu_n} A_{n,k}\frac{q^2}{4}\frac{1}{\pi}\int_0^{\pi/4}\left[\frac{\sin^2\theta}{c_{\mathrm{MQAM}}\tilde{b}_n+\sin^2\theta}\right]^k d\theta. \tag{45}$$

Therefore,

$$P_{e,\mathrm{GDC}}^{\mathrm{MQAM}} = \sum_{n=1}^{\tilde{N}}\sum_{k=1}^{\mu_n} A_{n,k} P_{e,\mathrm{MRC}}^{\mathrm{MQAM}}(k,\tilde{b}_n). \tag{46}$$

Note that a similar structure, namely, "linear combination of the simple elementary SEP's," is evident from (46) for MQAM.

B. Canonical Forms for SEP's With H-S/MRC

For H-S/MRC, it can be seen from (32) that the number of distinct values of $\{b_n\}$ is $\tilde{N} = N - L + 1$. The distinct values of $\tilde{b}_n$s are given by

$$\tilde{b}_n = \begin{cases}\Gamma, & n=1\\ \Gamma\dfrac{L}{L+n-1}, & n=2,\ldots,\tilde{N}\end{cases} \tag{47}$$

and their multiplicities μ_ns are given by

$$\mu_n = \begin{cases} L, & n=1\\ 1, & n=2,\ldots,\tilde{N}.\end{cases} \tag{48}$$

Substituting (47) and (48) into (44) and (46) of Section IV-A, we arrive at the SEP's for coherent detection of MPSK and MQAM with H-S/MRC, which are given, respectively, by

$$P_{e,\mathrm{H\text{-}S/MRC}}^{\mathrm{MPSK}} = \sum_{k=1}^{L} A_{1,k} P_{e,\mathrm{MRC}}^{\mathrm{MPSK}}(k,\Gamma) + \sum_{n=2}^{N-L+1} A_{n,1} P_{e,\mathrm{MRC}}^{\mathrm{MPSK}}\left(1,\Gamma\frac{L}{L+n-1}\right) \tag{49}$$

$$P_{e,\mathrm{H\text{-}S/MRC}}^{\mathrm{MQAM}} = \sum_{k=1}^{L} A_{1,k} P_{e,\mathrm{MRC}}^{\mathrm{MQAM}}(k,\Gamma) + \sum_{n=2}^{N-L+1} A_{n,1} P_{e,\mathrm{MRC}}^{\mathrm{MQAM}}\left(1,\Gamma\frac{L}{L+n-1}\right). \tag{50}$$

The canonical structure for SEP with H-S/MRC is evident from (49) and (50) as a linear combination of the simple "elementary SEP's," as for the case of GDC in (44) and (46) of Section IV-A.

C. Canonical Forms for SEP's With SD

For SD, the number of distinct values of $\{b_n\}$, $\tilde{N}$, is equal to N. The distinct values are $\tilde{b}_n = \Gamma/n$ and the corresponding multiplicities are given by $\mu_n = 1$ for $n = 1,\ldots,N$. Substituting these values into (44) and (46) of Section IV-A, the SEP's for coherent detection of MPSK and MQAM with SD becomes

$$P_{e,\mathrm{SD}}^{\mathrm{MPSK}} = \sum_{n=1}^{N} A_{n,1} P_{e,\mathrm{MRC}}^{\mathrm{MPSK}}\left(1,\frac{\Gamma}{n}\right) \tag{51}$$

and

$$P_{e,\mathrm{SD}}^{\mathrm{MQAM}} = \sum_{n=1}^{N} A_{n,1} P_{e,\mathrm{MRC}}^{\mathrm{MQAM}}\left(1,\frac{\Gamma}{n}\right) \tag{52}$$

respectively. Note again that the SEP for SD is simply a weighted sum of the elementary SEP's, as for the case of GDC in (44) and (46) of Section IV-A. Since SD is a special case

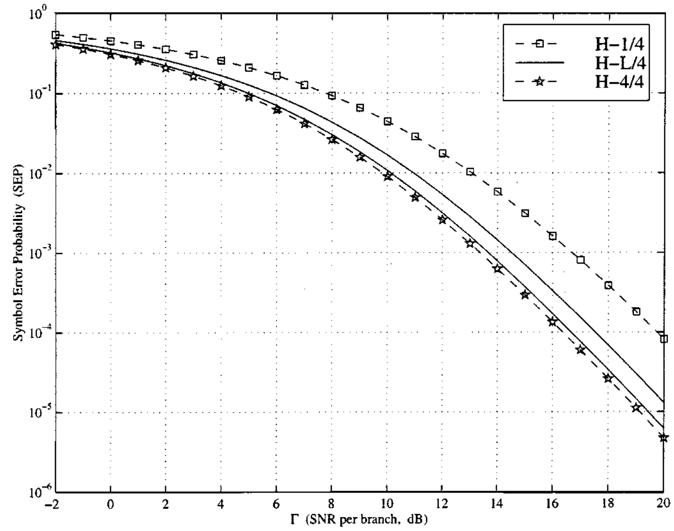

Fig. 1. The symbol error probability for coherent detection of 8-PSK with H-S/MRC as a function of average SNR per branch in decibels for various L with $N = 4$. The curves are parameterized by different H-$L/4$ starting from the highest curve representing H-1/4, and decrease monotonically to the lowest curve representing H-4/4.

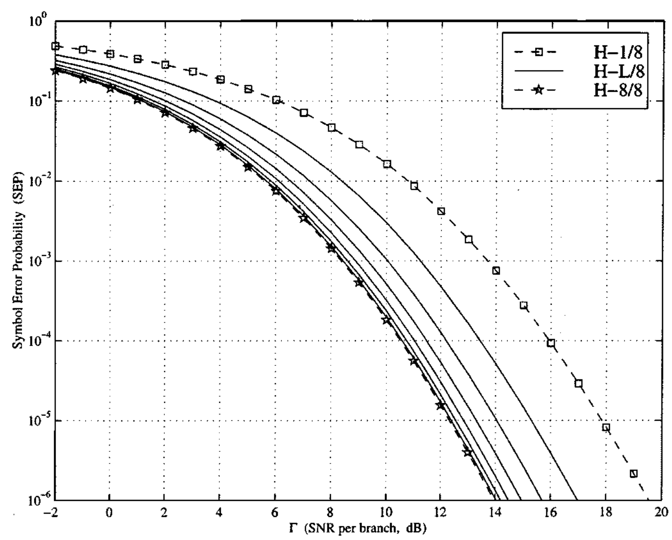

Fig. 2. The symbol error probability for coherent detection of 8-PSK with H-S/MRC as a function of average SNR per branch in decibels for various L with $N = 8$. The curves are parameterized by different H-$L/8$ starting from the highest curve representing H-1/8, and decrease monotonically to the lowest curve representing H-8/8.

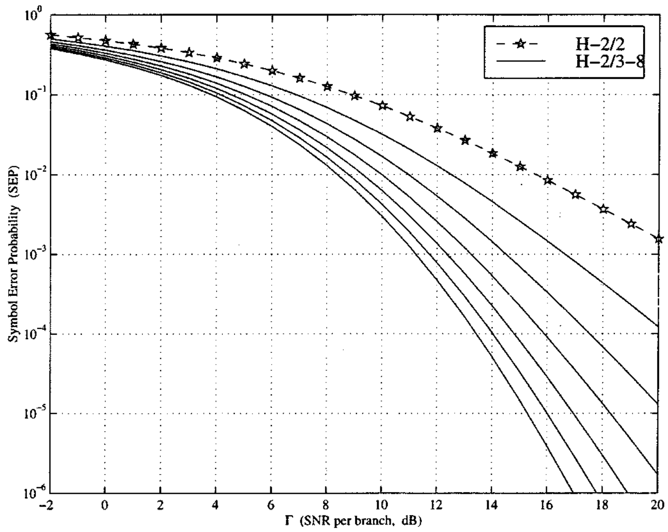

Fig. 3. The symbol error probability for coherent detection of 8-PSK with H-S/MRC as a function of average SNR per branch in decibels for various N with $L = 2$. The curves are parameterized by different H-2/N starting from the highest curve representing H-2/2, and decrease monotonically to the lowest curve representing H-2/8.

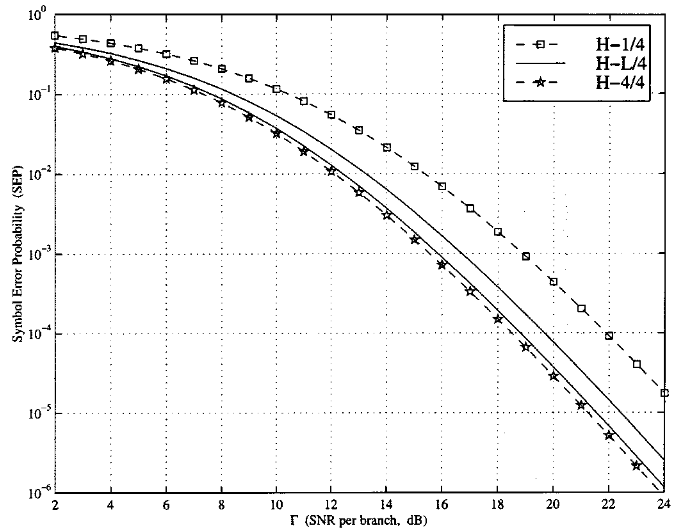

Fig. 5. The symbol error probability for coherent detection of 16-QAM with H-S/MRC as a function of average SNR per branch in decibels for various L with $N = 4$. The curves are parameterized by different H-$L/4$ starting from the highest curve representing H-1/4, and decrease monotonically to the lowest curve representing H-4/4.

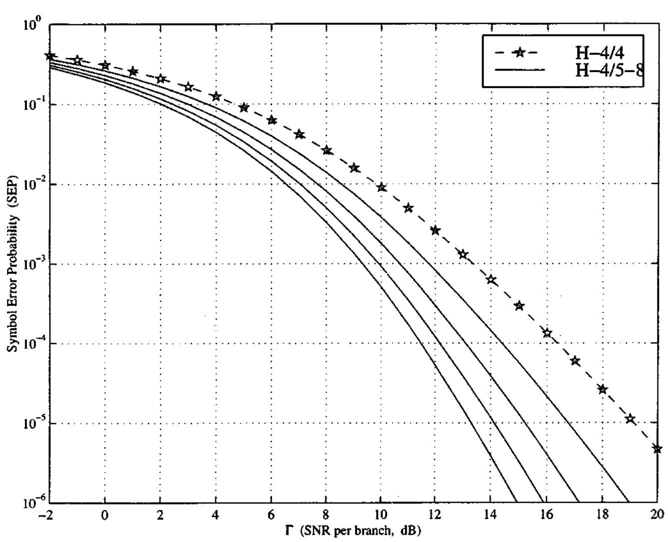

Fig. 4. The symbol error probability for coherent detection of 8-PSK with H-S/MRC as a function of average SNR per branch in decibels for various N with $L = 4$. The curves are parameterized by different H-4/N starting from the highest curve representing H-4/4, and decrease monotonically to the lowest curve representing H-4/8.

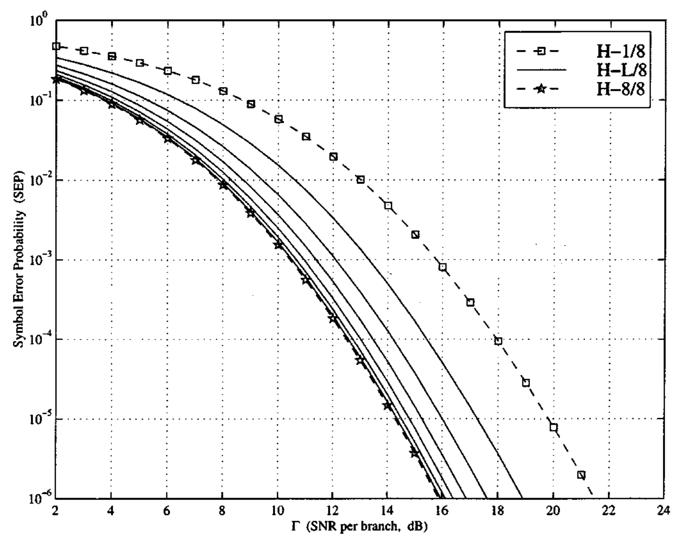

Fig. 6. The symbol error probability for coherent detection of 16-QAM with H-S/MRC as a function of average SNR per branch in decibels for various L with $N = 8$. The curves are parameterized by different H-$L/8$ starting from the highest curve representing H-1/8, and decrease monotonically to the lowest curve representing H-8/8.

of H-S/MRC with $L = 1$, (51) and (52) can also be obtained alternatively from the H-S/MRC results given by (49) and (50) of Section IV-B by setting $L = 1$.

V. Numerical Examples

In this section, the results derived in the previous section for H-S/MRC are illustrated. The notation H-L/N is used to denote H-S/MRC that selects and combines L out of N branches. Note that H-$1/1$ is a single branch receiver, and H-$1/N$ and H-N/N are N-branch SD and MRC, respectively.

Figs. 1 and 2 show the SEP for coherent detection of MPSK with $M = 8$ (8-PSK) versus average SNR per branch for various L with $N = 4$ and $N = 8$, respectively. Note that SD and MRC upper and lower bound, respectively, the SEP for H-S/MRC. It is seen that most of the gain of H-S/MRC is achieved for small L, e.g., H-S/MRC is within 1.1 dB of MRC when $L = N/2$.

Figs. 3 and 4 show the SEP for coherent detection of 8-PSK versus average SNR per branch for various N with $L = 2$ and $L = 4$, respectively. Note that, although the incremental gain with each additional antenna becomes smaller as N increases, the gain with each additional antenna is still significant even with $N = 8$. The results also show that, at a 10^{-4} SEP, H-2/8 requires 12.5 dB lower SNR than 2-branch MRC, and H-4/8 requires 4.5 dB lower SNR than 4-branch MRC.

Similar results for coherent detection of MQAM with $M = 16$ (16-QAM) are plotted in Figs. 5–8. These results show the same characteristics as 8-PSK illustrated in Figs. 1–4 except

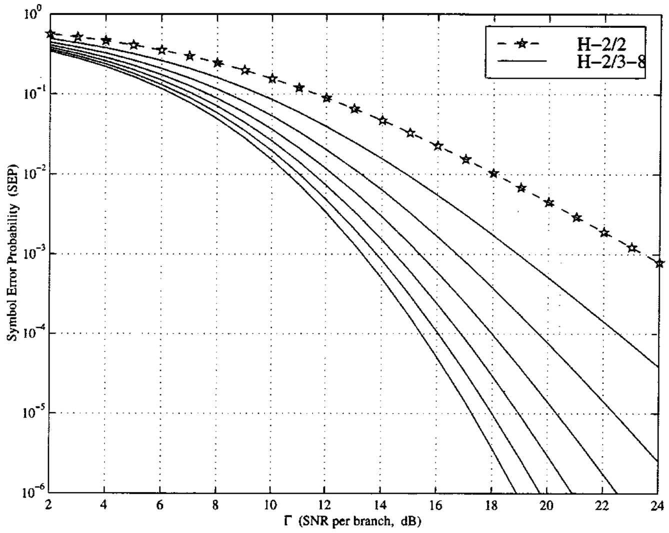

Fig. 7. The symbol error probability for coherent detection of 16-QAM with H-S/MRC as a function of average SNR per branch in decibels for various N with $L = 2$. The curves are parameterized by different H-2/N starting from the highest curve representing H-2/2, and decrease monotonically to the lowest curve representing H-2/8.

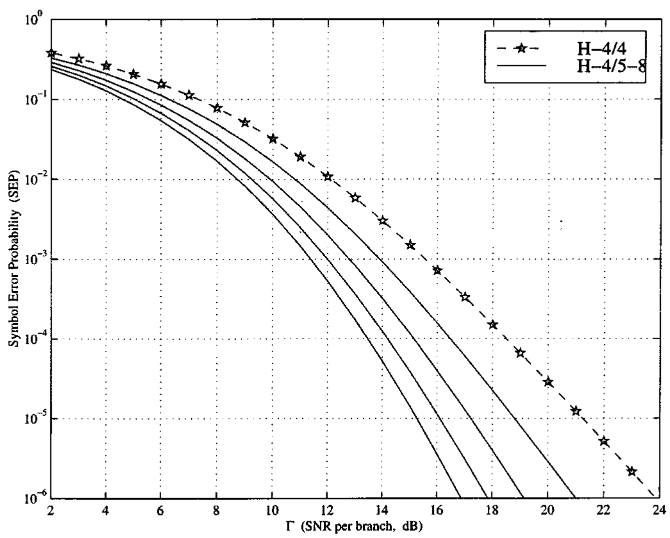

Fig. 8. The symbol error probability for coherent detection of 16-QAM with H-S/MRC as a function of average SNR per branch in decibels for various N with $L = 4$. The curves are parameterized by different H-4/N starting from the highest curve representing H-4/4, and decrease monotonically to the lowest curve representing H-4/8.

that the SNR per branch is about 2 dB higher with 16-QAM to achieve the same SEP as 8-PSK.

VI. Conclusions

We derived exact expressions for the SEP for coherent detection of MPSK and MQAM with H-S/MRC in multipath-fading wireless environments. With H-S/MRC, L out of N diversity branches are selected and combined using MRC. This technique provides improved performance over L branch MRC when additional diversity is available, without requiring additional electronics and/or power. We considered independent Rayleigh fading on each diversity branch with equal signal-to-noise ratios, averaged over the fading. We analyzed this system using a novel "virtual branch" technique which resulted in a simple derivation of the SEP for *arbitrary* L and N. The key idea was to transform the dependent ordered-branch variables into a new set of i.i.d. *virtual branches*, and express the combiner output SNR as a linear combination of the i.i.d. virtual branch SNR variables. We further obtained a *canonical structure* for the SEP of H-S/MRC as a weighted sum of the elementary SEP's. The elementary SEP's are the SEP's using MRC with i.i.d. branches in Rayleigh fading, or equivalently the SEP's of the nondiversity (single-branch) system in Nakagami fading, whose closed-form expressions are well-known. Numerical results for 8-PSK and 16-QAM showed that H-S/MRC, even with $L \ll N$, can achieve performance close to that of N-branch MRC.

Appendix
Canonical Expansion

The canonical expansion of

$$f(x) = \prod_{n=1}^{\tilde{N}} \left(\frac{c_n}{c_n + x} \right)^{\mu_n} \tag{53}$$

where the $\{-c_n\}$ are the $\tilde{N}$ distinct poles of $f(x)$, each having algebraic multiplicity μ_n, in terms of the elementary functions of the form $c_n/(c_n + x)$, is

$$f(x) = \sum_{n=1}^{\tilde{N}} \sum_{k=1}^{\mu_n} A_{n,k} \left(\frac{c_n}{c_n + x} \right)^k. \tag{54}$$

The coefficients of the canonical expansion are given by

$$A_{n,k} = \frac{1}{c_n^k (\mu_n - k)!} f_n^{(\mu_n - k)}(0), \quad n = 1, \ldots, \tilde{N}, \quad k = 1, \ldots, \mu_n \tag{55}$$

where $f_n^{(k)}(0)$ denotes the kth derivative of $f_n(x) = x^{\mu_n} f(x - c_n)$ evaluated at $x = 0$.

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Moe Z. Win (S’85–M’87–SM’97) received the B.S. degree (*magna cum laude*) from Texas A&M University, College Station, and the M.S. degree from the University of Southern California (USC), Los Angeles, in 1987 and 1989, respectively, both in electrical engineering. As a Presidential Fellow at USC, he received both an M.S. degree in applied mathematics and the Ph.D. degree in electrical engineering in 1998.

In 1987, he joined the Jet Propulsion Laboratory (JPL), California Institute of Technology, Pasadena. From 1994 to 1997, he was a Research Assistant with the Communication Sciences Institute at USC, where he played a key role in the successful creation of the Ultra-Wideband Radio Laboratory. Since 1998, he has been with the Wireless Systems Research Department, AT&T Laboratories-Research, Middletown, NJ, where he is a Principal Technical Staff Member. His main research interests are the application of communication, detection, and estimation theories to a variety of communications problems including time-varying channels, diversity, equalization, synchronization, signal design, ultrawide-bandwidth communication, and optical communications.

Dr. Win is a member of Eta Kappa Nu, Tau Beta Pi, Pi Mu Epsilon, Phi Theta Kappa, and Phi Kappa Phi. He was a University Undergraduate Fellow at Texas A&M University, where he received, among others awards, the Academic Excellence Award. At USC, he received several awards including the Outstanding Research Paper Award and the Phi Kappa Phi Student Recognition Award. He was the recipient of the IEEE Communications Society Best Student Paper Award at the Fourth Annual IEEE NetWorld+Interop ’97 Conference. He has been involved actively in chairing and organizing sessions and has served as a member of the Technical Program Committee in a number of IEEE conferences. He currently serves as the Technical Program Chair for the IEEE Communication Theory Symposium of ICC-2003 and IEEE Conference on Ultra Wideband Systems and Technologies (2002), and Technical Program Vice-Chair for IEEE International Conference on Communications (2002). He served as the Tutorial Chair for IEEE Semiannual International Vehicular Technology Conference (Fall-2001) and the Technical Program Chair for the IEEE Communication Theory Symposium of Globecom-2000. He is the current Editor for *Equalization and Diversity* for the IEEE TRANSACTIONS ON COMMUNICATIONS and a Guest-Editor for the 2002 IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, Special Issue on ultra wide band radio in multi-access wireless communications.

Jack H. Winters (S’77–M’81–SM’88–F’96) received the B.S.E.E. degree from the University of Cincinnati, Cincinnati, OH, in 1977, and the M.S. and Ph.D. degrees in electrical engineering from The Ohio State University, Columbus, in 1978 and 1981, respectively.

Since 1981, he has been with AT&T Bell Laboratories, and now AT&T Labs-Research, where he is currently Division Manager of the Wireless Systems Research Department. He has studied signal processing techniques for increasing the capacity and reducing signal distortion in fiber optic, mobile radio, and indoor radio systems and is currently studying smart antennas, adaptive arrays, and equalization for wireless local area networks and mobile radio systems.